GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

ECE2026 – Fall 2014
Quiz #2 (Written Exam 1)
October 3, 2014

NAME: _______KEY___________ **GT Username:** _____________________
LAST FIRST (e.g., gtinit0101)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):
(Failing to circle the correct section will cost you 3 points)

	Mon	Tue	Wed	Thu
9:35-10:55				L06 (Bhatti)
12:05-13:25		L07 (Causey)		
13:35-14:55		L09 (Causey)		L10 (Bhatti)
15:05-16:25	L01 (Moore)	L11 (Davenport)	L02 (Juang)	L12 (Walkenhorst)
16:35-17:55	L03 (Aghasi)	L13 (Davenport)	L04 (Juang)	L14 (Walkenhorst)

Important Notes:

- Write your name on the front page ONLY. **DO NOT** UNSTAPLE the test.
- Closed book, but a calculator is permitted.
- One page (8.5”x11”) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Partial credit for incorrect answers may be granted ONLY when you JUSTIFY your reasoning CLEARLY.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If more space is needed for scratch work, use the backs of previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	-3	
Total		

PROBLEM Fall-14-Q.2.1:
Consider the following MATLAB code:

```
tt = -0.6:1/1e4:1.2;
xx = real(abs(sqrt(3)+j)*exp(j*(6000*pi*tt - 1000*tt.^3 + 21*2*pi)));
soundsc(xx,1e4);
```

If you run this code you will hear some sound.

(a) The variable **xx** represents a continuous-time signal $x(t)$. Write a compact mathematical expression for this signal.

$x(t) = \quad 2\cos(6000\pi t - 1000t^3)$

(b) What is the highest instantaneous frequency (in Hz) that is being played from the signal and at what time in terms of **tt**?

$$\phi(t) = 6000\pi t - 1000t^3 \qquad \omega_{inst}(t) = \tfrac{d}{dt}\phi(t) = 6000\pi - 3000t^2$$

$$\tfrac{d}{dt}\omega_{inst}(t) = -6000t = 0 \rightarrow t = 0; \tfrac{d^2}{dt^2}\omega_{inst}(t) = -6000 < 0$$

$$\omega_{inst}(t)\big|_{t=0} = 6000\pi \rightarrow f_{\max} = 3000Hz$$

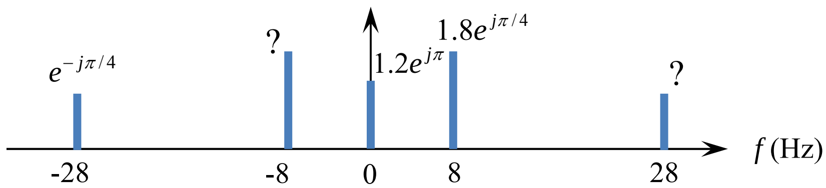
$f_{\max} = \quad 3000Hz$ at **tt** $= \quad 0$

(c) Suppose the signal lasted beyond the time limit in the code. What would be the instantaneous frequency (in Hz) heard at **tt = 1.7725 =** $\sqrt{\pi}$?

$$\omega_{inst}(t)\big|_{t=\sqrt{\pi}} = 6000\pi - 3000\pi = 3000\pi \quad \rightarrow \quad f_{inst} = 1500Hz$$

$f_{inst} = \ 1500Hz$

PROBLEM Fall-14-Q.2.2:



The spectrum of a real signal $x(t)$ is shown as above.

(a) Write the signal $x(t)$ as sum of sinusoids in the standard form: $x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(\omega_k t + \varphi_k)$.

$$x(t) = -1.2 + 3.6\cos(16\pi t + \pi/4) + 2\cos(56\pi t + \pi/4)$$

(b) Consider another signal $y(t)$, $y(t) = -2x(t) + 2\cos(20\pi t + \pi/5) + 4\cos(56\pi t + \pi/4)$ where $x(t)$ is given in (a). This signal can also be represented by a Fourier series: $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$.

Write ***all*** non-zero Fourier coefficients of $y(t)$ in the table below; make sure to include the correct harmonic numbers.

k	b_k
-5	$e^{-j\pi/5}$
-4	$3.6e^{j3\pi/4}$
0	2.4
4	$3.6e^{-j3\pi/4}$
5	$e^{j\pi/5}$

$$y(t) = 2.4 - 7.2\cos(16\pi t + \pi/4) - 4\cos(56\pi t + \pi/4) + 2\cos(20\pi t + \pi/5) + 4\cos(56\pi t + \pi/4)$$
$$= 2.4 + 7.2\cos(16\pi t - 3\pi/4) + 2\cos(20\pi t + \pi/5)$$

With frequencies 16π and 20π, the fundamental frequency is 4π and the two components are the 4th and the 5th harmonics

$$y(t) = 2.4 + 7.2\cos(16\pi t - 3\pi/4) + 2\cos(20\pi t + \pi/5)$$
$$= 2.4 + 7.2\cos(4(4\pi)t - 3\pi/4) + 2\cos(5(4\pi)t + \pi/5)$$

