GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 2

DATE: 20-Oct-17 COURSE: ECE-2026

NAME: SOLUTIONS TSquareID: ____________
LAST, FIRST ex: gtJohnA

Circle your correct recitation section number - failing to do so will cost you 2 points

Recitation time	Mon	Tue	Wed	Thu
09:30:10:45				L06 Harper
12:00-13:15		L07 Causey		L08 Harper
13:30-14:45		L09 Yang		L10 Stuber
15:00-16:15	L01 Juang	L11 Yang	L02 Causey	L12 Stuber
16:30-17:45	L03 Marenco		L04 Causey	

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- Two pages $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK ON ALL PROBLEMS TO RECEIVE CREDIT**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π instead of 1.257)**
- **ALL RADIAN ANSWERS SHOULD BE IN THE RANGE (-π, π].**

Problem	*Value*	*Score*
1	16	
2	20	
3	24	
No/Wrong Recitation Circled	-2	
Total		

PROBLEM 1:

(a) Express the signal $y(t)$ in standard reduced sinusoidal form (i.e., $\sum_{i=0}^{N} A_i \cos(\omega_i t + \phi_i)$ where N is unspecified with $A_i \geq 0$ and $-\pi < \phi_i \leq \pi$) (8 points)

$$y(t) = \Re e\{6e^{j(2\pi(100)t-\pi/3)}\} + 3e^{j200\pi t+j\pi/3} + 3e^{-j200\pi t-j\ \ /3}$$

$$y(t) = \Re e\left\{6e^{j2\pi(100)t-\frac{\pi}{3}}\right\} + 3e^{j200\pi t+\frac{j\pi}{3}} + 3e^{-j200\pi t-\frac{j\pi}{3}} \rightarrow 6\cos\left(200\pi t - \frac{\pi}{3}\right) + 6\cos\left(200\pi t + \frac{\pi}{3}\right)$$

Phasor Addition: $6e^{-\frac{j\pi}{3}} + 6e^{\frac{j\pi}{3}} = 6e^{j0} \rightarrow y(t) = 6\cos(200\pi t)$

$y(t) = 6\cos(200\pi t)$

(b) Express $x(t) = \cos(300\pi t + \pi/9)\cos(2100\pi t)$ as a sum of two sinusoids

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2)$$

and determine the fundamental frequency f_0.(8 points)

Center the 150 Hz signal on the 1050 hz signal giving two sinusoids at 1200 Hz and 900 Hz. $f_0 = 300\ Hz$

$x(t) = \frac{1}{2}\cos(1800\pi t - \pi/9) + \frac{1}{2}\cos(2400\pi t + \pi/9)$

$x(t) = \frac{1}{2}\cos(1800\pi t - \pi/9) + \frac{1}{2}\cos(2400\pi t + \pi/9)$

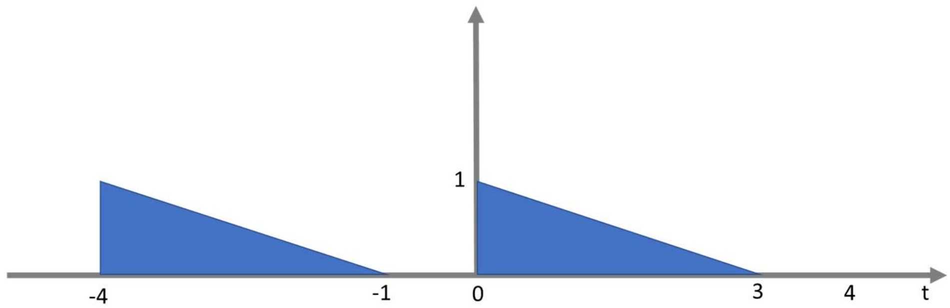
f_0 = 300 Hz

PROBLEM 2: (Parts a and b can be solved independently)

(a) A signal x(t) has Fourier Series coefficients that are computed over one period as:

$$a_k = \frac{1}{4}\int_0^3 \left(1 - \frac{1}{3}t\right) e^{-j\frac{\pi}{2}kt} dt$$

(i) Plot x(t) over the interval $-4 \le t \le 4$. (5 points) (properly label all points)



(ii) Find a_0 (5 points)

a_0 is the area of a triangle * 1/4: $a_0 = \frac{1}{2}(1)(3) * \frac{1}{4} = 3/8$

a_0 =3/8

(b) A signal x(t) with period of $T_0 = 0.008$ sec is defined as follows

$$x(t) = \sum_{k=-M}^{M} a_k e^{\frac{j2\pi}{T_0}kt}$$

with Fourier series coefficients expressed as:

$$a_k = \begin{cases} \frac{j^k}{k^2} & odd\ values\ of\ k \\ 0 & even\ values\ of\ k \end{cases} ;\ a_0 = 2e^{j\pi}$$

