GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 1

DATE: 2-October-15 COURSE: ECE-2026

NAME: SOLUTIONS TSquareID: ____________
LAST, FIRST ex: gtAyellow

Circle your correct **recitation section** number - failing to do so will cost you 3 points

Recitation time	Mon	Tue	Wed	Thu
09:35:10:55				L06 Altaf
12:05-13:25		L07 Altaf		L08 Bloch
13:35-14:55		L09 Barry		L10 Bloch
15:05-16:25	L01 Juang	L11 Barry	L02 Rozell	L12 Yeredor
16:35-17:55	L03 Causey		L04 Rozell	L14 Yeredor

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}''\times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK TO RECEIVE CREDIT**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. **Circle** your answers, or write them in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π instead of 1.257)**
- **ALL RADIAN ANSWERS SHOULD BE IN THE RANGE (-π, π].**

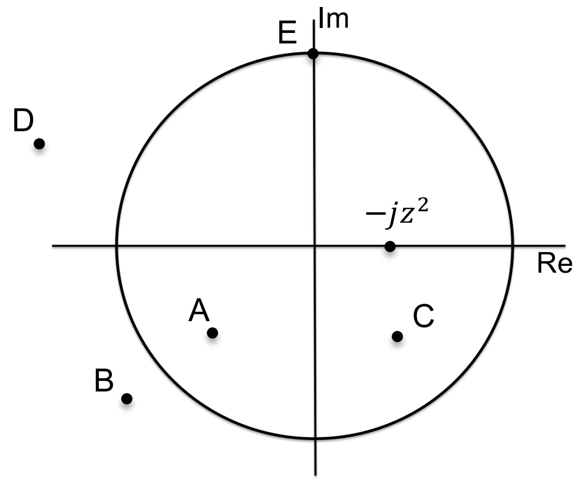
Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No/Wrong Recitation Circled	-3	
Total		

PROBLEM 1:

Parts a, b, and c can be solved independently of each other.

(a) Consider a complex number defined as $z = re^{j\theta}$ (where $\theta \in (-\pi, 0]$). The location of $-jz^2$ is shown on the complex plane below (where the circle represents the unit circle of radius 1). Consider the following lettered operations on the complex number z (A-E). Place the appropriate letter on the provided complex plane in the approximate location that it should be. (They do not have to be exact, but they should still be correct relative to the position of $-jz^2$). (10 points)

A. (5 points)	z
B. (1.25 points)	$1/z^*$
C. (1.25 points)	$z^* e^{-j\pi}$
D. (1.25 points)	$z^* + e^{-j\pi}$
E.(1.25 points)	z/z^*



The most critical part is finding z. We know $r < 1 \text{ and } \theta \in (-\pi, 0]$

$$-jz^2 = pe^{j0}$$
$$e^{-\frac{j\pi}{2}} r^2 e^{j2\theta} = pe^{j0}$$
$$r^2 e^{j2\theta} = pe^{\frac{j\pi}{2}}$$
$$r = \sqrt{p},\ \ \theta = \frac{\pi}{4} \text{ OR } r = \sqrt{p},\ \ \theta = \frac{-3\pi}{4}$$

For our conditions: $r = \sqrt{p},\ \ \theta = \frac{-3\pi}{4}$

(b) Let $2\cos\left(10\pi t + \frac{\pi}{3}\right) = A\cos(\omega_0 t + \varphi) + 2\cos\left(10\pi t - \frac{\pi}{4}\right) + 3\sin(10\pi t)$. Find $A, \omega_0,$ and φ. (9 points)

Use phasor addition (calculator): $Ae^{j\varphi} = 2e^{\frac{j\pi}{3}} - 3e^{-\frac{j\pi}{2}} - 2e^{-\frac{j\pi}{4}} \approx 6.16e^{j0.52\pi}$

$A =$ ______6.16__________ $\omega_0 =$ ____10π___________ $\varphi =$ __0.52π_____________

