GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2022
Quiz #1

June 15, 2022

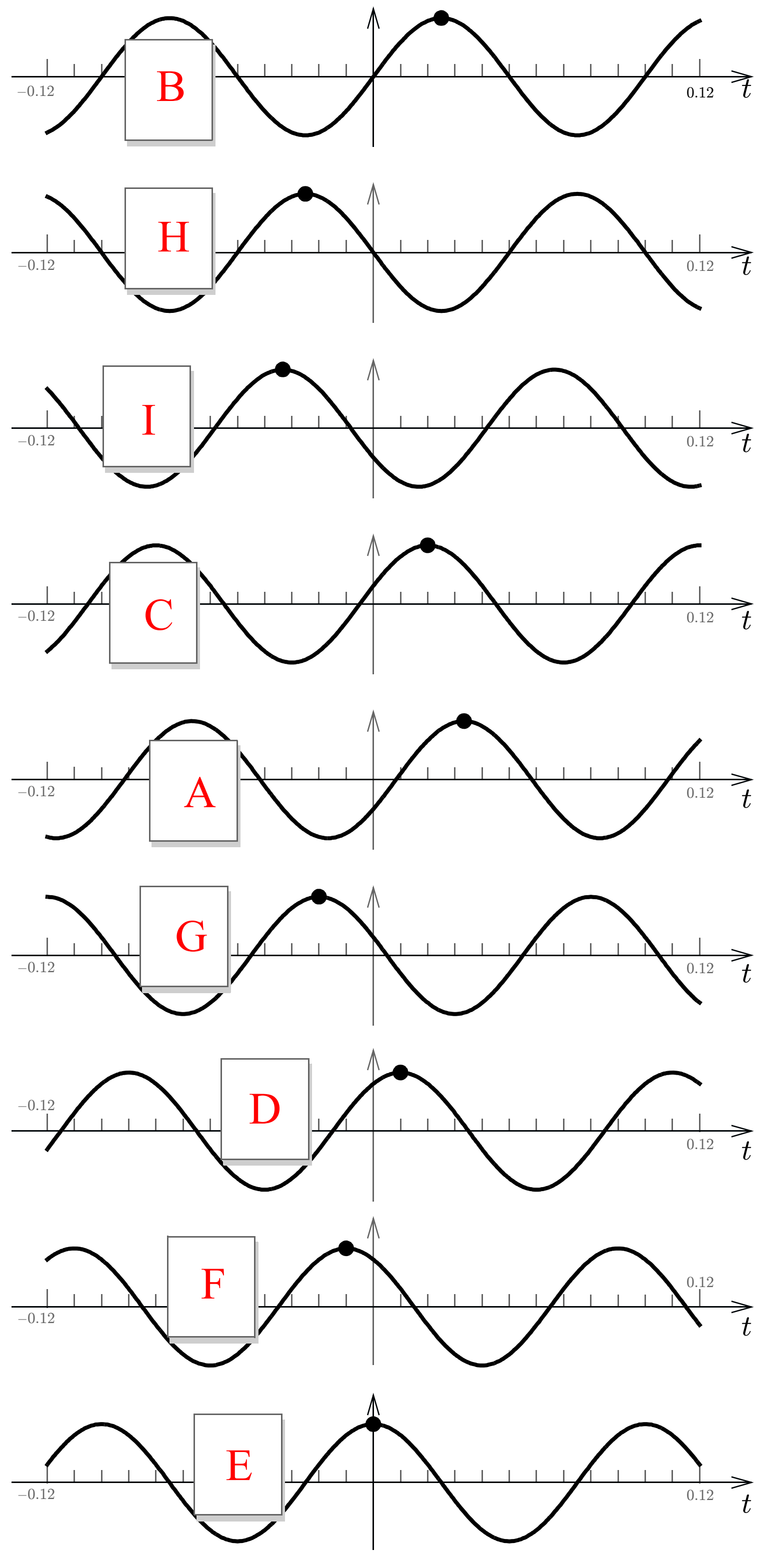
NAME: ANSWER KEY ______ ______ GT username: A ______
(FIRST) (LAST) (e.g., gtxyz123)

Important Notes:

- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

PROB. Su22-Q1.1.



