ECE 2026

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

JUNE 22, 2026

Name: ______________________________ FIRST ______________________________ LAST

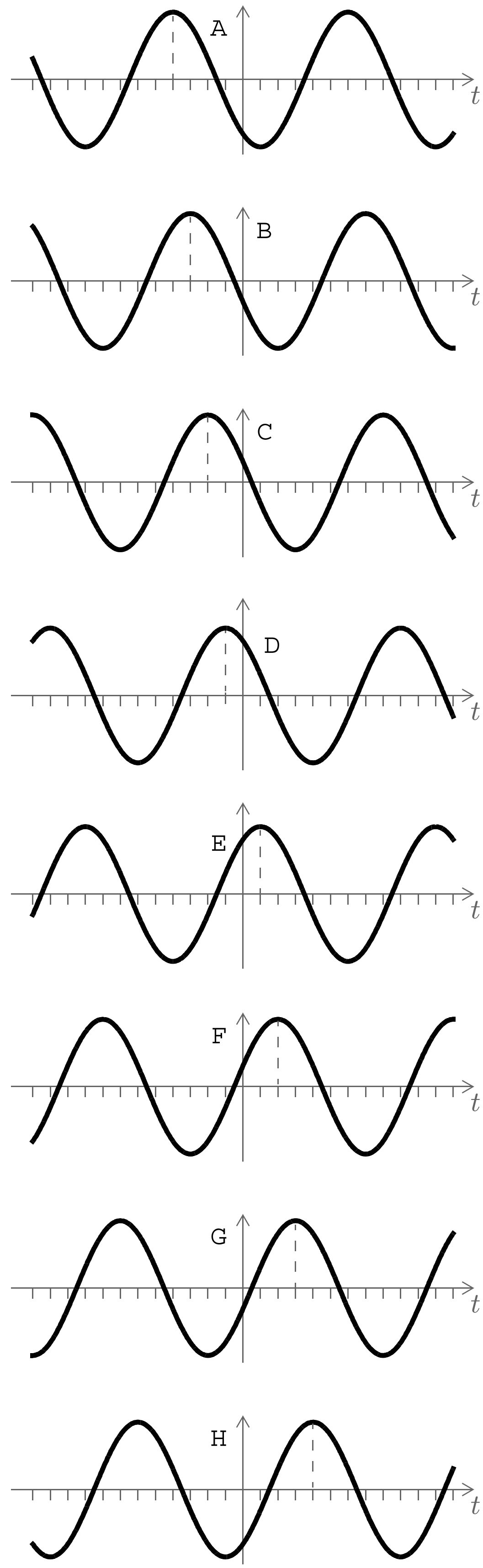
1. The quiz is closed book, closed notes,
   except for one double-sided handwritten sheets of notes.
2. No electronics (no watches/calculators/readers/phones/etc.)
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. A blank sheet is attached in case you need more room.
6. Only the *front* side of each page will be scanned and graded, use the back sides as scrap only.
7. Express all angles as a fraction of π. For example, write 0.1π instead of $18°$ or 0.3142 radians.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
TOTAL:	100	

PROB. Su26-Q1.1.

Consider the eight sinusoids shown in the figure, labeled A through H; they all have the same frequency and amplitude, they differ only in their phases. The axes are not labeled.

Match each equation below to its corresponding plot. Indicate answers by writing a letter (from $\{A ... H\}$) in each answer box:



(i) $\boxed{}$ $\sin(2\pi(t - 3) + 0.7\pi)$

(ii) $\boxed{}$ $\cos(2\pi t + 202.6\pi)$

(iii) $\boxed{}$ $2\cos^2(\pi t - 88.8\pi) - 1$

(iv) $\boxed{}$ $-0.5je^{j(2\pi t + 0.3\pi)} + 0.5je^{-j(2\pi t + 0.3\pi)}$

(v) $\boxed{}$ $-\cos(2\pi(t + 0.3))$

(vi) $\boxed{}$ $\cos(2\pi(t - 2026.4))$

(vii) $\boxed{}$ $\cos(2\pi(t - 0.6))$

(viii) $\boxed{}$ $\sin(1.1\pi - 2\pi t)$

PROB. Su26-Q1.2. (Parts (a) and (b) are unrelated.)

(a) Find two integers N, both from the set $\{1,\ 2,\ \ldots\ 100\}$, for which $\sum_{k=-N}^{N} e^{jk2\pi/15} = 0$.

