GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2023
Exam #2

NAME: ________________________________ GTemail: ______________________
FIRST LAST ex: GPburDELL@gatech.edu

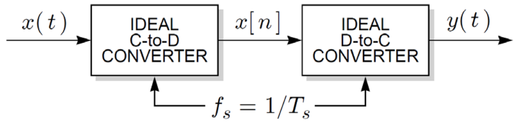
- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to received partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

Print Name (First Last) ________________________________

PROBLEM 1:

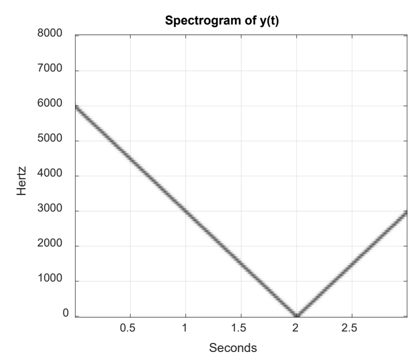
Parts a and b (10 points each) can be solved independently of each other.



(a) If $x(t) = \cos(60\pi t - \pi/4) + 6\cos(100\pi t - \pi/3)\cos(260\pi t)$ and $f_s = 80$ Hz, find $y(t)$.

$y(t) =$

(b) Let $x(t) = \cos(-\pi A t^2 + 2\pi B t + 0.2\pi)$ for $t \geq 0$. It is zero for negative time. Let $f_s = 16000$ Hz. The spectrogram for the output $y(t)$ is shown for $0 < t < 3$ seconds only. Find the positive constants $A > 0$ and $B > 0$. Additionally find the first time instance, t_a ($t_a > 0$), beyond which the instantaneous frequency of $x(t)$ and $y(t)$ will no longer be equal.



$A =$	$B =$	$t_a =$

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PROBLEM 2:

Parts a and b (10 points each) can be solved independently of each other.

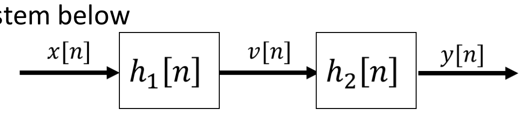
(a) Suppose there is a linear and time-invariant system with unknown impulse response $h[n]$, an input $x[n]$, and an output $y[n]$. We know the following:

When the input to the system is: $x_1[n] = \delta[n] + 3\delta[n-1] + \delta[n-2] + 3\delta[n-3]$
The output is: $y_1[n] = 3\delta[n] + 3\delta[n-1] - 15\delta[n-2] + 3\delta[n-3] - 18\delta[n-4]$

If we define the input to the system as: $x_2[n] = 2x_1[n] - 3x_1[n-1]$, find the corresponding output $y_2[n]$.

$y_2[n] =$

(b) Consider the cascaded LTI system below



where $h_1[n] = \frac{1}{3}\sum_{k=2}^{4} \delta[n-k]$ and $y[n] = 3v[n] - 6v[n-1]$. Find the overall difference equation relating the input $x[n]$ and output $y[n]$ that is valid for all n.

$y[n] =$

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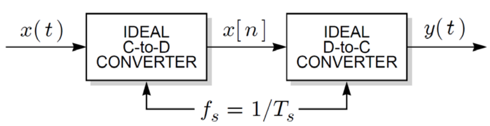
PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

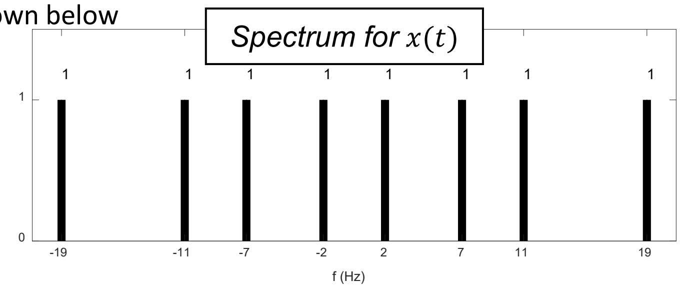
(a) An LTI system with output $y[n]$ has impulse response: $h[n] = 6\delta[n-2] + 3\delta\,[n-5] - 9\delta[n-8]$. If the input to the system is a scaled and delayed unit step function $x[n] = -2u[n-3]$, find $y[3]$ and $y[10]$.

