GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Fall 2012
Quiz #2

October 12, 2012

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section in the chart below (otherwise you lose 3 points!):

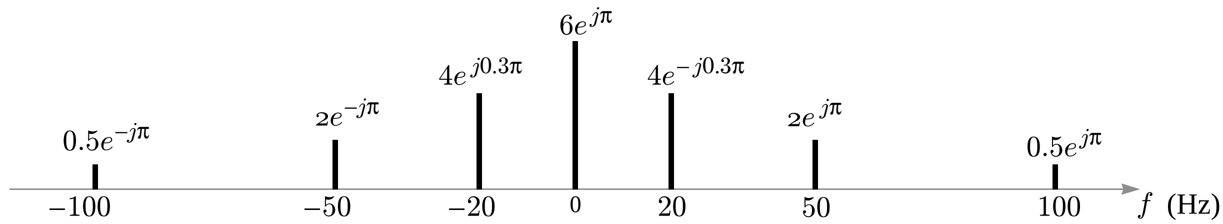
	Mon	Tue	Wed	Thu
9:30 – 11am				L06 (Fekri)
12 – 11:30pm		L07 (Al-Regib)		L08 (Fekri)
1:30 – 3pm		L09 (Al-Regib)		L10 (Rozell)
3 – 4:30pm	L01 (Juang)	L11 (Davenport)	L02 (Zajic)	L12 (Rozell)
4:30 – 6pm	L03 (Baxley)	L13 (Davenport)	L04 (Zajic)	
6 – 7:30pm	L05 (Baxley)			

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5" $\times$ 11") of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	–3	
Total		

PROB. Fall-12-Q2.1. Consider the periodic signal $x(t)$ whose spectrum is shown below:



(a) The fundamental period of $x(t)$ is $T_0 =$ _____ seconds.

The remainder of this problem considers the new signal that results after adding[1] a sinusoid:

$$y(t) = x(t) + A\cos(2\pi f_b t + \theta).$$

(b) The sum will not be periodic when $f_b =$ _____ Hz. (There are multiple answers; give one.)

(c) In order for the sum $y(t)$ to be periodic with a fundamental frequency of 50 Hz, we need:

$A =$ _____, $f_b =$ _____ Hz, and $\theta =$ _____ radians.

(d) If $A = 1$, $f_b = 100$ Hz, and $\theta = 0$, then the sum $y(t) = x(t) + \cos(200\pi t)$ can be represented using the Fourier series representation:

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t/T_0}$$

where $T_0 =$ _____ seconds, and

where the subset $\{a_0, a_1 \ldots a_5\}$ of the coefficients $\{a_k\}$ are (specify in *polar* form):

