GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
QUIZ #2

DATE: 11-Oct-13 COURSE: ECE-2026

NAME: ______________________________ GT#: ______________
LAST, FIRST ex: gtaburDEll

Circle your correct **recitation section** number - failing to do so will cost you 3 points

L01: Mon - (Juang) L02: Wed - (Bloch) L03: Mon - (Casinovi)

L04: Wed - (Bloch) L05: Tues - (Bhatti) L06: Thurs - (Coyle)

L07: Tues - (Bhatti) L08: Thurs - (Coyle) L09: Tues - (AlRegib)

L10: Thurs - (Ma) L11: Tues - (Causey) L12: Thurs - (Ma)

L13: Tues - (Causey) L14: Thurs - (AlRegib)

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning sufficiently to receive partial credit. Explanations are also required (as applicable) to receive full credit for an answer. (i.e., ***show enough work so graders understand your approach to solving the problem***)
- You must **write your answer in the boxes provided on the exam paper itself**. **Only answers in these boxes will be graded as the final solution**. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
Total	100	

PROBLEM fa-13-Q.2.1:

Suppose we have a periodic signal $x(t)$ represented through Fourier synthesis as follows:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j(2\pi/6)kt}$$

It is also known that the Fourier series coefficients for this representation of $x(t)$ are given by the integral:

$$a_k = \frac{1}{6} \int_{-1}^{1} (1 - |t|) \, e^{-j(2\pi/6)kt} dt$$

$$\text{where } |t| = \begin{cases} t & when\, t \geq 0 \\ -t & when\, t < 0 \end{cases}$$

(a) Determine the fundamental period T_0 of the signal $x(t)$.

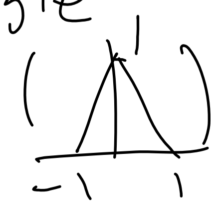
From equation $T_0 = 6$

$\boxed{T_0 = 6}$

(b) Determine the DC value of $x(t)$. Give your answer as a number.

Area of triangle

$\frac{1}{6} \cdot$ area of (

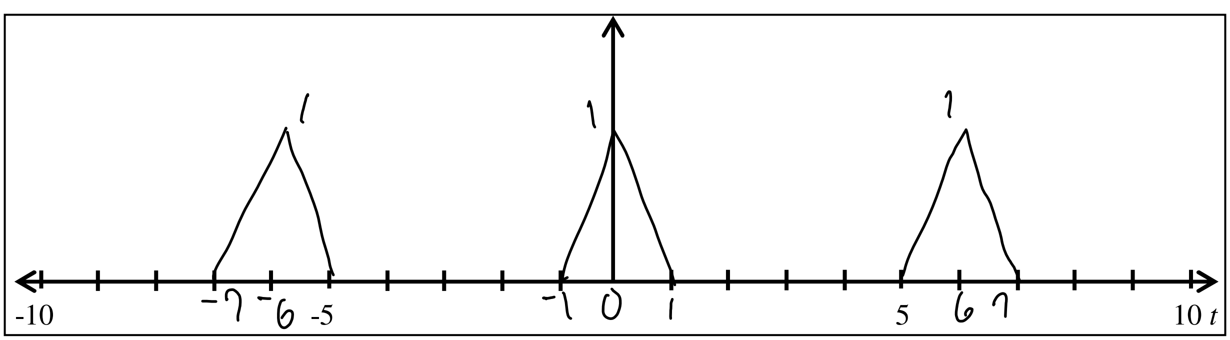


) $= \frac{1}{6} \cdot \left(\frac{1}{2} \cdot 2 \cdot 1\right)$

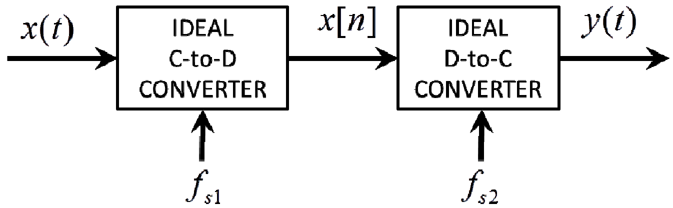
$= 1/6$

$\boxed{DC = 1/6}$

(c) In the plot area below, draw a plot of $x(t)$ over the range of $-10 \leq t \leq 10$ seconds. Label your plot carefully.