$N =$ $\boxed{}$, $\in\{1...100\}$

$N =$ $\boxed{}$. $\in\{1...100\}$

(b) Sampling $x(t) = \cos(24\pi t)$ results in $x[n] = x(\frac{n}{f_s})$.

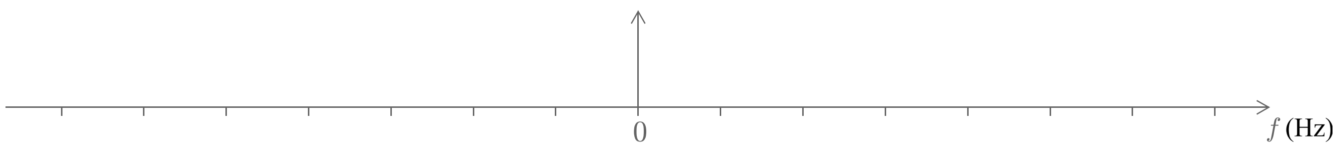
Find the smallest sampling rate f_s so that both of the following conditions are met:

- f_s is a positive integer
- $x[n]$ is periodic with fundamental period $N_0 = 21$

$f_s =$ $\boxed{}$ Hz. positive integer

PROB. Su26-Q1.3. (Parts (a) and (b) are unrelated.)

(a) In the space below, carefully sketch the spectrum for $x(t) = 12\cos(4\pi t)\sin(44\pi t + 0.6\pi)$, taking care to label both the frequency and complex coefficient for each spectral line:

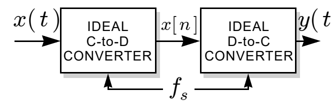


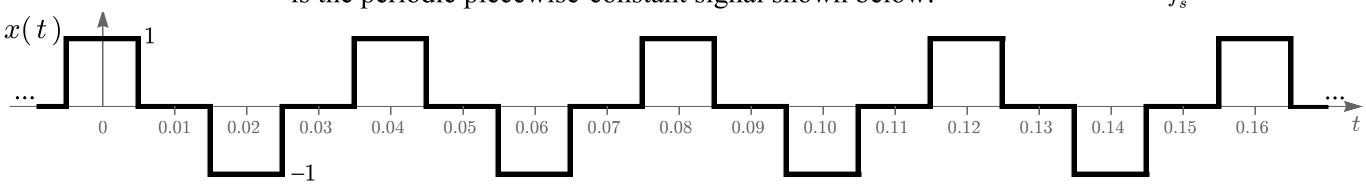
(b) Find the unspecified parameters $A > 0$ and $F > 0$ and $\varphi \in (-\pi, \pi]$ so that the following equation is true for all time t:

$$\cos(2026\pi t + 2\varphi) + \mathrm{Re}\{(je^{j2\varphi})e^{j2\pi F t}\} = A\cos(2026\pi t + \varphi).$$

$A = \boxed{} > 0,$ $F = \boxed{\text{ Hz}} > 0,$ $\varphi = \boxed{} \in (-\pi, \pi].$

PROB. Su26-Q1.4. Consider the sampling/reconstruction system with input $x(t)$ and output $y(t)$ shown here, where the input $x(t)$ is the periodic piecewise-constant signal shown below:





(a) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ ☐ Hz.

(b) The dc coefficient in the Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$ is $a_0 =$ ☐ .

(c) TRUE ☐ FALSE ☐ A sampling rate of $f_s = 1{,}000{,}000{,}000$ Hz (1 GHz) is fast enough to ensure that $y(t) = x(t)$.

(d) For each sampling rate below, specify an equation for the corresponding output $y(t)$.
Simplify as much as possible. (For example, use *standard form* for sinusoids.)

(i) $f_s = 100$ Hz $\Rightarrow y(t) =$ ☐ .

(ii) $f_s = 50$ Hz $\Rightarrow y(t) =$ ☐ .

(iii) $f_s = 25$ Hz $\Rightarrow y(t) =$ ☐ .

(iv) $f_s = \dfrac{100}{3}$ Hz $\Rightarrow y(t) =$ ☐ .

(v) $f_s = 20$ Hz $\Rightarrow y(t) =$ ☐ .