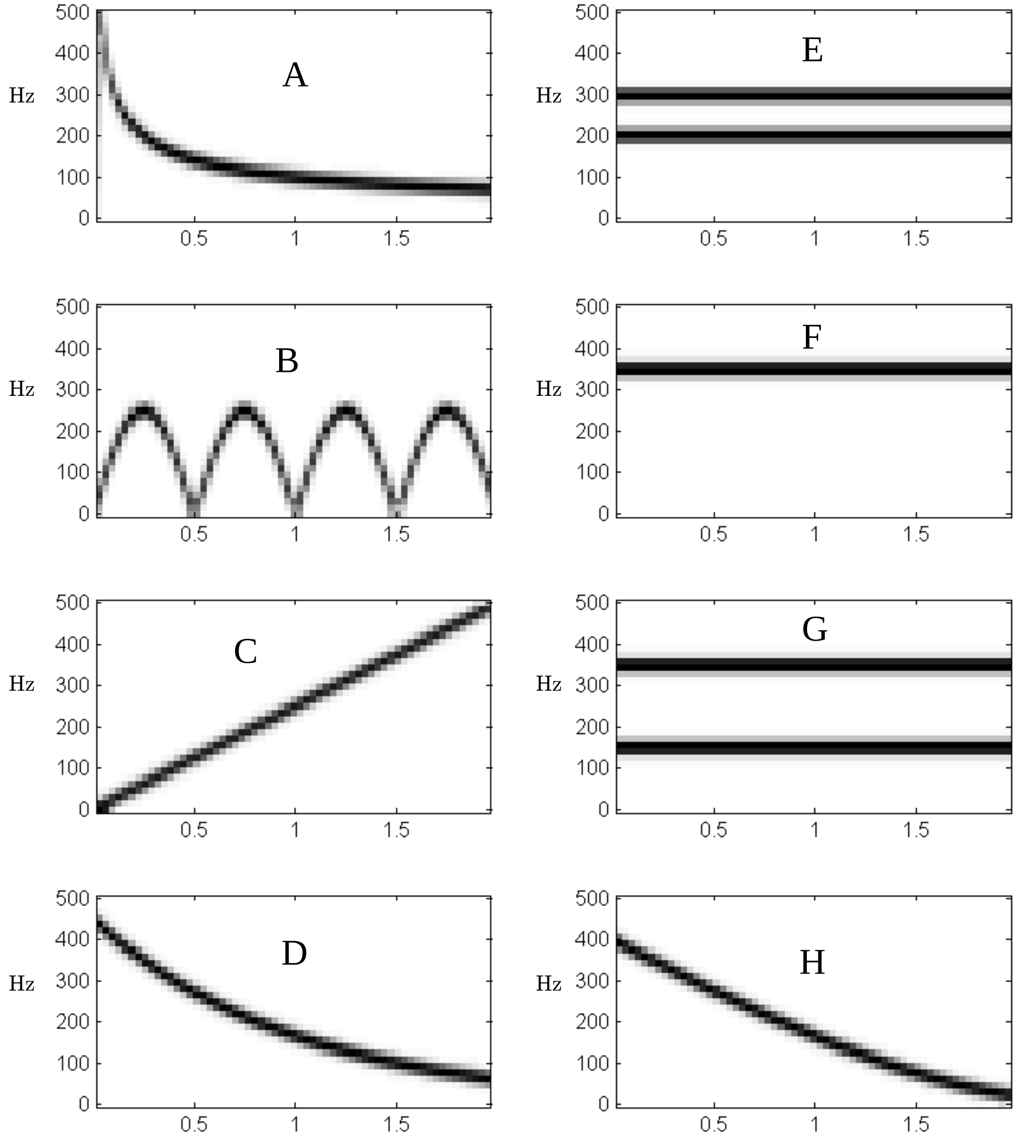
$a_0 =$ _____ $a_1 =$ _____ $a_2 =$ _____

$a_3 =$ _____ $a_4 =$ _____ $a_5 =$ _____ .

---

1. *Adding* a sinusoid with one phase is the same as *subtracting* a sinusoid with a different phase.

PROB. Fall-12-Q2.2. Shown below are the spectrograms (labeled A through H) for eight signals over the time range $0 \le t \le 2$. The frequency axis for each plot has units of Hz.

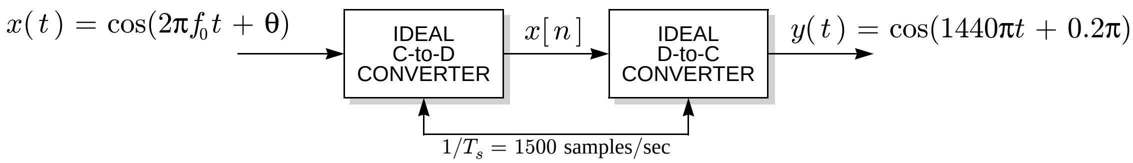


Identify which of the spectrograms corresponds to each of the signals described below by writing the corresponding letter (A through H) in the answer box.

(i)	☐	$x(t) = \cos(900\pi e^{-t})$	(v)	☐	$x(t) = \cos(250\cos(2\pi t))$
(ii)	☐	$x(t) = \cos(500\pi t)\cos(200\pi t)$	(vi)	☐	$x(t) = \cos(800\pi t + 4000\cos(0.2\pi t)))$
(iii)	☐	$x(t) = \cos(250\pi t^2)$	(vii)	☐	$x(t) = \cos(400\pi t - \pi/3) + \cos(600\pi t - \pi/3)$
(iv)	☐	$x(t) = \cos(400\pi\sqrt{t})$	(viii)	☐	$x(t) = \cos(700\pi t) + \cos(700\pi t + \pi/3)$

PROB. Fall-12-Q2.3.

