GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2023
Quiz #1

June 14, 2023

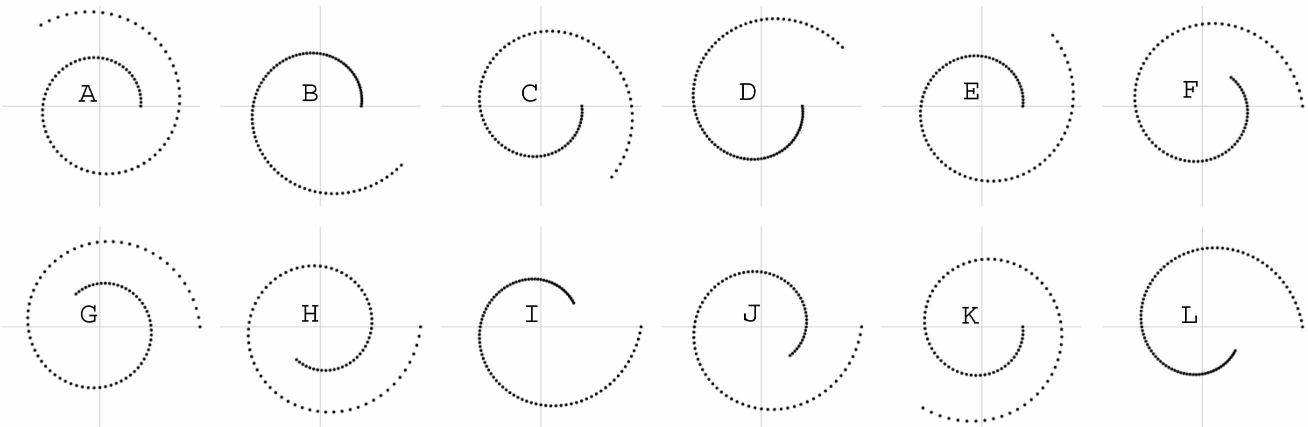
NAME: ______________ (FIRST) __________________ (LAST) GT username: ________________ (e.g., gtxyz123)

Important Notes:

- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5" × 11") of hand-written notes.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

PROB. Su23-Q1.1. Shown below are twelve different diagrams — labeled A through L — that show the locations of the powers $\{z^0, z^1, z^2, z^3, z^4, z^5, z^6, z^7, \ldots z^{89}, z^{90}\}$ in the complex plane, for twelve different values of z:



(The horizontal and vertical components represent the real and imaginary parts, respectively.
The axes are not labeled, the different plots are drawn with different scales. Only the shapes matter.)

(a) Match each diagram above to the corresponding value of z listed below.
Indicate your answer by writing a letter (from {A ... L}) into each answer box.

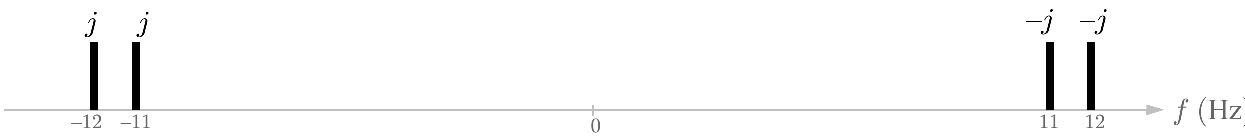
(i)		$z = 0.99e^{j0.02\pi}$
(ii)		$z = 0.99e^{-j0.02\pi}$
(iii)		$z = 1.01e^{j0.02\pi}$
(iv)		$z = 1.01e^{-j0.02\pi}$
(v)		$z = 0.99e^{j0.025\pi}$
(vi)		$z = 0.99e^{-j0.025\pi}$
(vii)		$z = 1.01e^{j0.025\pi}$
(viii)		$z = 1.01e^{-j0.025\pi}$
(ix)		$z = 0.99e^{j0.03\pi}$
(x)		$z = 0.99e^{-j0.03\pi}$
(xi)		$z = 1.01e^{j0.03\pi}$
(xii)		$z = 1.01e^{-j0.03\pi}$

(b) Explain your approach.

PROB. Su23-Q1.2.

