# GEORGIA INSTITUTE OF TECHNOLOGY SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING 

ECE 2026 - Summer 2024
Quiz \#1
June 17, 2024

NAME: $\qquad$ (FIRST) (LAST)

GT username: $\qquad$

## Important Notes:

- Do not unstaple the test.
- Closed book, except for one two-sided page ( $8.5^{\prime \prime} \times 11^{\prime \prime}$ ) of hand-written notes.
- No calculators or other electronics (no smartphones/readers/watches/tablets/laptops/etc.)
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of $\pi$. For example, write $0.1 \pi$ as opposed to $18^{\circ}$ or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the provided answer boxes.
- Do not write on the backs of pages, only the fronts will be graded.

| Problem | Value | Score Earned |
| :---: | :---: | :---: |
| 1 | 25 |  |
| 2 | 25 |  |
| 3 | 25 |  |
| 4 | 25 |  |
| Total |  |  |

PROB. Su24-Q1.1. Let $z_{1}=0.9 e^{j 2 \pi / 3}$ and $z_{2}=0.9 e^{-j \pi / 20}$.
Match each complex number listed below to its location in the complex plane. Indicate answer by writing a letter (from labeled A through $T$ ) into each answer box:


## PROB. Su24-Q1.2.

If the following equation is valid for all time $t$ :

$$
4 \cos (230 \pi t+0.2 \pi)+B \cos (240 \pi t) \cos (2 \pi F t+\theta)=A \cos \left(2 \pi f_{0} t+\varphi\right),
$$

and if the unspecified parameters are all positive, then it must be that:

$$
B=\square>0, \quad F=\square>0, \quad \theta=\square \in(-\pi, \pi],
$$

$$
A=\square>0, \quad f_{0}=\square>0, \quad \text { and } \varphi=\square \in(-\pi, \pi] .
$$

## PROB. Su24-Q1.3.

Shown on the right are ten spectrogram plots, labeled A through J.

Match each spectrogram plot to the corresponding signal listed at the bottom of the page.

Indicate answers by writing a letter (from A to J) into each answer box.
$\square x_{1}(t)=\cos (2000 \pi t)+\cos (3000 \pi t)$
$\square x_{2}(t)=16 \cos (400 \pi t) \cos (2400 \pi t)$
$\square x_{3}(t)=\sin \left(2 \pi\left(1000 t+200 t^{2}\right)\right)$
$\square x_{4}(t)=\sin \left(1000 \pi(t-1)^{2}\right)$
$\square x_{5}(t)=\cos \left(500 \pi(t-2)^{2}\right)$










$\square x_{6}(t)=\cos \left(2 \pi\left(-400 t-200 e^{t}\right)\right)$
$\square x_{7}(t)=\cos \left(2 \pi\left(1800 t+1200 e^{-t}\right)\right)$
$\square x_{8}(t)=3000 \cos \left(3000(t-1)^{3}\right)$
$\square x_{9}(t)=\pi^{2} \cos \left(200 t\left(32-t^{3}\right)\right)$
$\square x_{10}(t)=\sin (2000 \pi t)+\cos (2000 \pi t+0.3 \pi)$

PROB. Su24-Q1.4. Consider the sampling/reconstruction system with input $x(t)$ and $y(t)$ shown here, and assume that the spectrum of the input signal $x(t)$ is as shown below:

(a) The input $x(t)$ is periodic with fundamental frequency $f_{0}=\square \mathrm{Hz}$.
(b) To ensure that $y(t)=x(t)$, the sampling rate must satisfy $f_{\mathrm{s}}>\square$ samples/second.
(c) When $f_{s}=330 \mathrm{~Hz}$, the output $y(t)$ is periodic with fundamental frequency $f_{0}=\square \mathrm{Hz}$.
(d) When $f_{s}=71 \mathrm{~Hz}$, the output $y(t)$ is periodic with fundamental frequency $f_{0}=\square \mathrm{Hz}$.
(e) Find $f_{s}$ so that the output can be written as $y(t)=A \cos (240 \pi t+\varphi)$, and specify resulting $A$ and $\varphi$ :

$$
f_{s}=\square>{ }_{\text {samples } / \mathrm{s}}>0, \quad A=\square \in(-\pi, \pi] .
$$

# GEORGIA INSTITUTE OF TECHNOLOGY SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING 

ECE 2026 - Summer 2024
Quiz \#1
June 17, 2024

## VERSION A

NAME:
(FIRST)
(LAST)
GT username: $\qquad$

## Important Notes:

- Do not unstaple the test.
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- No calculators or other electronics (no smartphones/readers/watches/tablets/laptops/etc.)
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of $\pi$. For example, write $0.1 \pi$ as opposed to $18^{\circ}$ or 0.3142 radians.
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| :---: | :---: | :---: |
| 1 | 25 |  |
| 2 | 25 |  |
| 3 | 25 |  |
| 4 | 25 |  |
| Total |  |  |