(a) Suppose that a sinusoidal signal $x(t) = \cos(2\pi f_0 t + \theta)$ is ideally sampled and reconstructed, resulting in the sinusoidal output $y(t) = \cos(1440\pi t + 0.2\pi)$. Suppose that the sampling rate is 1500 samples/sec for both the C-to-D and D-to-C converters, as shown below:



There are many possible input signals of the form $x(t) = \cos(2\pi f_0 t + \theta)$ that might have resulted in this output $y(t)$, but only *four* have a frequency less than 3000 Hz. Name any three, taking care to restrict $0 < f_i < 3000$ Hz in all three cases:

$x_1(t) = \cos(2\pi f_1 t + \theta_1)$ where $f_1 =$ ☐ Hz and $\theta_1 =$ ☐ ,

$x_2(t) = \cos(2\pi f_2 t + \theta_2)$ where $f_2 =$ ☐ Hz and $\theta_2 =$ ☐ ,

$x_3(t) = \cos(2\pi f_3 t + \theta_3)$ where $f_3 =$ ☐ Hz and $\theta_3 =$ ☐ .

The remainder of this problem concerns the following piece of MATLAB code:

```
tt = 4:(1/2000):14;
xx = cos(2*pi*330*tt);
soundsc(xx, fsamp);
```

(b) If the duration of the sound coming from the speaker is 5 seconds, the value of `fsamp` must be

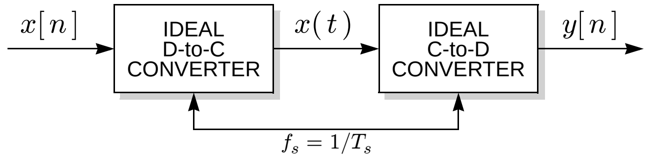
`fsamp` = ☐ .

(c) If a 990 Hz tone is heard coming from the speaker, the value of `fsamp` must be

`fsamp` = ☐ .

PROB. Fall-12-Q2.4. We often consider a C-to-D converter followed by an ideal D-to-C converter.

For a change of pace, this problem considers the same two blocks but in *reverse* order:



Assume that the input sequence is the discrete-time sinusoid $x[n] = 1.2\cos(0.02\pi n - 0.01\pi)$.

(a) If $f_s = 800$ samples/sec, then the output sequence is $y[n] = A\cos(\hat{\omega} n + \varphi)$ where:

$A =$ [ ], $\hat{\omega} =$ [ ], and $\varphi =$ [ ].

(b) Under what conditions on f_s
will the output sequence be identical to the input sequence (i.e., $y[n] = x[n]$)?

[ ]

(c) *Explain* your reasoning for your answer to part (b).

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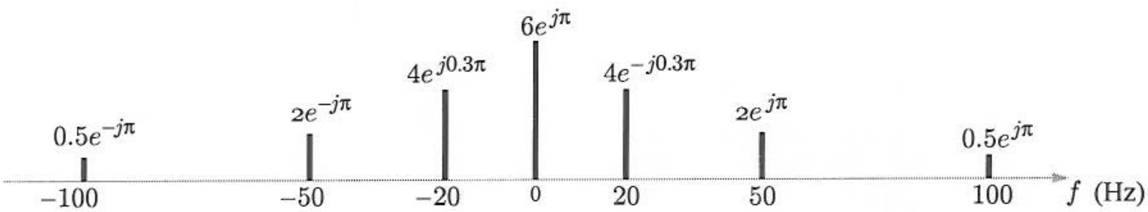
PROB. Fall-12-Q2.1. Consider the periodic signal $x(t)$ whose spectrum is shown below:



(a) The fundamental period of $x(t)$ is $T_0 =$ 0.1 seconds.

The remainder of this problem considers the new signal that results after adding[1] a sinusoid:

$$y(t) = x(t) + A\cos(2\pi f_b t + \theta).$$

(b) The sum will not be periodic when $f_b =$ $\sqrt{2}$ Hz. (There are multiple answers; give one.)

or anything irrational

(c) In order for the sum $y(t)$ to be periodic with a fundamental frequency of 50 Hz, we need:

$A =$ 8, $f_b =$ 20 Hz, and $\theta =$ 0.7π radians.

