GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2018
Quiz #1

June 13, 2018

NAME: _____ _____ GT username: _____
(FIRST) (LAST) (e.g., gtxyz123)

To avoid losing 3 points, circle your recitation section:

10:05 – 11:55am	L01 (Beck)
12:30 – 2:20pm	L02 (Beck)
2:35 – 4:25pm	L03 (Bhattacharjea)

Important Notes:

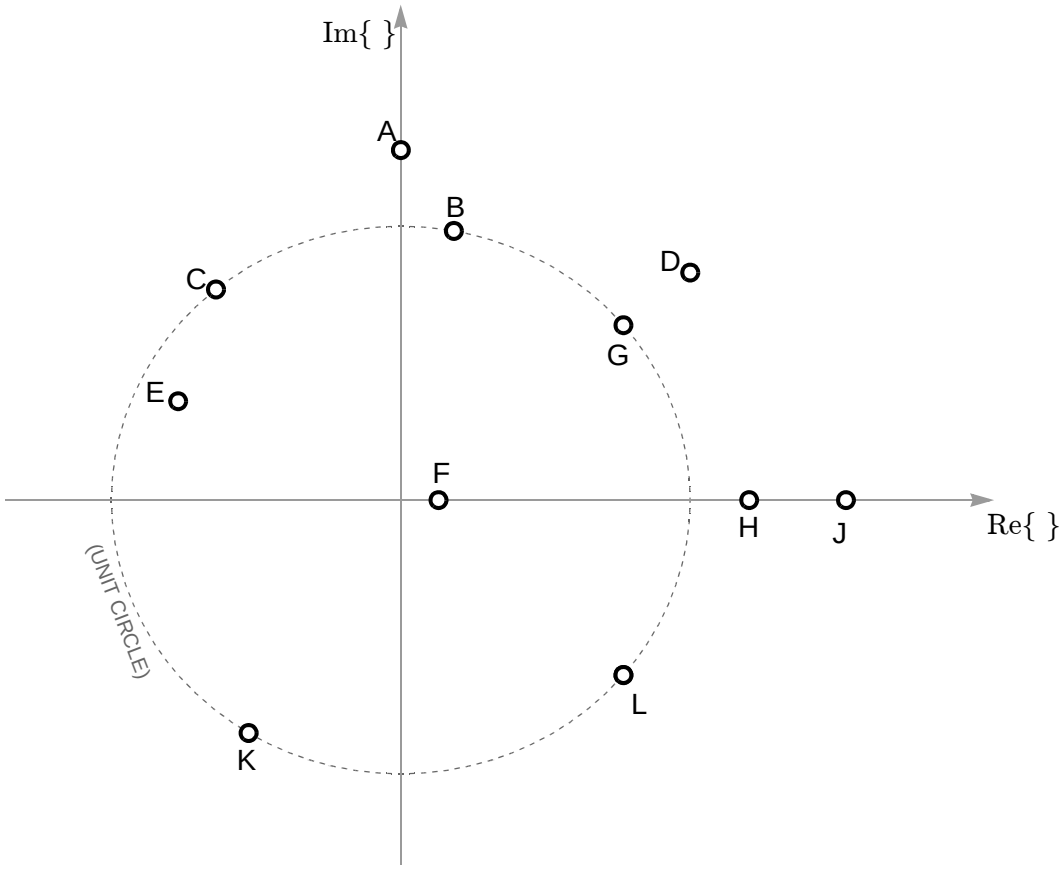
- Do not unstaple the test.
- One two-sided page (8.5" $\times$ 11") of hand-written notes permitted.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

PROB. Su18-Q1.1. Let $z = 2^j$. Here is a list of twelve numbers that are all dependent on z:

z	jz	z^*	$z + z^*$	$z - z^*$	$j - z$
$\mathrm{Re}\{z\} - \mathrm{Im}\{z\}$	$\frac{1}{z}$	$\frac{z}{\mathrm{Re}\{z\}}$	$\frac{z}{z^*}$	$\frac{\mathrm{Re}\{z\}}{\mathrm{Im}\{z\}}$	z^6

Shown below are the locations of these twelve numbers in the complex plane. Match each number above to its corresponding location in the complex plane below; indicate your answer by writing a letter from {A, B, C, ... L} in each answer box above. (A letter may be used more than once.)