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ECE 2026

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

JUNE 22, 2026

Name: SOLUTIONS (FIRST) VERSION A (LAST)

1. The quiz is closed book, closed notes,
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7. Express all angles as a fraction of π. For example, write 0.1π instead of $18°$ or 0.3142 radians.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
TOTAL:	100	

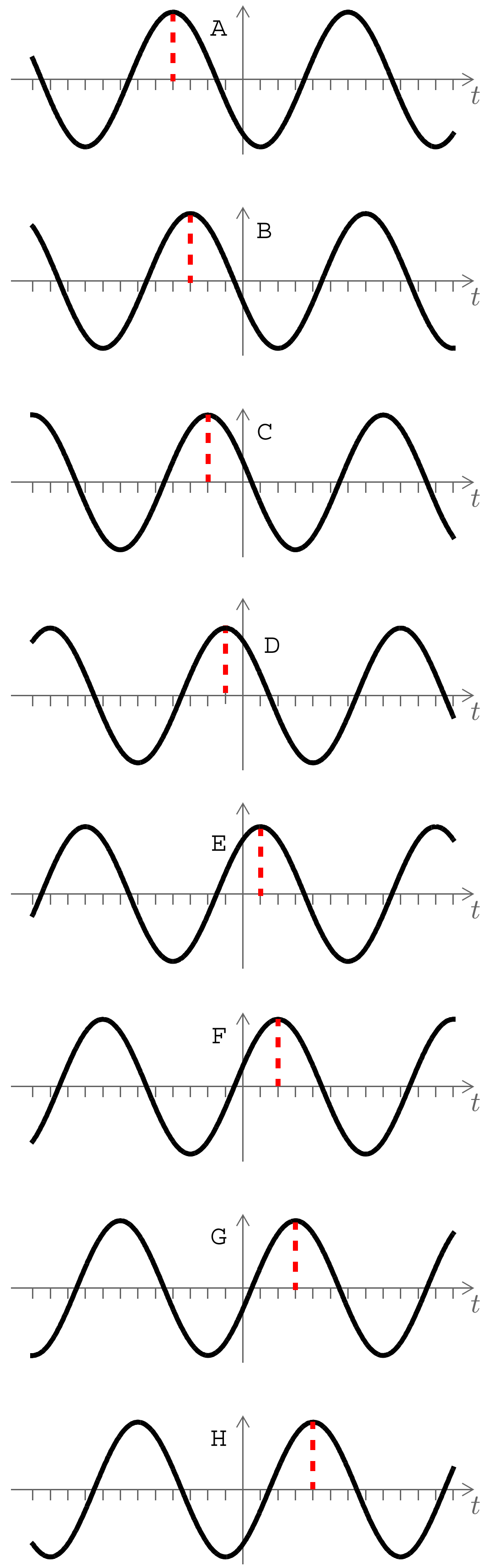
All sinusoids have frequency $f_0 = 1$ Hz, period $T_0 = 1$ second:

Manipulate each to the form $\cos(2\pi(t - t_0))$ where $|t_0| < 0.5$, then look for peak at t_0

PROB. Su26-Q1.1.

Consider the eight sinusoids shown in the figure, labeled A through H; they all have the same frequency and amplitude, they differ only in their phases. The axes are not labeled.

Match each equation below to its corresponding plot. Indicate answers by writing a letter (from {A ... H}) in each answer box:



(i) **D** $\sin(2\pi(t-3) + 0.7\pi) = \cos(2\pi(t + 0.1))$

(ii) **B** $\cos(2\pi t + 202.6\pi) = \cos(2\pi(t + 0.3))$

(iii) **C** $2\cos^2(\pi t - 88.8\pi) - 1 = \cos(2\pi(t + 0.2))$

(iv) **E** $-0.5je^{j(2\pi t + 0.3\pi)} + 0.5je^{-j(2\pi t + 0.3\pi)}$ $= \cos(2\pi(t - 0.1))$

(v) **F** $-\cos(2\pi(t + 0.3)) = \cos(2\pi(t - 0.2))$

(vi) **H** $\cos(2\pi(t - 2026.4)) = \cos(2\pi(t - 0.4))$

(vii) **A** $\cos(2\pi(t - 0.6)) = \cos(2\pi(t + 0.4))$

(viii) **G** $\sin(1.1\pi - 2\pi t) = \cos(2\pi(t - 0.3))$

PROB. Su26-Q1.2. (Parts (a) and (b) are unrelated.)