(c) Solve the following equation for A and ϕ. (6 points)

$$\sum_{k=2}^{7} \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}k\right)} + \sum_{k=8}^{13} \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}k\right)} + A\cos(\omega_0 t + \phi) + \sqrt{2} e^{j\omega_0 t} = 0$$

$e^{j\omega_0 t} A\cos(\phi)$

$$\sum_{k=0}^{7} \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}k\right)} - \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}\right)} + \sum_{k=8}^{14} \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}k\right)} - \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi(14)}{15}\right)} + A\cos(\omega_0 t + \phi) = 0$$

$e^{j\omega_0 t} A\cos(\phi)$

$$\sum_{k=0}^{14} \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}k\right)} - \left(\sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}\right)} + \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi(14)}{15}\right)}\right) + A\cos(\omega_0 t + \phi) = 0$$

$e^{j\omega_0 t} A\cos(\phi)$

$$\sum_{k=0}^{14} \sqrt{2} e^{j\left(\omega_0 t + \frac{2\pi}{15}k\right)} - e^{j\omega_0 t} 2\sqrt{2}\cos\left(\frac{2\pi}{15}\right) + A\cos(\omega_0 t + \phi) = 0$$

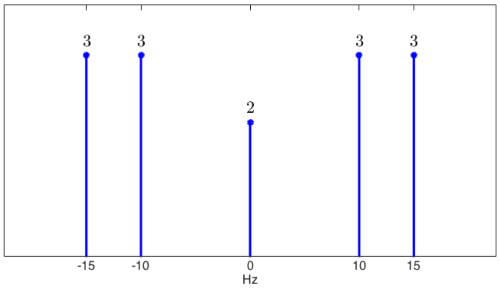
$e^{j\omega_0 t} A\cos(\phi)$

$$e^{j\omega_0 t} 2\sqrt{2}\cos\left(\frac{2\pi}{15}\right) = e^{j\omega_0 t} A\cos(\phi)$$

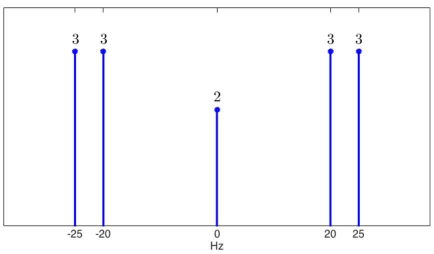
$A =$ ____$2\sqrt{2}$___________ ϕ______$2\pi/15$___________

PROBLEM 2:
Match the spectrums to the appropriate sinusoidal plots. Write your answers **in the boxes** provided.

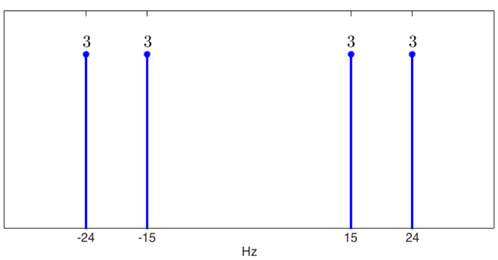
C



F



B



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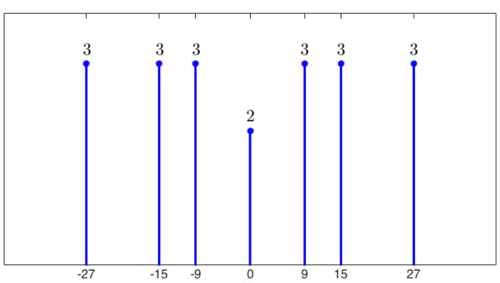
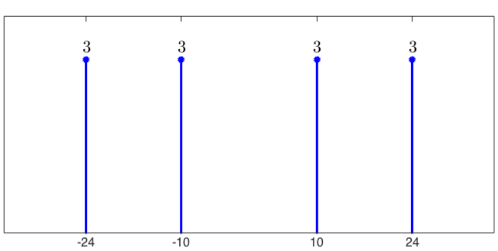
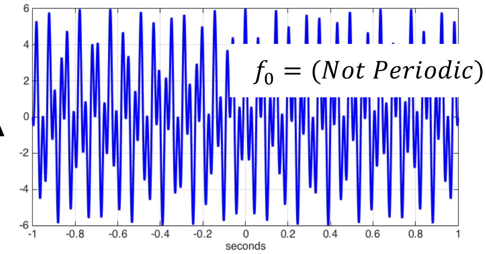