$y[3] =$	$y[10] =$

(b) The spectrum for an input signal $x(t)$ is shown below



Spectrum for $x(t)$



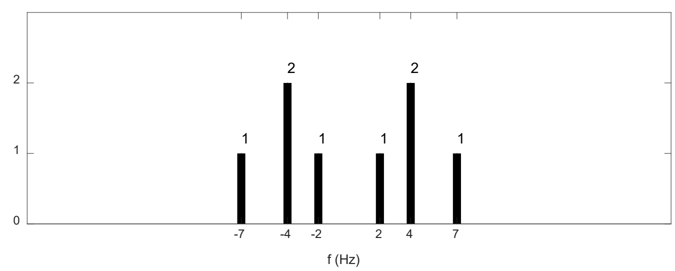
(i) Indicate the condition on f_s to avoid aliasing $x(t)$:

$f_s >$

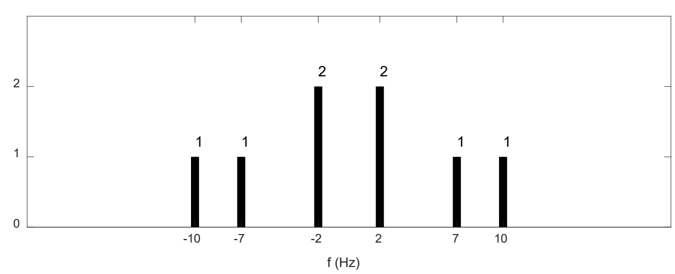
Find the sampling rate that would result in the following output spectrums for $y(t)$

Sampling Rate (f_s)	Spectrum for $y(t)$

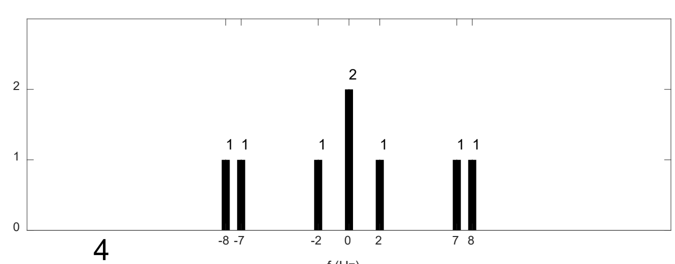
(ii) $f_s =$



(iii) $f_s =$



(iv) $f_s =$



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PROBLEM 1:

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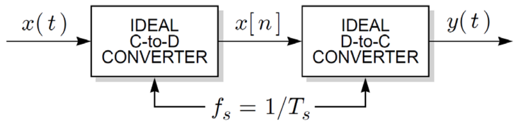


(a) If $x(t) = \cos(60\pi t - \pi/4) + 6\cos(100\pi t - \pi/3)\cos(260\pi t)$ and $f_s = 80$ Hz, find $y(t)$.

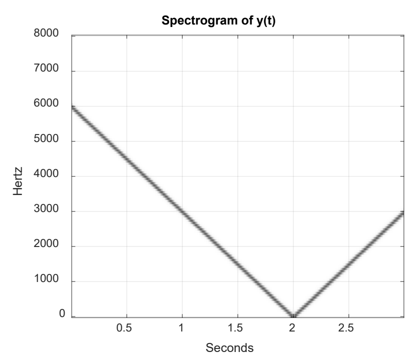
Solutions: $x(t) = \cos(60\pi t - \pi/4) + 3\cos(160\pi t + \pi/3) + 3\cos(360\pi t - \pi/3)$

For : $f_s = 80$

$$y(t) = \cos(60\pi t - \pi/4) + 1.5 + 3\cos(40\pi t - \pi/3)$$

$y(t) =$ $\cos(60\pi t - \pi/4) + 1.5 + 3\cos(40\pi t - \pi/3)$

(b) Let $x(t) = \cos(-\pi A t^2 + 2\pi B t + 0.2\pi)$ for $t \geq 0$. It is zero for negative time. Let $f_s = 16000$ Hz. The spectrogram for the output $y(t)$ is shown for $0 < t < 3$ seconds only. Find the positive constants $A > 0$ and $B > 0$. Additionally find the first time instance, t_a ($t_a > 0$), beyond which the instantaneous frequency of $x(t)$ and $y(t)$ will no longer be equal.



Solutions:

$$f_i(t) = -At + B \rightarrow f_i(0) = 6000, f_i(2) = 0$$

$$B = 6000; A = 3000$$

Once the chirp exceeds $\frac{f_s}{2}$ for the first time, the instantaneous frequency of $x(t)$ and $y(t)$ will not be equal.

$$f_i(t_a) = -At + B = -8000 \rightarrow -3000t_a + 6000 = -8000$$

$$t_a = \frac{(-8000 - 6000)}{-3000} = 4.67$$

$A =$ 3000	$B =$ 6000	$t_a =$ 4.67

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PROBLEM 2:

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When the input to the system is: $x_1[n] = \delta[n] + 3\delta[n-1] + \delta[n-2] + 3\delta[n-3]$
The output is: $y_1[n] = 3\delta[n] + 3\delta[n-1] - 15\delta[n-2] + 3\delta[n-3] - 18\delta[n-4]$