PROB. Su18-Q1.2.

Here is a list of 16 possible values for a frequency parameter f_0:

$f_0 = 0$ kHz	$f_0 = 4$ kHz	$f_0 = 8$ kHz	$f_0 = 12$ kHz
$f_0 = 1$ kHz	$f_0 = 5$ kHz	$f_0 = 9$ kHz	$f_0 = 13$ kHz
$f_0 = 2$ kHz	$f_0 = 6$ kHz	$f_0 = 10$ kHz	$f_0 = 14$ kHz
$f_0 = 3$ kHz	$f_0 = 7$ kHz	$f_0 = 11$ kHz	$f_0 = 15$ kHz

(a) Circle *all* of the values (if any) in the above list for which the following sum of sinusoids is zero (there may not be any, or there may be more than one):

$$\cos(2\pi f_0 t) + \cos(2\pi f_0 (t - \frac{1}{3000})) + \cos(2\pi f_0 (t - \frac{2}{3000})) = 0, \qquad \text{for all time } t.$$

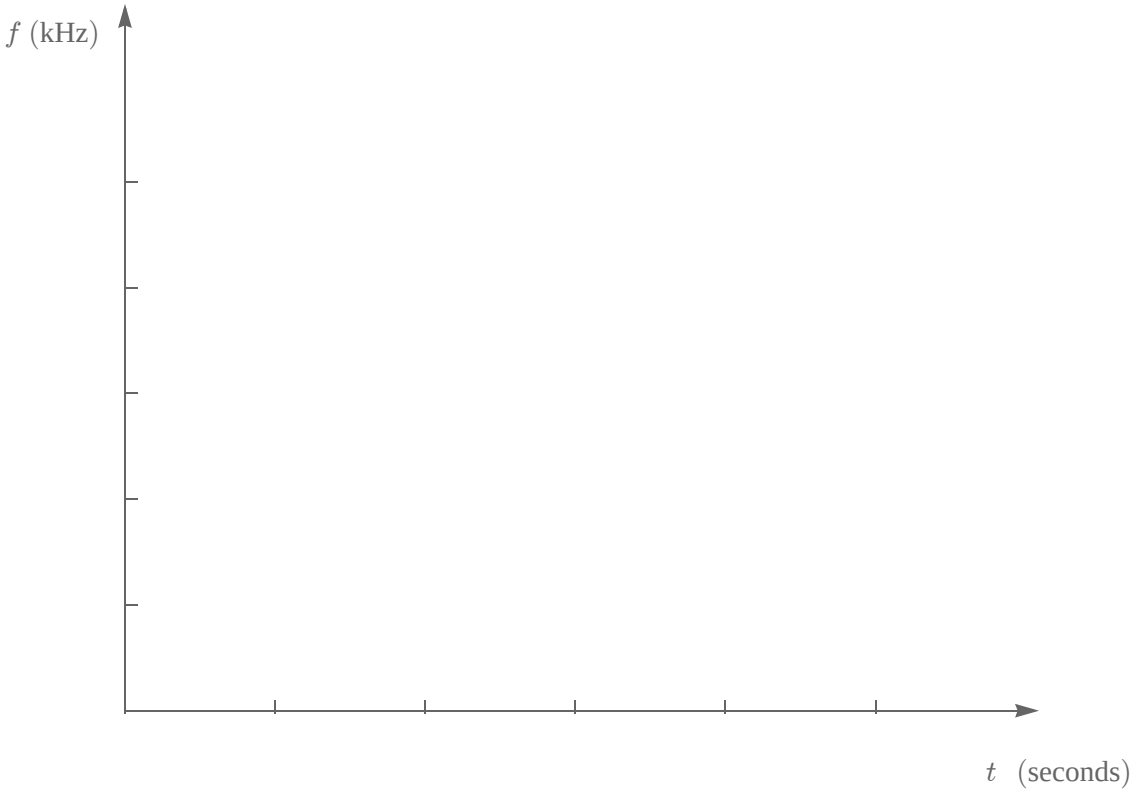
(b) Explain.