(a) Find two integers N, both from the set $\{1, 2, \ldots 100\}$, for which $\sum_{k=-N}^{N} e^{jk2\pi/15} = 0$.

The 15-th roots of unity sum to zero: $\sum_k e^{jk2\pi/15} = 0$

when the sum is over $k \in \{0, \ldots 14\}$

or more generally over any block of 15 consecutive integers.

$N = \boxed{7}$, $\in\{1...100\}$

$N = \boxed{22}$. $\in\{1...100\}$

$\Rightarrow$ the sum in question $\sum_{k=-N}^{N} e^{jk2\pi/15}$ will be zero when

the (odd) number of terms in the sum $(2N+1)$ is a multiple of 15:

$$
\begin{aligned}
2N+1 &= 15 \Rightarrow N = 7 \\
2N+1 &= 45 \Rightarrow N = 22 \\
2N+1 &= 75 \Rightarrow N = 37 \\
2N+1 &= 105 \Rightarrow N = 52 \\
2N+1 &= 135 \Rightarrow N = 67 \\
2N+1 &= 165 \Rightarrow N = 82 \\
2N+1 &= 195 \Rightarrow N = 97
\end{aligned}
$$

(seven answers < 100)

(b) Sampling $x(t) = \cos(24\pi t)$ results in $x[n] = x(\frac{n}{f_s})$.

Find the smallest sampling rate f_s so that both of the following conditions are met:

- f_s is a positive integer
- $x[n]$ is periodic with fundamental period $N_0 = 21$

$f_s = \boxed{63}$ Hz. positive integer

The period will be 21 when the digital frequency $\hat{\omega} = \frac{24\pi}{fs}$ has the form $2\pi\frac{k}{21}$ not multiple of 3 or 7

i.e., when $f_s = \frac{(12)(21)}{k} = \frac{(2)(2)(3)(3)(7)}{k}$.

The <u>biggest</u> k (minimizing f_s) that is not a multiple of 3 or 7

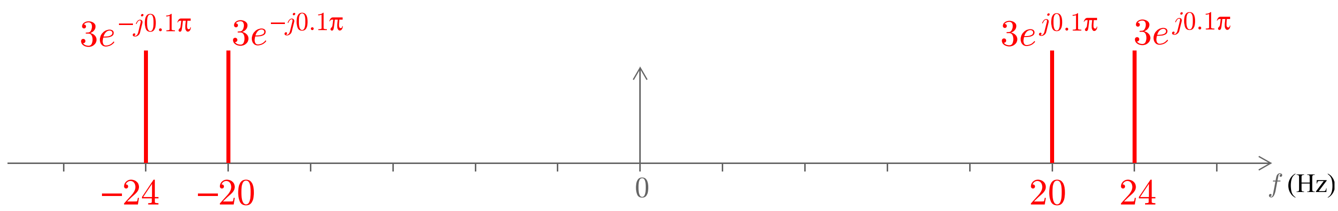
and keeps f_s an integer is $k = 4 \Rightarrow f_s = 63$.

PROB. Su26-Q1.3. (Parts (a) and (b) are unrelated.)

(a) In the space below, carefully sketch the spectrum for $x(t) = 12\cos(4\pi t)\sin(44\pi t + 0.6\pi)$, taking care to label both the frequency and complex coefficient for each spectral line:

$$
\begin{aligned}
x(t) &= 12\cos(4\pi t)\cos(44\pi t + 0.1\pi) \\
&= 6\cos(40\pi t + 0.1\pi) + 6\cos(48\pi t + 0.1\pi) \\
&= 3e^{j0.1\pi}e^{j40\pi t} + 3e^{-j0.1\pi}e^{-j40\pi t} + 3e^{j0.1\pi}e^{j48\pi t} + 3e^{-j0.1\pi}e^{-j48\pi t}
\end{aligned}
$$

Each complex exponential in the sum leads to a line in the spectrum:



(b) Find the unspecified parameters $A > 0$ and $F > 0$ and $\varphi \in (-\pi, \pi]$ so that the following equation is true for all time t:

$$\cos(2026\pi t + 2\varphi) + \mathrm{Re}\{(je^{j2\varphi})e^{j2\pi Ft}\} = A\cos(2026\pi t + \varphi).$$

$A = \boxed{\sqrt{2}} > 0$, $F = \boxed{1013 \text{ Hz}} > 0$, $\varphi = \boxed{-0.25\pi} \in (-\pi, \pi]$.