(cancel line @ 20 Hz)

(d) If $A = 1$, $f_b = 100$ Hz, and $\theta = 0$, then the sum $y(t) = x(t) + \cos(200\pi t)$ can be represented using the Fourier series representation:

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t/T_0}$$

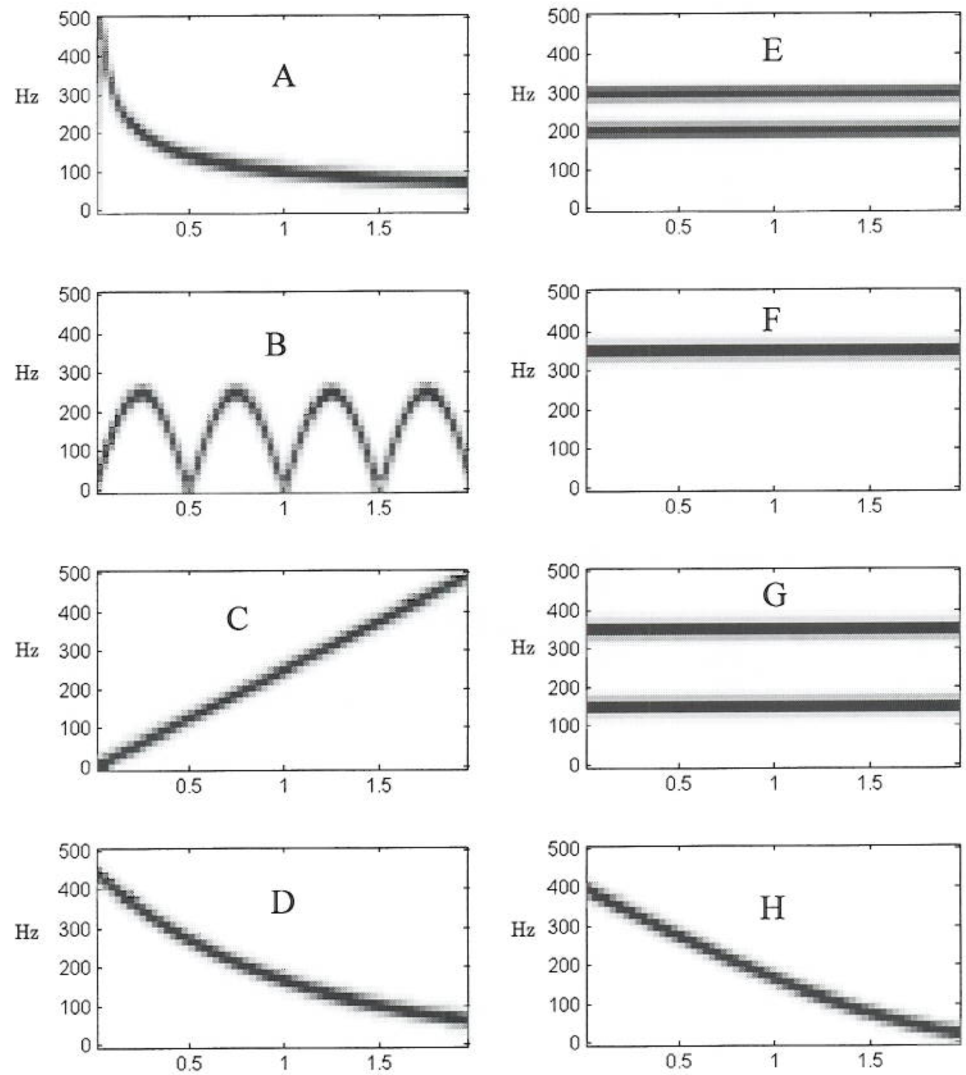
where $T_0 =$ 0.1 seconds, and

where the subset $\{a_0, a_1 \ldots a_5\}$ of the coefficients $\{a_k\}$ are (specify in *polar* form):

$a_0 =$	$6e^{j\pi}$	$a_1 =$	0	$a_2 =$	$4e^{-j0.3\pi}$
$a_3 =$	0	$a_4 =$	0	$a_5 =$	$2e^{j\pi}$

1. *Adding* a sinusoid with one phase is the same as *subtracting* a sinusoid with a different phase.

PROB. Fall-12-Q2.2. Shown below are the spectrograms (labeled A through H) for eight signals over the time range $0 \le t \le 2$. The frequency axis for each plot has units of Hz.



Identify which of the spectrograms corresponds to each of the signals described below by writing the corresponding letter (A through H) in the answer box.