PROBLEM fa-13-Q.2.2:



The input to an ideal C-to-D converter is a signal $x(t)$ defined as follows:

$$x(t) = 10 + \cos(480\pi t - \pi/3) + \cos(800\pi t + \pi/5)$$

(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$ in the ideal C-to-D converter.

$480\pi \Rightarrow 2\pi(240)$

$800\pi = 2\pi(\underline{400})$

highest frequency

Nyquist = 2 · highest freq.

$\boxed{800 \text{ Hz}}$

(b) If the sampling rate for the C-to-D converter is $f_{s1} = 500$ samples/sec., determine the discrete-time signal $x[n]$ as a sum of cosines. *Make sure that all sinusoidal frequencies in your answer are expressed in the range* $0 \le \omega \le \pi$ *radians.*

$\hat{w} = \dfrac{2\pi f}{f_s} \rightarrow$

$\hat{w}_1 = \dfrac{480\pi}{500} = \dfrac{24\pi}{25}$ (not aliased)

$\hat{w}_2 = \dfrac{800\pi}{500} = \dfrac{40\pi}{25} - 2\pi = \dfrac{-10\pi}{25}$ (folding)

$x[n] = 10 + \cos\left(\frac{24}{25}\pi n - \pi/3\right) + \cos\left(\frac{10\pi}{25} n - \pi/5\right)$

(c) If $f_{s1} = f_{s2} = 500$ samples/sec, determine the continuous-time signal $y(t)$.

$$\omega = \hat{\omega} f_s$$

$$\omega_1 = \frac{24\pi}{25} \cdot 500 = 480\pi$$

$$\omega_2 = \frac{10\pi}{25} \cdot 500 = 200\pi$$

$y(t) = 10 + \cos(480\pi t - \pi/3) + \cos(200\pi t - \pi/5)$

PROBLEM fa-13-Q.2.3:

(a) Suppose the following MATLAB® code is run:

```
tt = 0:(1/4500):9;
xx = cos(400*pi*tt.^2 + 500*pi*tt + pi/3);
spectrogram(xx,256,128,512,4500);
```

Find the equation for the **instantaneous frequency**, $f_i(t)$ (in Hz), of the signal xx.

$$\psi(t) = 400\pi t^2 + 500\pi t + \pi/3$$

$$f_i(t) = \frac{1}{2\pi}(800\pi t + 500\pi) = 400t + 250$$

$f_i(t) =$ $400t + 250$ Hz

(b) Suppose the following MATLAB code is run (on a supercomputer):

```
tt = 0:1/6400:100000;
xx = (2.^(3*pi))*sin(2*pi*1600*tt - pi/8);
soundsc(xx, fsamp)
```

f_s ← f_o $\hat{f}$

If the analog tone that results has a frequency of 3500 Hz, what is the value of the variable fsamp?

$$\hat{w} = \frac{2\pi f_o}{f_s} = \frac{2\pi(1600)}{6400} = \pi/2$$

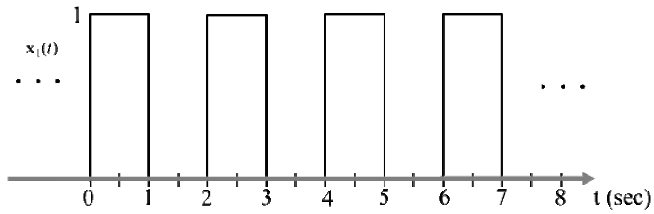
$$\hat{f} = \frac{\hat{w}\cdot f_{samp}}{2\pi} \rightarrow f_{samp}$$

$$= \frac{\hat{f}\cdot 2\pi}{\hat{w}} = \frac{(3500)\cdot 2\cancel{\pi}}{\cancel{\pi}/2} = 14000\,\text{Hz}$$

$\boxed{14{,}000\ \text{Hz}}$

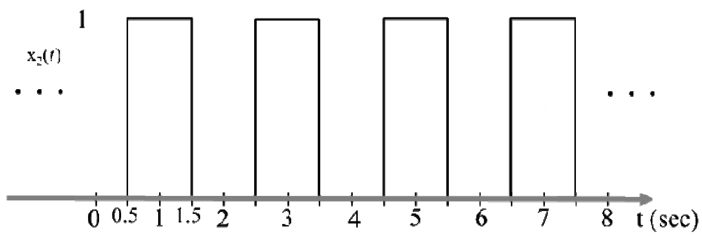
PROBLEM fa-13-Q.2.4:

Consider the periodic square wave $x_1(t)$ below with fundamental period $T_0 = 2$ seconds and Fourier Series coefficients a_k as shown below.