Consider the nine sinusoidal plots shown here, they all have the same amplitudes and frequencies, they differ only in their phases:

(a) The frequency of all nine sinusoids is

$f_0 =$ 10 Hz.

(b) Below is a list of nine sinusoid equations, labeled A through I. Match each equation to its corresponding plot.

Indicate your answers by writing a letter (from A through I) into each plot's answer box.

(The black dots might be helpful; they highlight the sinusoid peak in each case that is closest to time zero.)

(A) $x(t) = \cos(2\pi f_0 t - \frac{2\pi}{3})$

(B) $x(t) = \cos(2\pi f_0 t - 0.5\pi)$

(C) $x(t) = \cos(2\pi f_0 t - 0.4\pi)$

(D) $x(t) = \cos(2\pi f_0 t - 0.2\pi)$

(E) $x(t) = \cos(2\pi f_0 t)$

(F) $x(t) = \cos(2\pi f_0 t + 0.2\pi)$

(G) $x(t) = \cos(2\pi f_0 t + 0.4\pi)$

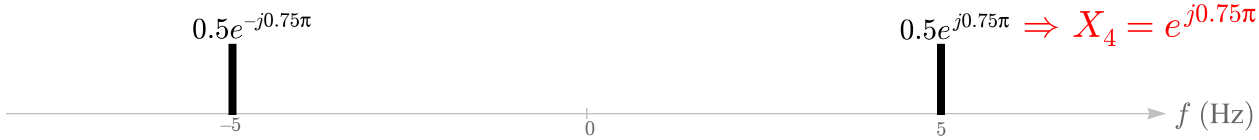
(H) $x(t) = \cos(2\pi f_0 t + 0.5\pi)$

(I) $x(t) = \cos(2\pi f_0 t + \frac{2\pi}{3})$

PROB. Su22-Q1.2.

Define four sinusoids as follows:

- Let $x_1(t) = \cos(10\pi t + \pi)$. $\Rightarrow X_1 = -1$
- Let $x_2(t) = x_1(t - 0.1)$. $\Rightarrow X_2 = +1$
- Let $x_3(t) = \text{Re}\{X_3 e^{j10\pi t}\}$, where $X_3 = \frac{1 + j}{\sqrt{2}}$. $\Rightarrow X_3 = e^{j0.25\pi}$
- Let $x_4(t)$ be the signal whose spectrum is shown in the sketch below:



The sum of all four sinusoids is a single sinusoid:

$$x_1(t) + x_2(t) + x_3(t) + x_4(t) = A\cos(10\pi t + \varphi),$$

where (in standard form)

$A =$ $\boxed{\sqrt{2}}$ > 0, and $\varphi =$ $\boxed{0.5\pi}$ $\in (-\pi, \pi]$.

Phasor addition:

$$
\begin{aligned}
Ae^{j\varphi} &= X_1 + X_2 + X_3 + X_4 \\
&= -1 + 1 + e^{j0.25\pi} + e^{j0.75\pi} \\
&= \sqrt{2}j \\
&= \sqrt{2}e^{j0.5\pi}
\end{aligned}
$$

PROB. Su22-Q1.3.

Let $x(t) = 30 + \cos(120\pi t)\cos(2\pi f_c t)$.

The parameter f_c is positive but otherwise unspecified, it may be different in each part below.

(a) When $f_c = 60$ Hz, the signal $x(t)$ is periodic with fundamental frequency $f_0 =$ 120 Hz,

and the first few Fourier series coefficients in the expansion $x(t) = \sum_k a_k e^{jk2\pi f_0 t}$ are:

$a_{-2} =$ 0 $a_{-1} =$ 0.25 $a_0 =$ 30.5 $a_1 =$ 0.25 $a_2 =$ 0

(b) The *smallest* positive value of f_c that makes the signal $x(t)$ periodic with fundamental frequency $f_0 = 8$ Hz is

$f_c =$ 4 Hz.

(hint: answer is a positive integer)

f_c	$f_0 = \gcd(\lvert 60 - f_c \rvert, \lvert 60 + f_c \rvert)$
1	$f_0 = \gcd(59, 61) = 1$
2	$f_0 = \gcd(58, 62) = 2$
3	$f_0 = \gcd(57, 63) = 3$
4	$f_0 = \gcd(56, 64) = 8$ ✓

(c) There are numerous positive values for f_c that makes the signal $x(t)$ periodic with fundamental frequency $f_0 = 40$ Hz. Name any two:

$f_c =$ Hz,

or $f_c =$ Hz.