Define three signals as follows:

- Let $x_1(t) = 2\cos(20\pi t + 0.5\pi) + 2\cos(22\pi t + 0.5\pi)$.
- Let $x_2(t) = B\sin(22\pi t)\cos(2\pi F t)$, where $B > 0$ and $F > 0$ are unspecified.
- Let $x_3(t)$ be the signal whose spectrum is shown in the sketch below:



If the sum of these three signals is a *single* sinusoid:

$$x_1(t) + x_2(t) + x_3(t) = A\cos(2\pi f_0 t + \varphi),$$

then it must be that:

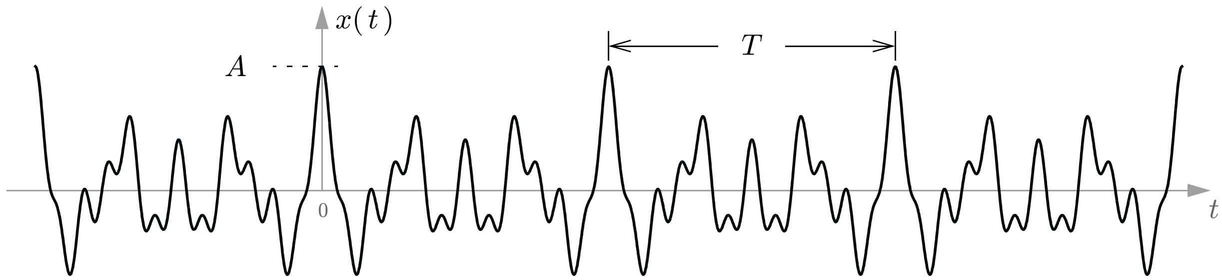
$B =$ [ ] > 0, $F =$ [ ] Hz > 0,

and where (in standard form)

$A =$ [ ] > 0, $f_0 =$ [ ] Hz > 0, and $\varphi =$ [ ] $\in (-\pi, \pi]$.

PROB. Su23-Q1.3.

Shown below is a plot of $x(t) = \sum_{k=1}^{4} \left(\frac{60k}{k+1} \right) \cos\left(\frac{480\pi t}{k} \right)$:



The scale is not labeled.

(a) The time between nearest peaks is $T =$ ☐ .

(b) The peak value is $A =$ ☐ .

(c) The Fourier series for this signal is $x(t) = \sum_k a_k e^{jk2\pi f_0 t}$,

where the fundamental frequency is $f_0 =$ ☐ Hz.

Identify which of the following FS coefficients are *nonzero* with an ✖ (*Hint:* only four are nonzero!):

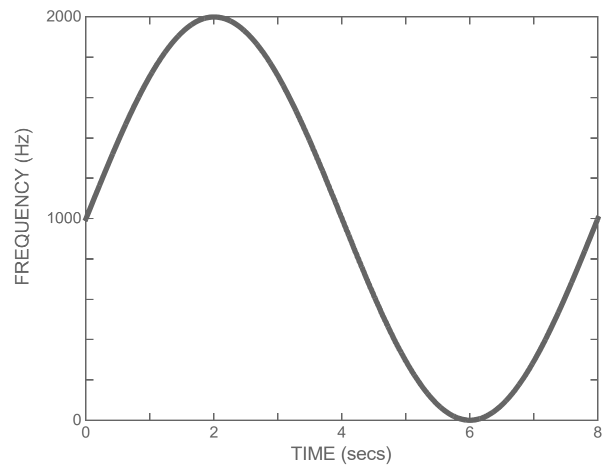
a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}

PROB. Su23-Q1.4. (The two parts of this problem are unrelated.)

(a) Find numerical values for the constants A, B, and C so that the spectrogram of the signal

$$x(t) = 2026\cos(\pi A t + B\cos(\pi C t))$$

looks like this:



$A =$

$B =$

$C =$

(b) A continuous-time sinusoid $x(t) = \cos(2\pi f_0 t)$ whose unspecified frequency f_0 is known to be in the range $700 < f_0 < 1700$ Hz is sampled with sampling rate $f_s = 8000$ Hz, resulting in a discrete-time sequence:

$$x[n] = x(\frac{n}{f_s}).$$

Find the unique value for the frequncy f_0 in the range $f_0 \in (700,\ 1700)$ for which the resulting discrete-time sequence is *periodic* with fundamental period $N_0 = 40$,
i.e., so that $N_0 = 40$ is the smallest positive integer N for which $x[n] = x[n + N]$ for all n.

$f_0 =$ Hz $\in (700,\ 1700)$.