If we define the input to the system as: $x_2[n] = 2x_1[n] - 3x_1[n-1]$, find the corresponding output $y_2[n]$

Solutions: Use linearity

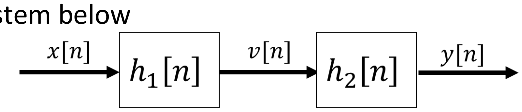
$$x_2[n] = 2x_1[n] - 3x_1[n-1] \rightarrow y_2[n] = 2y_1[n] - 3y_1[n-1]$$
$$y_2[n] = 2(3\delta[n] + 3\delta[n-1] - 15\delta[n-2] + 3\delta[n-3] - 18\delta[n-4])$$
$$-3(3\delta[n-1] + 3\delta[n-2] - 15\delta[n-3] + 3\delta[n-4] - 18\delta[n-5])$$

6	6	-30	6	-36	
	-9	-9	45	-9	54
6	-3	-39	51	-45	54

$$y_2[n] = 6\delta[n] - 3\delta[n-1] - 39\delta[n-2] + 51\delta[n-3] - 45\delta[n-4] + 54\delta[n-5]$$

$y_2[n] = 6\delta[n] - 3\delta[n-1] - 39\delta[n-2] + 51\delta[n-3] - 45\delta[n-4] + 54\delta[n-5]$

(b) Consider the cascaded LTI system below



where $h_1[n] = \frac{1}{3}\sum_{k=2}^{4}\delta[n-k]$ and $y[n] = 3v[n] - 6v[n-1]$. Find the overall difference equation relating the input $x[n]$ and output $y[n]$ that is valid for all n.

Solutions:
The overall impulse response is: $h_1[n] * h_2[n] = h[n]$

$$h[n] = \frac{1}{3}(\delta[n-2] + \delta[n-3] + \delta[n-4]) * (3\delta[n] - 6\delta[n-1])$$
$$= \delta[n-2] - \delta[n-3] - \delta[n-4] - 2\delta[n-5]$$

$$y[n] = x[n-2] - x[n-3] - x[n-4] - 2x[n-5]$$

$y[n] = x[n-2] - x[n-3] - x[n-4] - 2x[n-5]$

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PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) An LTI system with output $y[n]$ has impulse response: $h[n] = 6\delta[n-2] + 3\delta\,[n-5] - 9\delta[n-8]$. If the input to the system is a scaled and delayed unit step function $x[n] = -2u[n-3]$, find $y[3]$ and $y[10]$.

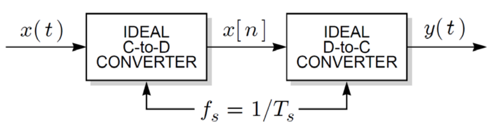
Solutions:

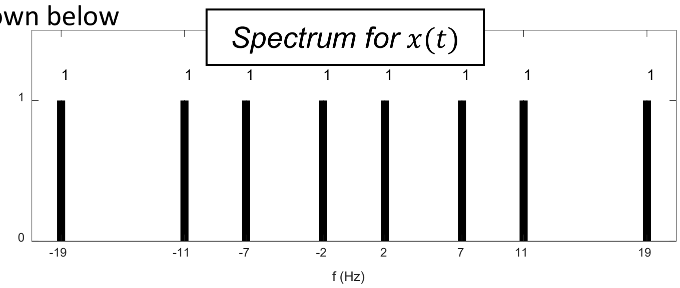
$$y[n] = 6x[n-2] + 3x[n-5] - 9x[n-8]$$
$$y[n] = 6u[n-5] + 3u[n-8] - 9u[n-11]$$

$$y[3] = -12u[3-5] - 6u[3-8] + 18u[3-11] = 0$$
$$y[10] = -12u[10-5] - 6u[10-8] + 18u[10-11] = -12 - 6 = -18$$

$y[3] = 0$	$y[10] = -18$

(b) The spectrum for an input signal $x(t)$ is shown below





(i) Indicate the condition on f_s to avoid aliasing $x(t)$: $f_s > 38$

Find the sampling rate that would result in the following output spectrums for $y(t)$

Sampling Rate (f_s)	Spectrum for $y(t)$
(ii) $f_s = 15$	$14 < fs < 22$; new freqs: 4; missing $11 \to -4$; $19 \to 4$
(iii) $f_s = 21$	$20 < fs < 22$; new freqs: 10; missing $11 \to -10$; $19 \to -2$
(iv) $f_s = 19$	$16 < fs < 22$; new freqs: 0,8; missing $11 \to -8$, $19 \to 0$