GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 2

DATE:18-Oct-19 COURSE: ECE-2026

NAME: Solutions CanvasID: ______________
LAST, FIRST ex: gtJohnA

Circle your correct recitation section number - failing to do so will cost you 2 points

Recitation time	Mon	Tue	Wed	Thu
09:30:10:45		L12 Farahmand		L06 Causey
12:00-13:15		L07 Farahmand		L08 Barry
13:30-14:45		L09 Farahmand		L10 Barry
15:00-16:15	L01 Juang	L11 Farahmand	L02 Casinovi	
16:30-17:45			L04 Casinovi	

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK ON ALL PROBLEMS TO RECEIVE CREDIT. PROBLEMS WITH NO WORK AND JUST ANSWERS MAY RECEIVE 0 CREDIT, EVEN IF THE ANSWER IS CORRECT. YOU MUST SHOW SOME NUMERICAL WORK, REASONING, OR EXPLANATION FOR YOUR ANSWER. (I.E., DON'T JUST PUT AN ANSWER AND LEAVE THE WORK AREA BLANK)**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)**
- **ALL RADIAN ANSWERS MUST BE IN THE RANGE $(-\pi, \pi]$ FOR CREDIT.**

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
No/Wrong Recitation Circled	-2	
Total		

PROBLEM 1: Parts a and b can be solved independently of each other

(a) A periodic signal $x(t)$ has Fourier Series coefficients a_k that are computed over one period as:

$$a_k = \frac{1}{T_0}\int_{-2}^{4} (4-t)\, e^{-j\frac{\pi}{4}kt}\, dt$$

(i) Find the fundamental period T_0 and a_0. (5 points)

$T_0 = 8$ (from the equation); $a_0 = \frac{1}{8}\left(\frac{1}{2}\right)(6)(6) = 2.25$

$T_0 =$ ___8 seconds__________	$a_0 =$ ___2.25_________

(ii) A new signal $y(t) = 2x(t-3)$ has Fourier Series coefficients defined as b_k. Express b_k in terms of a_k (Fourier Series of $x(t)$ in the equation above). (NOTE: Your answer will be in the form $b_k = Qa_k$ where you must define Q) (5 points)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{\frac{j\pi}{4}kt}$$

$$y(t) = 2x(t-3) = 2\sum_{k=-\infty}^{\infty} a_k\, e^{\frac{j\pi}{4}k(t-3)} = 2\sum_{k=-\infty}^{\infty} a_k\, e^{\frac{j\pi}{4}kt} e^{-\frac{j3\pi}{4}k} = \sum_{k=-\infty}^{\infty} 2e^{-\frac{j3\pi}{4}k} a_k\, e^{\frac{j\pi}{4}kt}$$

$$b_k = 2e^{-\frac{j3\pi}{4}k} a_k$$

$b_k =$ __$2e^{-\frac{j3\pi}{4}k}$ ______a_k

(b) Consider the following periodic signal

$$x(t) = -20 - 4\cos(12\pi t + 0.4\pi) + 2\cos(30\pi t - 0.6\pi) - 5\cos(42\pi t - 0.3\pi)$$

Find its fundamental frequency f_0 and fill in the table below of its Fourier series coefficients a_k. (10 points)

$$f_0 = GCD(6,15,21) = 3$$

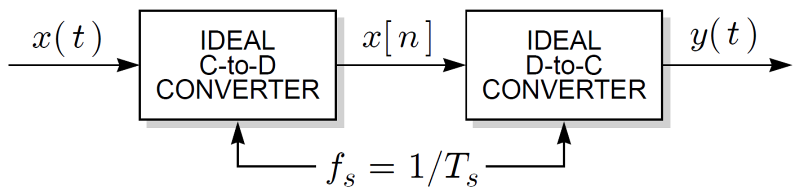
$$k = 2, 5, 7$$

$f_0 = 3$ Hz

k	0	1	2	3	4	5	6	7	8
a_k	$20e^{-j\pi}$	0	$2e^{-j0.6\pi}$	0	0	$e^{-j0.6\pi}$	0	$2.5e^{j0.7\pi}$	0

PROBLEM 2: All parts can be solved independently of each other

Consider the ideal C-to-D / D-to-C system in the figure below for all problems below.