(i) **D** $x(t) = \cos(900\pi e^{-t})$ → $-450e^{-t}$

(ii) **G** $x(t) = \cos(500\pi t)\cos(200\pi t)$ → @ 150, 350 Hz

(iii) **C** $x(t) = \cos(250\pi t^2)$ → $250t$

(iv) **A** $x(t) = \cos(400\pi\sqrt{t})$ → $\frac{100}{\sqrt{t}}$

(v) **B** $x(t) = \cos(250\cos(2\pi t))$ → $-250\sin(2\pi t)$

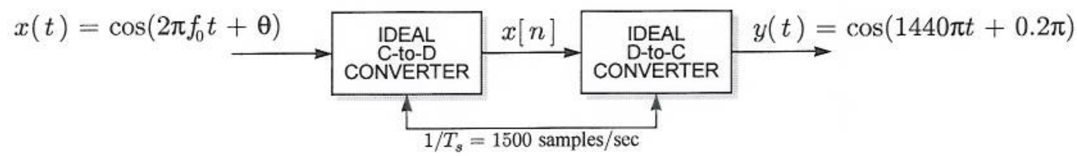
(vi) **H** $x(t) = \cos(800\pi t + 4000\cos(0.2\pi t))$ → $400 - 400\sin(0.2\pi t)$

(vii) **E** $x(t) = \cos(400\pi t - \pi/3) + \cos(600\pi t - \pi/3)$ → @ 200, 300 Hz

(viii) **F** $x(t) = \cos(700\pi t) + \cos(700\pi t + \pi/3)$ → @ 350 Hz

PROB. Fall-12-Q2.3.

(a) Suppose that a sinusoidal signal $x(t) = \cos(2\pi f_0 t + \theta)$ is ideally sampled and reconstructed, resulting in the sinusoidal output $y(t) = \cos(1440\pi t + 0.2\pi)$. Suppose that the sampling rate is 1500 samples/sec for both the C-to-D and D-to-C converters, as shown below:



There are many possible input signals of the form $x(t) = \cos(2\pi f_0 t + \theta)$ that might have resulted in this output $y(t)$, but only *four* have a frequency less than 3000 Hz. Name any three, taking care to restrict $0 < f_i < 3000$ Hz in all three cases:

$720 + \ell f_s$

$x_1(t) = \cos(2\pi f_1 t + \theta_1)$ where $f_1 =$ **720** Hz and $\theta_1 =$ **0.2π**,

$x_2(t) = \cos(2\pi f_2 t + \theta_2)$ where $f_2 =$ **2220** Hz and $\theta_2 =$ **0.2π**,

$x_3(t) = \cos(2\pi f_3 t + \theta_3)$ where $f_3 =$ **780** Hz and $\theta_3 =$ **-0.2π**.

OR $f_4 = 2280$ & $\theta_4 = -0.2\pi$

The remainder of this problem concerns the following piece of MATLAB code:

```
tt = 4:(1/2000):14;
xx = cos(2*pi*330*tt);
soundsc(xx, fsamp);
```

(b) If the duration of the sound coming from the speaker is 5 seconds, the value of `fsamp` must be

`fsamp` = **4000**.

$$f_{samp} = \frac{\text{length(xx)}}{\text{duration}} = \frac{20\,000}{5}$$

(c) If a 990 Hz tone is heard coming from the speaker, the value of `fsamp` must be

`fsamp` = **6000**.

$$f_{samp} = \frac{(2000)(990)}{(330)}$$

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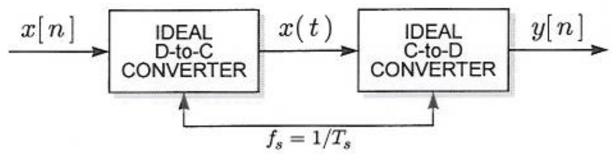

Assume that the input sequence is the discrete-time sinusoid $x[n] = 1.2\cos(0.02\pi n - 0.01\pi)$.

(a) If $f_s = 800$ samples/sec, then the output sequence is $y[n] = A\cos(\hat{\omega}n + \varphi)$ where:

$A =$ 1.2 , $\hat{\omega} =$ 0.02π , and $\varphi =$ -0.01π .

(b) Under what conditions on f_s will the output sequence be identical to the input sequence (i.e., $y[n] = x[n]$)?

(NO CONDITIONS)

(c) *Explain* your reasoning for your answer to part (b).

$y[n] = x[n]$ <u>always</u>.

By definition of an ideal D-to-C converter, sampling its output produces its input.