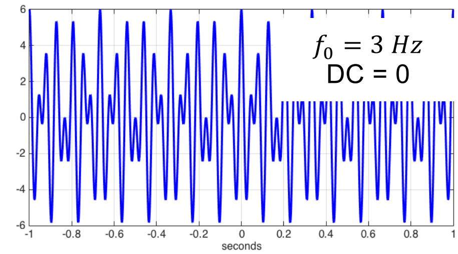
D



A



B



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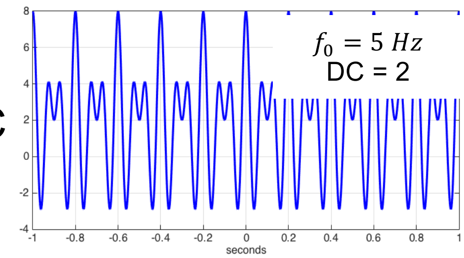
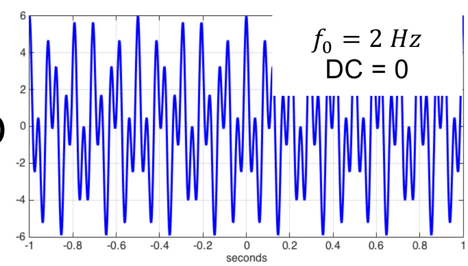


D



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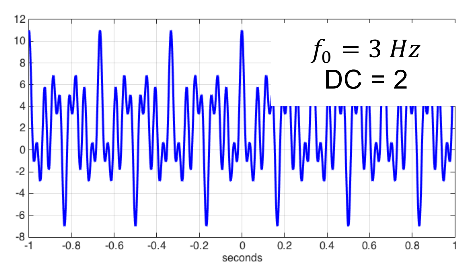

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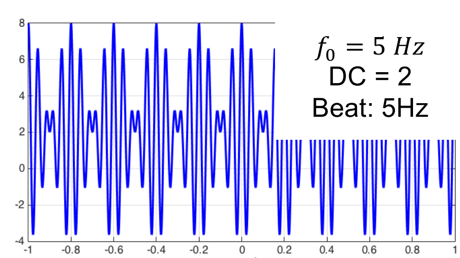


PROBLEM 3:
Assume that the Fourier series coefficients for $x(t)$ are given by the integral:

$$a_k = \frac{1}{6}\int_{-2}^{2}(1+|t|)\,e^{-\frac{j\pi}{3}kt}\,dt$$

Put your answers in the boxes provided.

(a) Determine the fundamental period T_0 of the signal $x(t)$. (4 points)

Can be read from the equation $a_k = \frac{1}{T_0}\int_0^{T_0}(1+|t|)e^{-\frac{j2\pi}{T_0}kt}dt$

