# GEORGIA INSTITUTE OF TECHNOLOGY 

SCHOOL of ELECTRICAL \& COMPUTER ENGINEERING
QUIZ \#2

DATE: 14-Oct-11
COURSE: ECE-2025

NAME:


GT username:

> (ex: gp.burdell3)

3 points 3 points
3 points
Recitation Section: Circle the date \& time when your Recitation Section meets (not Lab):

|  | L05:Tues-9:30am (Richards) | L06:Thur-9:30am (Casinovi) |  |
| :--- | :--- | :--- | :--- |
|  | L07:Tues-Noon (Richards) |  | L08:Thur-Noon (Casinovi) |
|  | L09:Tues-1:30pm (Chang) |  | L10:Thur-1:30pm (Coyle) |
| L01:M-3pm (Barry) | L11:Tues-3pm (Chang) | L02:W-3pm (Clements) | L12:Thur-3pm (Baxley) |
| L03:M-4:30pm (Barry) |  | L04:W-4:30pm (Clements) | L14:Thur-4:30pm (Baxley) |

- Write your name on the front page ONLY. DO NOT unstaple the test.
- Closed book, but a calculator is permitted.
- One page ( $8 \frac{1^{\prime \prime}}{} \times 11^{\prime \prime}$ ) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- You must write your answer in the space provided on the exam paper itself.

Only these answers will be graded. Circle your answers, or write them in the boxes provided. If more space is needed for scratch work, use the backs of previous pages.

| Problem | Value | Score |
| :---: | :---: | :---: |
| 1 | 25 |  |
| 2 | 25 |  |
| 3 | 25 |  |
| 4 | 25 |  |
| No/Wrong Rec | -3 |  |

## PROBLEM Fall-11-Q.2.1:

Shown below are spectrograms (labeled as $\mathbf{S 1} \mathbf{- S 6 )}$ for six signals. The (vertical) frequency axis for each plot has units of Hz ; the horizontal axis is time, $0 \leq t \leq 14 \mathrm{~s}$. For each signal description below, identify the corresponding spectrogram. Write each answer in the box provided.
(a) $\square$ $x(t)=\cos (-1700 \cos (2 \pi t / 42))$
(b) $\square$ $x(t)=\cos (2 \pi 40 t+400 \pi \exp (-t / 5))$
(c) $\square$ $x(t)=\cos (2 \pi 15 t) \cos (2 \pi 20 t)$
(d) $\square$ $x(t)=\cos (2 \pi 5 t+\pi / 4)+\cos (2 \pi 35 t)+\cos (2 \pi 5 t-3 \pi / 4)$
(e) $\square$ $x(t)=\cos \left(2.5 \pi t^{2}\right)$
(f) $\square$ $x(t)=\cos (2 \pi 20 t)+\cos (2 \pi 15 t)$






## PROBLEM Fall-11-Q.2.2:

Suppose that a periodic signal $x(t)$ is defined by the plot below (only the section $-3 \leq t \leq 3$ is shown).

(a) Determine the fundamental frequency of $x(t)$ in radians/second.

$$
\omega_{0}=\square
$$

(b) Since $x(t)$ is periodic, it has a Fourier series given by $x(t)=\sum_{k=-\infty}^{\infty} a_{k} e^{j \omega_{0} k t}$. Determine the numerical value of $a_{0}$.

$$
a_{0}=\square
$$

(c) Define a new signal $y(t)$ that is related to the signal above by the following formula:

$$
y(t)=4 \cos (4 \pi t / 3+\pi / 4)+2 x(t)-1 .
$$

This new signal is also periodic with the same period as $x(t)$, having a Fourier series $y(t)=\sum_{k=-\infty}^{\infty} b_{k} e^{j \omega_{0} k t}$. Fill in the table below with appropriate expressions for $b_{k}$. Do not try to calculate numeric answers here. Each $b_{k}$ should be written in terms of the coefficients $a_{k}$ for the signal $x(t)$ described above.