(a) If $x(t) = 20 + \cos(68\pi t - 0.2\pi) + \cos(170\pi t - 0.8\pi) - \cos(238\pi t + 0.8\pi)$ and $f_s = 595$ Hz, then $x[n] = x[n + N_0]$ (i.e., is periodic) where its fundamental period is $N_0 =$ 35 samples (5 points)

$$f_0 = GCD(34,85,119) = 17\ Hz \rightarrow\ N_0 = \frac{f_s}{GCD(f_0, f_s)} = \frac{595}{GCD(17,595)} = \frac{595}{17} = 35 \text{ samples}$$

(b) If $x(t) = \cos(68\pi t - 0.2\pi) + 2\cos(170\pi t - 0.8\pi)\cos(238\pi t)$ and $f_s = 85$ Hz, find $y(t)$. (5 points)

$$y(t) = \cos(68\pi t - 0.2\pi) + \cos(68\pi t + 0.8\pi) + \cos(68\pi t - 0.8\pi) = \cos(68\pi t - 0.8\pi)$$

$y(t) = \cos(68\pi t - 0.8\pi)$

(c) Suppose that $f_s = 1500\ Hz$ and $y(t) = 2\cos(300\pi t - \pi/5)$. Find three possible inputs for $x(t)$ (i.e., $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$, $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$, $x_3(t) = A_3\cos(\omega_3 t + \phi_3)$) that would have resulted in the same output signal $y(t)$ with the restriction that the **input frequencies be between 0 and 2000 Hz.** (10 points)

One is unaliased: $x_1(t) = 2\cos\left(300\pi t - \frac{\pi}{5}\right)$

One is folded: $x_2(t) = 2\cos\left(2700\pi t + \frac{\pi}{5}\right)$

One is aliased: $x_3(t) = 2\cos\left(3300\pi t - \frac{\pi}{5}\right)$

$x_1(t) = 2\cos\left(300\pi t - \frac{\pi}{5}\right)$

$x_2(t) = 2\cos\left(2700\pi t + \frac{\pi}{5}\right)$

$x_3(t) = 2\cos\left(3300\pi t - \frac{\pi}{5}\right)$

PROBLEM 3: All parts can be solved independently of each other

(a) Consider the FM signal $x(t) = 20\cos(Bt + 10\cos(50\pi t))$. Find the lower bound on B so that the theoretical instantaneous frequency, $f_i(t)$, is greater than zero (i.e., $f_i(t) > 0$). (5 points)

$$f_i(t) = \frac{1}{2\pi}(B - 500\pi\sin(50\pi t)) > 0 \rightarrow B > 500\pi\sin(50\pi t) \rightarrow B > 500\pi$$

$B > 500\pi$

(b) Consider the AM signal $x(t) = 10\cos(300\pi t)\cos(2\pi f_c t)$. If $x(t)$ is sampled with sampling frequency $f_s = 1500\ Hz$, what is the upper bound on f_c to avoid aliasing? (5 points)

Max frequency = $f_c + 150 < 750 \rightarrow f_c < 500\ Hz$

600

$f_c < 500\ Hz$

(c) A chirp signal is defined as $x(t) = \cos(\psi(t))$ where $\psi(t) = 130\pi t^2 + 970\pi t - \frac{\pi}{6}$ for $t \geq 0$. Assume we input this signal into MATLAB with a sampling frequency $f_s = 3700\ Hz$ and then play the audio back at the same frequency $f_s = 3700\ Hz$. What frequencies would be heard at the following times $t = 5$ seconds (i.e., $f_{@t=5}$) and $t = 50$ (i.e., $f_{@t=50}$) seconds. Explain your reasoning. (10 points)

$$f_i(t) = 130t + 485$$

@t=5 seconds: $f_i(5) = 130(5) + 485 = 1135\ Hz$ NO aliasing, so 1135 Hz will be heard

@t=50 seconds: $f_i(50) = 130(50) + 485 = 6985$ – Aliasing (with folding). Aliases are separated by the sampling frequency f_s so the frequency that will be heard is: $6985 - (2)3700 = -415\ Hz$, so 415 Hz will be heard.

$f_{@t=5} = 1135$ Hz

$f_{@t=50} = 415$ Hz

Explanation:

No Aliasing @t=5.

Aliasing (with folding) @t=50. Compute the frequency heard by subtracting the sampling rate f_s until it is under $\frac{f_s}{2}$.