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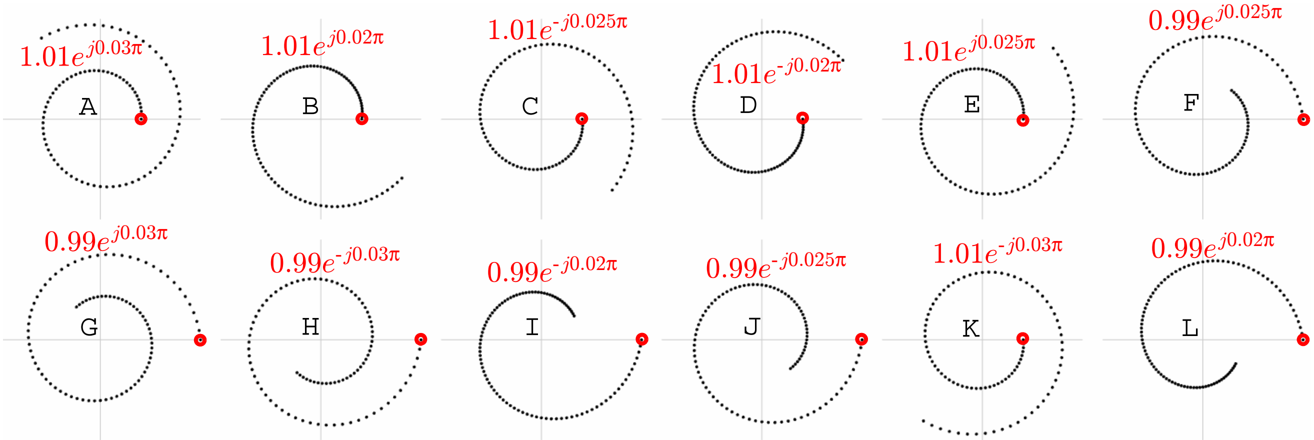
PROB. Su23-Q1.1. Shown below are twelve different diagrams — labeled `A` through `L` — that show the locations of the powers $\{z^0, z^1, z^2, z^3, z^4, z^5, z^6, z^7, \ldots z^{89}, z^{90}\}$ in the complex plane, for twelve different values of z:



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(a) Match each diagram above to the corresponding value of z listed below. Indicate your answer by writing a letter (from {`A` ... `L`}) into each answer box.

(i)	L	$z = 0.99e^{j0.02\pi}$
(ii)	I	$z = 0.99e^{-j0.02\pi}$
(iii)	B	$z = 1.01e^{j0.02\pi}$
(iv)	D	$z = 1.01e^{-j0.02\pi}$
(v)	F	$z = 0.99e^{j0.025\pi}$
(vi)	J	$z = 0.99e^{-j0.025\pi}$
(vii)	E	$z = 1.01e^{j0.025\pi}$
(viii)	C	$z = 1.01e^{-j0.025\pi}$
(ix)	G	$z = 0.99e^{j0.03\pi}$
(x)	H	$z = 0.99e^{-j0.03\pi}$
(xi)	A	$z = 1.01e^{j0.03\pi}$
(xii)	K	$z = 1.01e^{-j0.03\pi}$

- starting point is $z^0 = 1$, real, as highlighted

- subsequent powers determines magnitude:

  expand $\Rightarrow |z| = 1.01$
  contract $\Rightarrow |z| = 0.99$

- rotation *direction* determines sign of phase

  ↺ $\Rightarrow$ phase positive
  ↻ $\Rightarrow$ phase negative

- #revolutions $\in \{ \frac{1}{2\pi}90(0.020\pi) = 0.9$
  $\frac{1}{2\pi}90(0.025\pi) = 1.125$
  $\frac{1}{2\pi}90(0.030\pi) = 1.35 \}$

  determines
  phase magnitude $\in \{0.02\pi, 0.025\pi, 0.03\pi\}$

(b) Explain your approach.

PROB. Su23-Q1.2.

Define three signals as follows:

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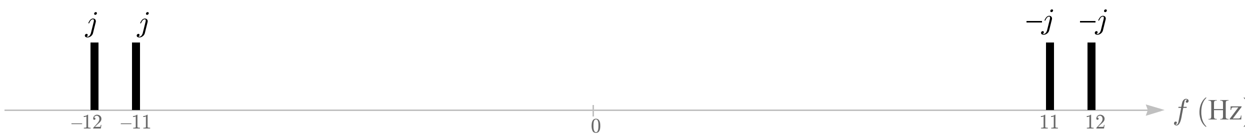


If the sum of these three signals is a *single* sinusoid:

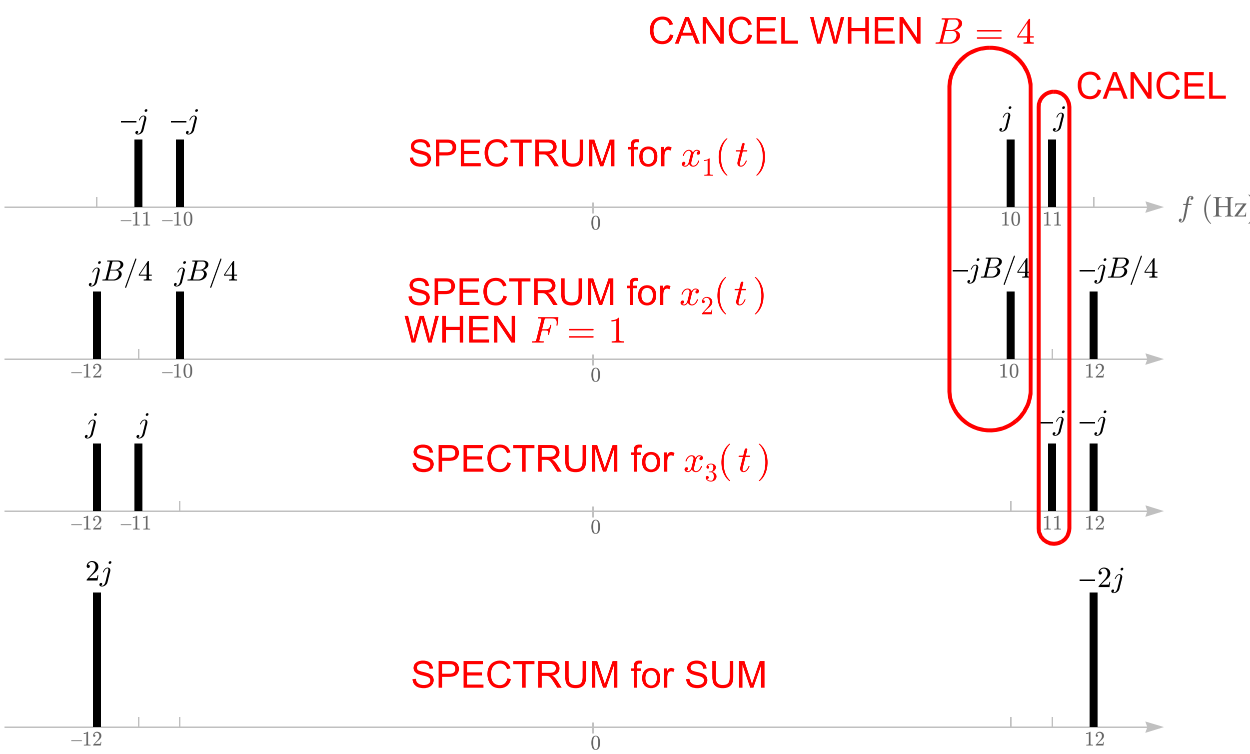
$$x_1(t) + x_2(t) + x_3(t) = A\cos(2\pi f_0 t + \varphi),$$

then it must be that:

$B =$ 4 > 0, $F =$ 1 Hz > 0,

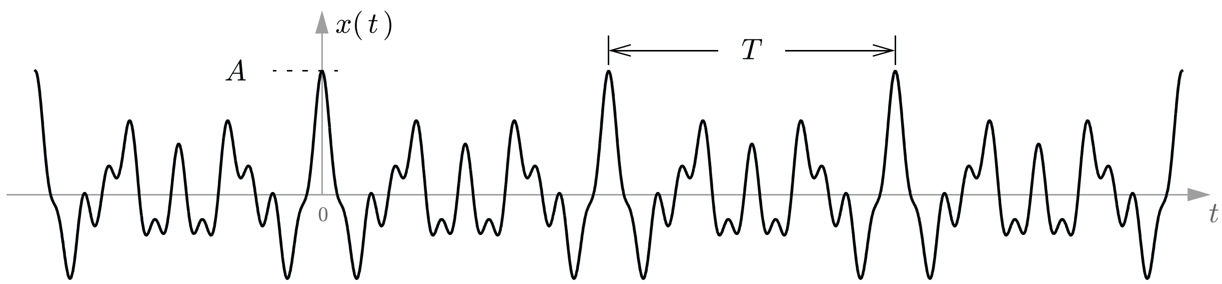
and where (in standard form)

$A =$ 4 > 0, $f_0 =$ 12 Hz > 0, and $\varphi =$ -0.5π $\in(-\pi, \pi]$.