Signal: $z(t)$

| $b_{k}$ | Value |
| :---: | :---: |
| $b_{3}$ |  |
| $b_{2}$ |  |
| $b_{1}$ |  |
| $b_{0}$ |  |
| $b_{-1}$ |  |
| $b_{-2}$ |  |
| $b_{-3}$ |  |

## PROBLEM Fall-11-Q.2.3:



For parts (a) and (b) below, the input to the C/D converter is a signal $x(t)$ whose spectrum is shown here. The frequency $f$ is in hertz.

(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$.

$$
f_{\text {Nyquist }}=\square \mathrm{Hz}
$$

(b) If the sampling rate is $f_{s_{1}}=600 \mathrm{samples} / \mathrm{sec}$., plot all of the spectrum components of the discrete-time signal $x[n]$ over the range of frequencies $-\pi \leq \hat{\omega} \leq \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.

(c) Note that in the diagram above, $f_{s_{1}}$ may not be equal to $f_{s_{2}}$. We consider such a situation in this part of the problem. Suppose that a student writes the following Matlab code to generate a sine wave, where the variable assignment $f f$ has been left for you to fill in:

```
ff = ?;
tt = 0:1/2400:10000;
xx = sqrt(pi) * sin(2*pi*ff*tt-pi/4);
soundsc(xx,1600);
```

Determine the value of $f f$ that should be used to play the vector xx as a 400 Hz sinusoid.

$$
\mathrm{ff}=\square
$$

## PROBLEM Fall-11-Q.2.4:

The diagram in the figure below depicts a cascade connection of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

(a) Suppose that System $\# 1$ is an FIR filter whose filter coefficients are $\left\{b_{k}\right\}=\{0,0,-0.4,0,0.2\}$. Determine the impulse response, $h_{1}[n]$, of the first system. Give your answer as a stem plot.

(b) Suppose that System \#2 is defined by the Matlab code below, where the variable xx is the signal $x(t)$ and the variable yy is the signal $y(t)$ :
yy $=\operatorname{conv}\left(\left[\begin{array}{lll}1 & 0 & -1\end{array}\right], x x\right)$;
Write the difference equation that relates the output $y[n]$ of this second system to its input $x[n]$.
(c) Using the descriptions of System \#1 and System \#2 in parts (a) and (b) above, determine the impulse response $h[n]$ of the overall cascaded system. Give your answer as a sum of shifted deltas.

# GEORGIA INSTITUTE OF TECHNOLOGY 

SCHOOL of ELECTRICAL \& COMPUTER ENGINEERING
QUIZ \#2
name: ANSWER KEY
LAST,
FIRST

GT username: VERSION \#1
(ex: gpburdell3)

3 points
3 points
3 points
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| L11:Tues-3pm (Chang) | L02:W-3pm (Clements) | L12:Thur-3pm (Baxley) |
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## PROBLEM Fall-11-Q.2.1:

Shown below are spectrograms (labeled as $\mathbf{S 1} \mathbf{- S 6 )}$ for six signals. The (vertical) frequency axis for each plot has units of Hz ; the horizontal axis is time, $0 \leq t \leq 14 \mathrm{~s}$. For each signal description below, identify the corresponding spectrogram. Write each answer in the box provided.
(a)

S6 $x(t)=\cos (-1700 \cos (2 \pi t / 42))$
(b)

S4 $x(t)=\cos (2 \pi 40 t+400 \pi \exp (-t / 5))$
(c)

S5 $x(t)=\cos (2 \pi 15 t) \cos (2 \pi 20 t)$
(d)

S1 $x(t)=\cos (2 \pi 5 t+\pi / 4)+\cos (2 \pi 35 t)+\cos (2 \pi 5 t-3 \pi / 4)$
(e) S3 $x(t)=\cos \left(2.5 \pi t^{2}\right)$
(f) $\mathbf{S 2} x(t)=\cos (2 \pi 20 t)+\cos (2 \pi 15 t)$







## PROBLEM Fall-11-Q.2.2:

Suppose that a periodic signal $x(t)$ is defined by the plot below (only the section $-3 \leq t \leq 3$ is shown).