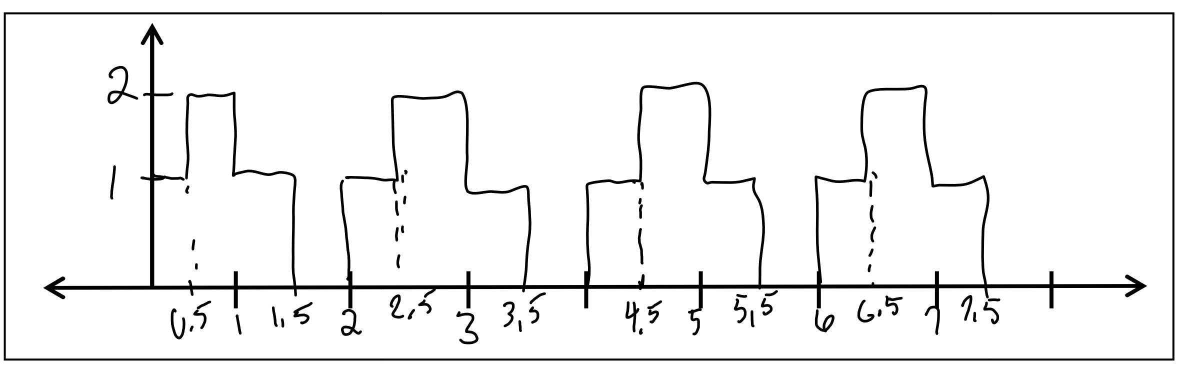
$$x_1(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j(2\pi/2)kt}$$

Assume we also have a second periodic square wave $x_2(t)$ with a fundamental period $T_0 = 2$ seconds shown below:



(a) In the plot area provided below, sketch a plot of the periodic signal $y(t) = x_1(t) + x_2(t)$ over the range $0 \le t \le 8$. (Label your plot carefully)

Diredt Summation



(b) It can be seen that $x_2(t) = x_1(t - 0.5)$. This implies that the Fourier series coefficients for $x_2(t)$ can be expressed in terms of the Fourier series coefficients of $x_1(t)$ (i.e., a_k). Assuming that b_k represents the Fourier series coefficients for $x_2(t)$, fill in the box below to determine b_k in terms of the Fourier series coefficients for a_k. (Hint: **<u>DO NOT perform integration here.</u>**)

$$x_1(t-0.5) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/2)k(t-0.5)}$$
$$= \sum_{k=-\infty}^{\infty} \underbrace{a_k e^{-j\frac{\pi}{2}k}}_{b_k} e^{j(2\pi/2)kt}$$

$b_k =$ $a_k e^{-j\frac{\pi}{2}k} = a_k (-j)^k$

(c) The periodic signal (from part (a)) is:

$$y(t) = x_1(t) + x_2(t) = x_1(t) + x_1(t - 0.5)$$

This implies that the Fourier series coefficients for $y(t)$ can be expressed in terms of the Fourier series coefficients for $x_1(t)$ (i.e., a_k). Assuming that c_k represents the Fourier series coefficients for $y(t)$, fill in the box below to determine c_k in terms of the Fourier series coefficients for a_k.
(Hint: **<u>DO NOT perform integration here.</u>**)

$$\Rightarrow x_1(t) + x_2(t) = \sum a_k e^{j(2\pi/2)kt} + \sum b_k e^{j(2\pi/2)kt}$$
$$= \sum \underbrace{\left(a_k + a_k e^{-j\frac{\pi}{2}k}\right)}_{c_k} e^{j(2\pi/2)kt}$$

$c_k =$ $a_k\left(1 + e^{-j\frac{\pi}{2}k}\right) = a_k\left(1 + (-j)^k\right)$