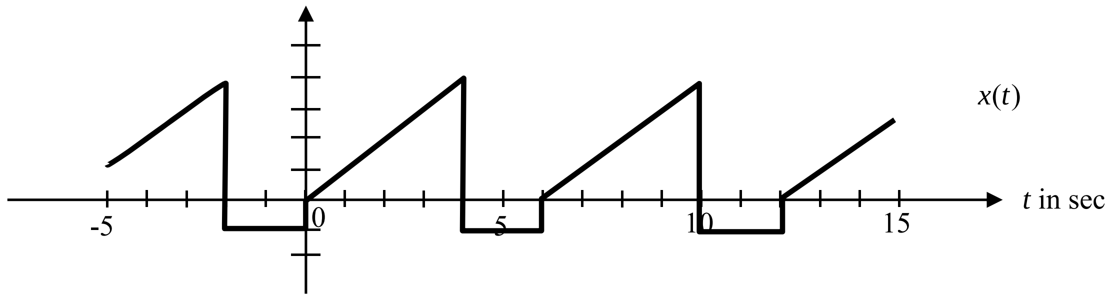
PROBLEM Fall-14-Q.2.3:

A periodic signal $x(t)$ with a period $T_0 = 6$ is described over one period, $-2 < t \le 4$, by the equation:

$$x(t) = \begin{cases} -1, & -2 < t \le 0 \\ t, & 0 < t \le 4 \end{cases}$$

This signal can be represented by the Fourier series, $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, which is valid for $-\infty \le t \le \infty$.

(a) Sketch the signal $x(t)$ over the time interval $-5 < t \le 15$.



(b) Determine the DC coefficient of the Fourier Series, a_0.

The area under the curve averged over one period is: $\left\{\frac{(4\times4)}{2} + (-1)\times 2\right\}/6 = 1$

$$a_0 = \quad 1$$

(c) The Fourier coefficient of the 12th harmonic component of the signal $x(t)$ above is denoted by a_{12}. Let $y(t) = x(t+0.1)$ and its Fourier series representation be given as $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$.

Find the 12th harmonic coefficient, b_{12} ; express the result in relation with a_{12}.

$$y(t) = x(t+0.1) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 (t+0.1)} = \sum_{k=-\infty}^{\infty} \left(a_k e^{j0.1k\omega_0}\right) e^{jk\omega_0 t}$$

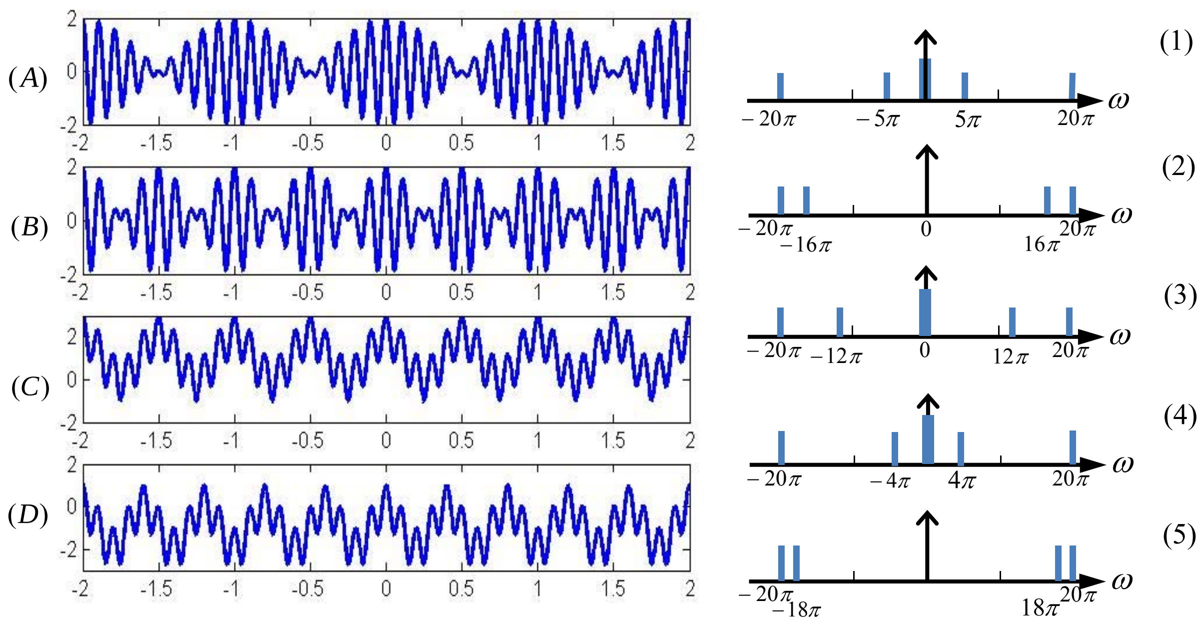
$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \left(a_k e^{j0.1k\omega_0}\right) e^{jk\omega_0 t}$$

$$T_0 = 6 \rightarrow \omega_0 = 2\pi/6 = \pi/3 \qquad b_{12} = a_{12} e^{j0.1\times12\times\pi/3} = a_{12} e^{j0.4\pi}$$

$$b_{12} = a_{12} e^{j0.4\pi}$$

PROBLEM Fall-14-Q.2.4:

The waveforms of four real signals are plotted in the following panels on the left, marked (A), (B), (C), and (D). Their spectra are shown on the right among seven possible diagrams, indexed from 1 to 5. Find the corresponding spectrum for each of the signals on the left. State your reason below each answer box. (Note: The phase is not specified in the spectral plots and the line at $\omega = 0$ can have a phase value of π.)



$(A) \leftrightarrow$ 5	$(B) \leftrightarrow$ 2	$(C) \leftrightarrow$ 4	$(D) \leftrightarrow$ 1