(a) Determine the fundamental frequency of $x(t)$ in radians/second.

$$
\omega_{0}=2 \pi / 3
$$

(b) Since $x(t)$ is periodic, it has a Fourier series given by $x(t)=\sum_{k=-\infty}^{\infty} a_{k} e^{j \omega_{0} k t}$. Determine the numerical value of $a_{0}$.

$$
a_{0}=1
$$

(c) Define a new signal $y(t)$ that is related to the signal above by the following formula:

$$
y(t)=4 \cos (4 \pi t / 3+\pi / 4)+2 x(t)-1
$$

This new signal is also periodic with the same period as $x(t)$, having a Fourier series $y(t)=\sum_{k=-\infty}^{\infty} b_{k} e^{j \omega_{0} k t}$. Fill in the table below with appropriate expressions for $b_{k}$. Do not try to calculate numeric answers here. Each $b_{k}$ should be written in terms of the coefficients $a_{k}$ for the signal $x(t)$ described above.

| Signal: $z(t)$ |  |
| :---: | :---: |
| $b_{k}$ | Value |
| $b_{3}$ | $2 a_{3}$ |
| $b_{2}$ | $2 a_{2}+2 e^{j \pi / 4}$ |
| $b_{1}$ | $2 a_{1}$ |
| $b_{0}$ | $2 a_{0}-1$ |
| $b_{-1}$ | $2 a_{-1}$ |
| $b_{-2}$ | $2 a_{-2}+2 e^{-j \pi / 4}$ |
| $b_{-3}$ | $2 a_{-3}$ |

PROBLEM Fall-11-Q.2.3:


For parts (a) and (b) below, the input to the C/D converter is a signal $x(t)$ whose spectrum is shown here. The frequency $f$ is in hertz.

(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$.

$$
f_{\text {Nyquist }}=1350 \mathrm{~Hz}
$$

(b) If the sampling rate is $f_{s_{1}}=600$ samples/sec., plot all of the spectrum components of the discrete-time signal $x[n]$ over the range of frequencies $-\pi \leq \hat{\omega} \leq \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.

(c) Note that in the diagram above, $f_{s_{1}}$ may not be equal to $f_{s_{2}}$. We consider such a situation in this part of the problem. Suppose that a student writes the following MATLAB code to generate a sine wave, where the variable assignment $f f$ has been left for you to fill in:

```
ff = ?;
tt = 0:1/2400:10000;
xx = sqrt(pi) * sin(2*pi*ff*tt-pi/4);
soundsc(xx,1600);
```

Determine the value of $f f$ that should be used to play the vector $x x$ as a 400 Hz sinusoid.

$$
f f=600
$$

## PROBLEM Fall-11-Q.2.4:

The diagram in the figure below depicts a cascade connection of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

(a) Suppose that System \#1 is an FIR filter whose filter coefficients are $\left\{b_{k}\right\}=\{0,0,-0.4,0,0.2\}$. Determine the impulse response, $h_{1}[n]$, of the first system. Give your answer as a stem plot.


Plot is of $h_{1}[n]=-0.4 \delta[n-2]+0.2 \delta[n-4]$
(b) Suppose that System \#2 is defined by the MATLAB code below, where the variable xx is the signal $x(t)$ and the variable yy is the signal $y(t)$ :
$y y=\operatorname{conv}\left(\left[\begin{array}{lll}1 & 0 & -1\end{array}\right], x x\right)$;

Write the difference equation that relates the output $y[n]$ of this second system to its input $x[n]$.
$y[n]=x[n]-x[n-2]$
(c) Using the descriptions of System \#1 and System \#2 in parts (a) and (b) above, determine the impulse response $h[n]$ of the overall cascaded system. Give your answer as a sum of shifted deltas.
$h[n]=-0.4 \delta[n-2]+0.6 \delta[n-4]-0.2 \delta[n-6]$