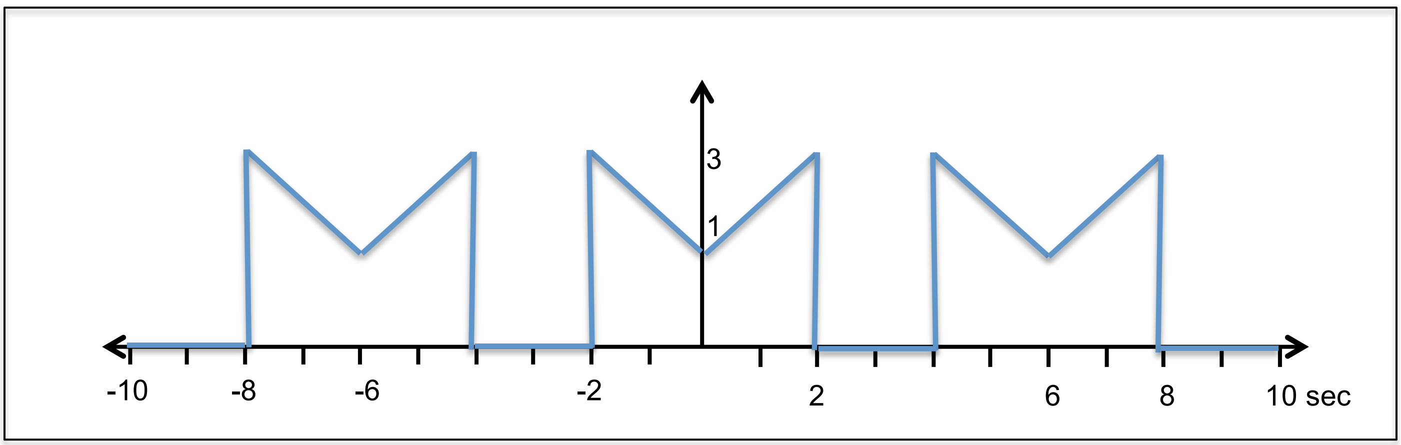
T_0 =6 sec

(b) Determine the DC value (a_0) $x(t)$. Give your answer as a number. (4 points)

$$a_0 = \frac{1}{6}\int_{-2}^{2}(1+|t|)\,dt = 4/3$$

a_0 =4/3

(c) In the plot area below, draw a plot of $x(t)$ over the range of $-10 \le t \le 10$ seconds. Label your plot carefully. (8 points)



(d) Assume that a new signal is created from $x(t)$ as follows:

$$y(t) = 3x(t-0.2)$$

Determine the fundamental period T_0 of $y(t)$ and the DC value of $y(t)$. (4 points)

Scaling and shifting has NO impact on T_0
Scaling increases DC by a factor of 3

$T_0 = 6$

DC = 4

PROBLEM 4:

Parts a and b can be solved independently of each other.

(a) Assume we have an input signal defined as $x(t) = \cos(2\pi f t)$.

(a.1) If the input frequency is $f = 300$ Hz, what is the lower bound on the sampling rate (f_s) to avoid aliasing as we have discussed in class? (3 points)

2 X the highest frequency = 2*300 = 600

$f_s > 600$

(a.2) Assume that input frequency f is unknown and the sampling rate is set to $f_s = 300$ Hz resulting in the discrete signal $x[n] = \cos(0.3\pi n)$. List three possible input frequencies for f in the range of $0 < f \leq 500$ Hz. (9 points)

$\hat{\omega} = \frac{2\pi f}{f_s} \to 0.3\pi = \frac{2\pi f}{300} \to f = 45Hz$

Other aliases are found by adding/subtracting $f_s = 300$ to 45 Hz, therefore: $f = 45 + 300 = 345\ Hz$ OR $f = -45 + 300 = 255\ Hz$ in the range $0 < f \leq 500$ Hz.

f =_45__ or _255______ or _345_____ Hz

(b) Suppose the following MATLAB code is run:

```
tt = 0:(1/4050):3;
xx = cos(600*pi*tt.^2 + 400*pi*tt + pi/3);
```

(b.1) How many samples (*N*) are contained in the vector xx? (4 points)

Sampling rate is 4050 Samples/sec. The time vector is computed over 3 seconds so: N=4050*3+1 (we add one because Matlab includes the point at 0)

N= 12151

(b.2) Find the equation for the instantaneous frequency, $f_i(t)$ (in Hz) of the signal represented by the vector xx. (6 points)

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left(600\pi t^2 + 400\pi t + \frac{\pi}{3}\right)$$
$$f_i(t) = 600t + 200$$

$f_i(t)$= $600t + 200$

(b.3) Will the signal xx alias at any point during the interval from 0 to 3 seconds? If NO, explain why it will NOT alias. If YES, explain why it WILL alias and find the time (in seconds) at which point it will alias. (Circle one and then provide your explanation) (YES OR NO) (3 points)

No. The max frequency obtained in 3 seconds is (600)*3+200 = 2000 Hz. The Nyquist frequency in this case is 2025 Hz so no aliasing occurs.