(i) If $M = 8$, what is the minimum sampling rate (f_s) needed for proper reconstruction? (5 points)

$T_0 = 0.008 \rightarrow f_0 = 125\ Hz \rightarrow f_s > 2 * 7f_0 = 2 * (7 * 125) = 1750\ H$

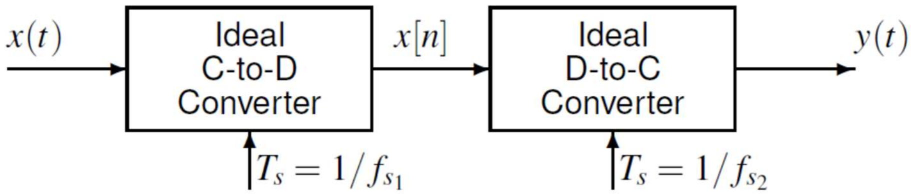
f_s > 1750 Hz

(ii) If $M = 2$, expand x(t) as a sum of sinusoids with each sinusoid expressed in standard form (i.e., $A\cos(\omega_0 t + \phi)$) (5 points)

$$a_0 = -2; a_{\pm 1} = e^{\pm j\frac{\pi}{2}}; a_{\pm 2} = 0 \rightarrow x(t) = -2 + e^{\frac{j\pi}{2}} e^{j2\pi(125)t} + e^{-\frac{j\pi}{2}} e^{-j2\pi(125)t} = -2 + 2\cos\left(250\pi t + \frac{\pi}{2}\right)$$

$x(t) = -2 + 2\cos\left(250\pi t + \frac{\pi}{2}\right)$

PROBLEM 3: (Each part of this problem may be answered independently)
Consider the ideal sampling and reconstruction system shown below for parts (a) and (b):



(a) Suppose that $f_{s_1} = 2000\ Hz$ and $x[n] = 6\cos(0.4\pi n + \pi/3)$. Find two possible inputs for $x(t)$ (i.e., $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$) that would have resulted in the same discrete-time signal $x[n]$ with the restriction that the **input frequencies be between 0 and 2000 Hz.** (8 points)

One is unaliased: $x_1(t) = 6\cos\big((0.4\pi * 2000)t + \pi/3\big) = 6\cos\left(800\pi t + \frac{\pi}{3}\right)$

One is folded: $x_2(t) = 6\cos\big((1.6\pi * 2000)t - \pi/3\big) = 6\cos(3200\pi t - \pi/3)$

$x_1(t) = 6\cos\left(800\pi t + \frac{\pi}{3}\right)$

$x_2(t) = 6\cos\left(3200\pi t - \frac{\pi}{3}\right)$

(b) Suppose that $f_{s_1} = f_{s_2} = f_s$ and $x(t) = \cos(800\pi t) + \cos\left(3400\pi t - \frac{\pi}{3}\right)$. Find the **largest** f_s such that $y(t) = A\cos(800\pi t + \phi)$. Also, find A and ϕ (8 points)

Fold 1700Hz signal into 400 Hz signal: $f_s = 2100 Hz$

$$Ae^{j\phi} = 1e^{j0} + 1e^{\frac{j\pi}{3}} = \sqrt{3}e^{\frac{j\pi}{6}} \approx 1.73e^{-j0.167\pi}$$

$f_s =$ _$2100\ Hz$_____

$A = \sqrt{3} \approx 1.73$_____

$\phi = \frac{\pi}{6} \approx 0.167\pi$_

(c) Consider the following MATLAB code: (8 points)

```
tt=0:1/3000:tstop; % time in seconds
psi=2*pi*(450*tt+250*tt.^2);
xx = cos(psi);
spectrogram(xx, 256, 200, 512, 3000,'yaxis')
```

(i) What is the instantaneous frequency represented in the spectrogram at 20 seconds (i.e., $f_i(20)$)

$\psi(t) = 2\pi(450t + 250t^2) \rightarrow f_i(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} = 450 + 500t$

$f_i(20) = 450 + 500(20) = 10450 \rightarrow 10450 - 9000 = 1450\ Hz$

$f_i(20) =$ _$1450\ Hz$____

(ii) What is the largest value the variable `tstop` can be to avoid seeing folding in the spectrogram?

$f_i(t_{stop}) = 450 + 500(t_{stop}) < 1500 \rightarrow t_{stop} < 2.1$ sec

`tstop <   __2.1 sec___`