PROB. Su24-Q1.1. Let $z_{1}=0.9 e^{j 2 \pi / 3}$ and $z_{2}=0.9 e^{-j \pi / 20}$.
Match each complex number listed below to its location in the complex plane. Indicate answer by writing a letter (from labeled A through $T$ ) into each answer box:


## PROB. Su24-Q1.2.

If the following equation is valid for all time $t$ :

$$
4 \cos (230 \pi t+0.2 \pi)+B \cos (240 \pi t) \cos (2 \pi F t+\theta)=A \cos \left(2 \pi f_{0} t+\varphi\right)
$$

and if the unspecified parameters are all positive, then it must be that:

$$
\begin{aligned}
& B=\gg 0, \quad F=\int_{\mathrm{Hz}} \gg 0, \quad \theta=0.8 \pi \quad \in(-\pi, \pi], \\
& A=4>0, \quad f_{0}={ }^{4} \quad 125_{\mathrm{Hz}}>0, \quad \text { and } \varphi=0.8 \pi \quad \in(-\pi, \pi] .
\end{aligned}
$$

Spectrum for left-hand side of equation looks like this:

$\Rightarrow$ Cancellation when $F=5 \mathrm{~Hz}$ and $B=8$ and $\theta=0.8 \pi$, yielding only a sinusoid at $f_{0}=125 \mathrm{~Hz}$ :


$$
\Rightarrow A=0.5 B=4 \text { and } \varphi=\theta=0.8 \pi
$$

## PROB. Su24-Q1.3.

Shown on the right are ten spectrogram plots, labeled A through J.

Match each spectrogram plot to the corresponding signal listed at the bottom of the page.

Indicate answers by writing a letter (from A to J) into each answer box.










$\mathrm{D} x_{6}(t)=\cos \left(2 \pi\left(-400 t-200 e^{t}\right)\right)$
$\mathrm{G} x_{7}(t)=\cos \left(2 \pi\left(1800 t+1200 e^{-t}\right)\right)$
$\square x_{8}(t)=3000 \cos \left(3000(t-1)^{3}\right)$
$\mathrm{E} \quad x_{4}(t)=\sin \left(1000 \pi(t-1)^{2}\right)$
$\mathrm{B} \quad x_{5}(t)=\cos \left(500 \pi(t-2)^{2}\right)$
$\mathrm{H} \quad x_{1}(t)=\cos (2000 \pi t)+\cos (3000 \pi t)$

A $x_{2}(t)=16 \cos (400 \pi t) \cos (2400 \pi t)$
$\mathrm{F} x_{3}(t)=\sin \left(2 \pi\left(1000 t+200 t^{2}\right)\right)$
$\mathrm{I} x_{9}(t)=\pi^{2} \cos \left(200 t\left(32-t^{3}\right)\right)$
$\mathrm{C} x_{10}(t)=\sin (2000 \pi t)+\cos (2000 \pi t+0.3 \pi)$

PROB. Su24-Q1.4. Consider the sampling/reconstruction system with input $x(t)$ and $y(t)$ shown here, and assume that the spectrum of the input signal $x(t)$ is as shown below:

(a) The input $x(t)$ is periodic with fundamental frequency $f_{0}=\square 60 \mathrm{~Hz}$.

$$
f_{0}=\operatorname{gcd}(120,180)
$$

(b) To ensure that $y(t)=x(t)$, the sampling rate must satisfy $f_{\mathrm{s}}>3360$ samples/second.
(c) When $f_{s}=330 \mathrm{~Hz}$, the output $y(t)$ is periodic with fundamental frequency $f_{0}=303 \mathrm{~Hz}$.

$$
f_{0}=\operatorname{gcd}(120,180-30)
$$

(d) When $f_{s}=71 \mathrm{~Hz}$, the output $y(t)$ is periodic with fundamental frequency $f_{0}=1 \mathrm{~Hz}$.

$$
\begin{aligned}
f_{0} & =\operatorname{gcd}\left(\left|120-\ell_{1} 71\right|,\left|180-\ell_{2} 71\right|\right) \\
& =\operatorname{gcd}(|120-(2) 71|,|180-(3) 71|) \\
& =\operatorname{gcd}(22,33)
\end{aligned}
$$

(e) Find $f_{s}$ so that the output can be written as $y(t)=A \cos (240 \pi t+\varphi)$, and specify resulting $A$ and $\varphi$ : There are two solutions, either is acceptable:

$$
\begin{aligned}
& f_{s}=\begin{array}{c}
300 \\
\text { samples/s }
\end{array}>0,
\end{aligned} A=\begin{gathered}
2 \sqrt{2} \\
f_{s}=\begin{array}{c}
360 \\
\text { samples/s }
\end{array}
\end{gathered}>0, \quad \varphi=-0.25 \pi \in(-\pi, \pi] .
$$