(hint: both positive integers)

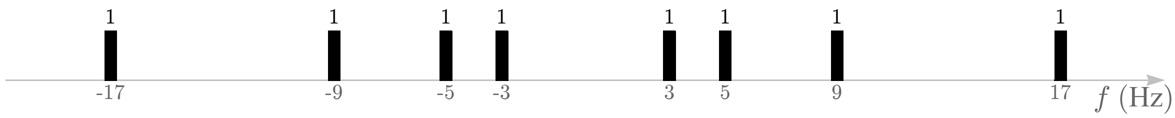
All answers:

$\ell 120 \pm 20$ Hz for integer ℓ

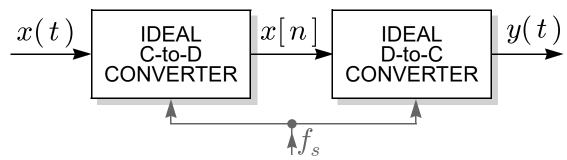
$\in \{20, 100, 140, 220, 260, 340, 380, 460, 500, ...\}$

PROB. Su22-Q1.4.

Consider the spectrum shown below:



Suppose we sample the signal $x(t)$ having the above spectrum with sampling rate f_s, and then feed the samples to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output $y(t)$, as shown here:



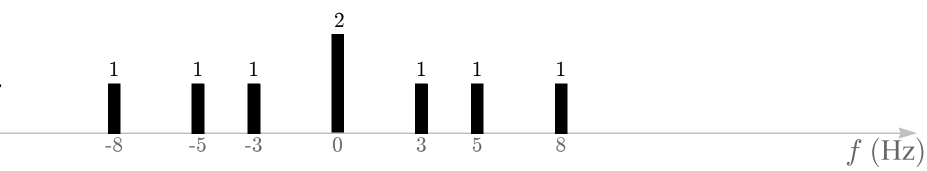
(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s >$ 34 Hz.

(b) Shown below is the spectrum for the D-C output $y(t)$, for each of five different values of the sampling rate.

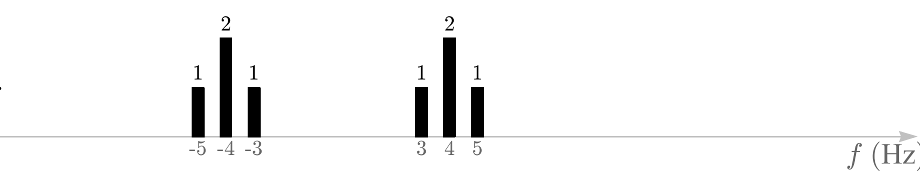
Find the largest sampling rate f_s that leads to each output spectrum, and write it in the corresponding answer box.

Hint: all answers are integers chosen from the set $f_s \in \{1, 2, 3, 4, \ldots 100\}$ Hz.

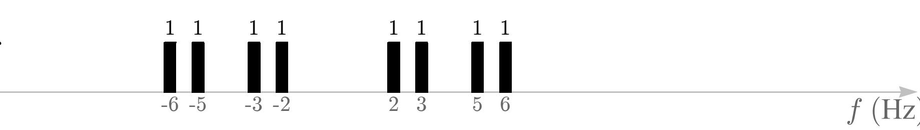
(i) $f_s =$ 17 Hz.



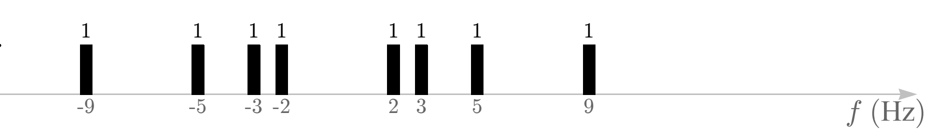
(ii) $f_s =$ 13 Hz.



(iii) $f_s =$ 15 Hz.



(iv) $f_s =$ 19 Hz.



(v) $f_s =$ 11 Hz.