The corresponding phasor equation: $e^{j2\varphi} + je^{j2\varphi} = Ae^{j\varphi}$

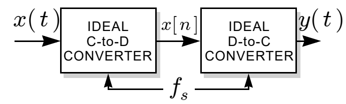
Multiply both sides by $e^{-j2\varphi}$: $1 + j = Ae^{-j\varphi}$

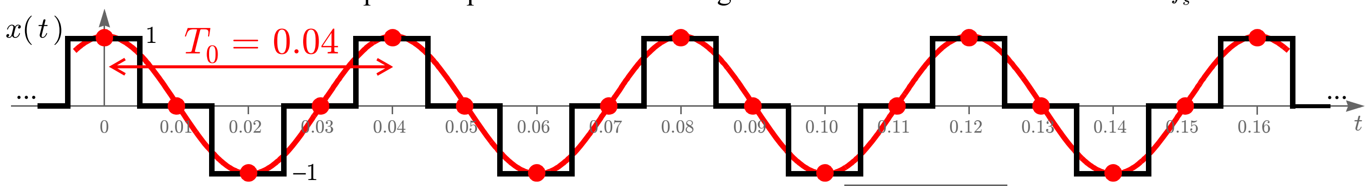
Conjugate both sides: $Ae^{j\varphi} = 1 - j = \sqrt{2}\,e^{-j0.25\pi}$

Procedure for (d) is shown explictly in the figure for case (i) but is the same for all 5 cases:

(1) draw dots at times $t = n/f_s$;
(2) identify sequence as a digital sinusoid;
(3) connect dots by substituting $n = f_s t$

PROB. Su26-Q1.4. Consider the sampling/reconstruction system with input $x(t)$ and output $y(t)$ shown here, where the input $x(t)$ is the periodic piecewise-constant signal shown below:





(a) The input $x(t)$ is periodic with fundamental frequency $f_0 = \boxed{25}$ Hz. $= \dfrac{1}{T_0}$

(b) The dc coefficient in the Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$ is $a_0 = \boxed{0}$.

(c) TRUE ☐ FALSE ☒ A sampling rate of $f_s = 1{,}000{,}000{,}000$ Hz (1 GHz) is fast enough to ensure that $y(t) = x(t)$.

$x(t)$ not bandlimited, it doesn't "have" an $f_{\max}$, can't sample fast enough

(d) For each sampling rate below, specify an equation for the corresponding output $y(t)$.
Simplify as much as possible. (For example, use *standard form* for sinusoids.)

(i) $f_s = 100$ Hz

$\Rightarrow x[n] = \ldots\ \underline{1}\ 0\ -1\ 0\ 1\ 0\ -1\ 0\ \ldots$

$= \cos(0.5\pi n)$

$\Rightarrow y(t) = \cos(0.5\pi f_s t)$

$\Rightarrow y(t) = \boxed{\cos(50\pi t)}$.

(ii) $f_s = 50$ Hz

$\Rightarrow x[n] = \ldots\ \underline{1}\ -1\ \ 1\ -1\ \ 1\ -1\ \ 1\ \ldots$

$= \cos(\pi n)$

$\Rightarrow y(t) = \cos(\pi f_s t)$

$\Rightarrow y(t) = \boxed{\cos(50\pi t)}$.

(iii) $f_s = 25$ Hz

$\Rightarrow x[n] = \ldots\ \underline{1}\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ldots$

$\Rightarrow y(t) = 1$

$\Rightarrow y(t) = \boxed{1}$.

(iv) $f_s = \dfrac{100}{3}$ Hz

$\Rightarrow x[n] = \ldots\ \underline{1}\ 0\ -1\ 0\ 1\ 0\ -1\ 0\ \ldots$

$= \cos(0.5\pi n)$

$\Rightarrow y(t) = \cos(0.5\pi f_s t)$

$\Rightarrow y(t) = \boxed{\cos(\frac{50\pi}{3} t)}$.

(v) $f_s = 20$ Hz

$\Rightarrow x[n] = \cos(0.5\pi n)$

$\Rightarrow y(t) = \cos(0.5\pi f_s t)$

$\Rightarrow y(t) = \boxed{\cos(10\pi t)}$.

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