PROB. Su23-Q1.3.

Shown below is a plot of $x(t) = \sum_{k=1}^{4} \left(\frac{60k}{k+1}\right) \cos\left(\frac{480\pi t}{k}\right)$:



The scale is not labeled.

(a) The time between nearest peaks is $T =$ 0.05 .

The gcd of contributing frequencies at $\frac{240}{k} \in \{240, 120, 80, 60\}$

is the fundamental freq $\Rightarrow f_0 = 20$ Hz

$\Rightarrow$ fundamental period $T_0 = \frac{1}{f_0} = 0.05$

(b) The peak value is $A =$ 163 .

Evaluate at time 0 $\Rightarrow x(0) = \sum_{k=1}^{4} \left(\frac{60k}{k+1}\right) \cos(0) = \frac{60}{2} + \frac{120}{3} + \frac{180}{4} + \frac{240}{5}$

$= 30 + 40 + 45 + 48 = 163$

(c) The Fourier series for this signal is $x(t) = \sum_k a_k e^{jk2\pi f_0 t}$,

where the fundamental frequency is $f_0 =$ 20 Hz.

Identify which of the following FS coefficients are *nonzero* with an **X** (*Hint:* only four are nonzero!):

Contributing frequencies $\{240, 120, 80, 60\} = \{12f_0, 6f_0, 4f_0, 3f_0\}$

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
			X	X		X						X

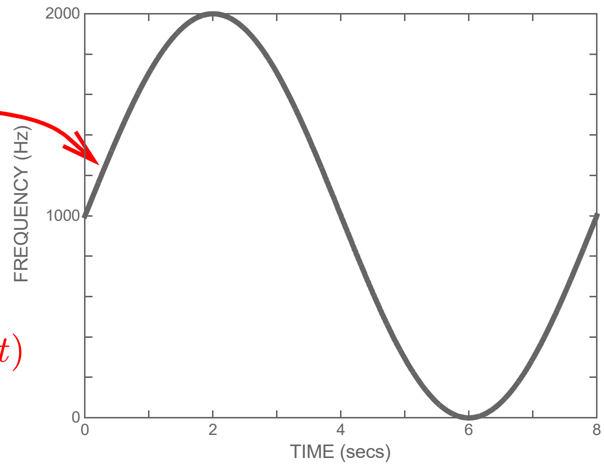
PROB. Su23-Q1.4. (The two parts of this problem are unrelated.)

(a) Find numerical values for the constants A, B, and C so that the spectrogram of the signal

$$x(t) = 2026\cos(\pi A t + B\cos(\pi C t))$$

looks like this:

The equation for this sinusoid is



$A =$ 2000

$B =$ –8000

$C =$ 0.25

$$f_i(t) = 1000 + 1000\sin(0.25\pi t)$$

Multiply by 2π and integrate

$$\Rightarrow \varphi(t) = 2000\pi t - 8000\sin(0.25\pi t)$$
$$= \pi A t + B\cos(\pi C t)$$

(b) A continuous-time sinusoid $x(t) = \cos(2\pi f_0 t)$ whose unspecified frequency f_0 is known to be in the range $700 < f_0 < 1700$ Hz is sampled with sampling rate $f_s = 8000$ Hz, resulting in a discrete-time sequence:

$$x[n] = x(\frac{n}{f_s}).$$

Find the unique value for the frequncy f_0 in the range $f_0 \in (700, 1700)$ for which the resulting discrete-time sequence is *periodic* with fundamental period $N_0 = 40$, *i.e.*, so that $N_0 = 40$ is the smallest positive integer N for which $x[n] = x[n+N]$ for all n.

Substite $t = \frac{n}{8000} \Rightarrow x[n] = \cos(\frac{2\pi f_0 n}{8000})$.

$f_0 =$ 1400 Hz $\in (700, 1700)$.

Factor digital frequency as a ratio of integers times 2π:

$$\hat{\omega} = \frac{2\pi f_0}{8000} = \frac{k}{N}(2\pi);$$

when fraction is reduced (k, N share no common factors), the denom N is the period.

$$\Rightarrow f_0 = k\frac{8000}{N} = 200k, \text{ where } k \in \{1, 3, 7, 9, 11 \ldots\} \text{ shares no factors with 40.}$$

Of these, only $k = 7$ puts f_0 in desired range $\Rightarrow$ $f_0 = 200(7) = 1400$ Hz.