PROB. Su18-Q1.3. The following MATLAB code generates a spectrogram plot:

```
fs = 10000;
t = 0:1/fs:1;
A = 6000; B = -5000;
x = cos(2*pi*A*t - pi*B*(t.^2));
plotspec(x,fs);
% spectrogram(x,512,[],[],fs,'yaxis');
```

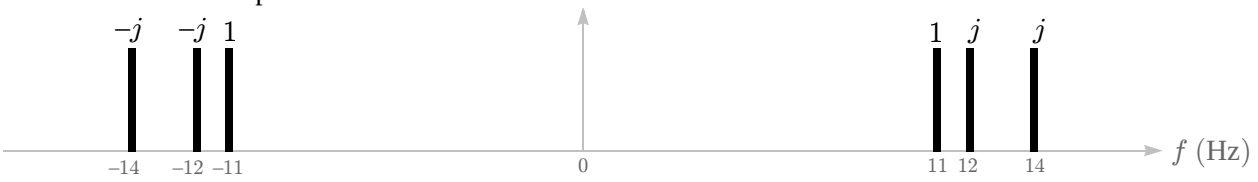
(Both the plotspec and spectrogram command would generate the same picture.)

Predict what the resulting plot will look like by carefully sketching it in the space below. Be sure to label both the x axis (in units of seconds) and the y axis (in units of kHz)!

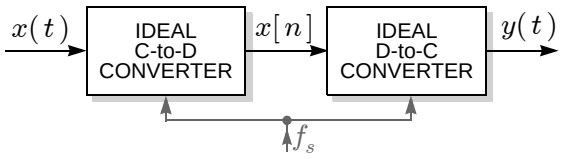


PROB. Su18-Q1.4.

Consider the spectrum shown below:



Suppose we sample the signal $x(t)$ having the above spectrum with sampling rate f_s, and then feed the samples to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output $y(t)$, as shown here:



(a) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ ☐ Hz.

(b) For which* of the sampling rates listed below will the output $y(t)$ be the **same** as the input $x(t)$?

(c) For which* of the sampling rates listed below will the output $y(t)$ be a **constant**, independent of t?

(d) For which* of the sampling rates listed below will the output $y(t)$ be a **single** sinusoid, *i.e.*, of the form $y(t) = A\cos(\omega_0 t + \varphi)$ for some parameters $A > 0$, $\omega_0 > 0$, and φ?

(e) For which* of the sampling rates listed below will the output $y(t)$ be the **sum of two** sinusoids, *i.e.*, of the form $y(t) = A_1\cos(\omega_1 t + \varphi_1) + A_2\cos(\omega_2 t + \varphi_2)$ for $A_1 > 0$, $A_2 > 0$, and $\omega_1 \neq \omega_2$?

answers for part (b)	answers for part (c)	answers for part (d)	answers for part (e)
$f_s = 15$ Hz	$f_s = 15$ Hz	$f_s = 15$ Hz	$f_s = 15$ Hz
$f_s = 16$ Hz	$f_s = 16$ Hz	$f_s = 16$ Hz	$f_s = 16$ Hz
$f_s = 17$ Hz	$f_s = 17$ Hz	$f_s = 17$ Hz	$f_s = 17$ Hz
$f_s = 18$ Hz	$f_s = 18$ Hz	$f_s = 18$ Hz	$f_s = 18$ Hz
$f_s = 19$ Hz	$f_s = 19$ Hz	$f_s = 19$ Hz	$f_s = 19$ Hz
$f_s = 20$ Hz	$f_s = 20$ Hz	$f_s = 20$ Hz	$f_s = 20$ Hz
$f_s = 21$ Hz	$f_s = 21$ Hz	$f_s = 21$ Hz	$f_s = 21$ Hz
$f_s = 22$ Hz	$f_s = 22$ Hz	$f_s = 22$ Hz	$f_s = 22$ Hz
$f_s = 23$ Hz	$f_s = 23$ Hz	$f_s = 23$ Hz	$f_s = 23$ Hz
$f_s = 24$ Hz	$f_s = 24$ Hz	$f_s = 24$ Hz	$f_s = 24$ Hz
$f_s = 25$ Hz	$f_s = 25$ Hz	$f_s = 25$ Hz	$f_s = 25$ Hz
$f_s = 26$ Hz	$f_s = 26$ Hz	$f_s = 26$ Hz	$f_s = 26$ Hz
$f_s = 27$ Hz	$f_s = 27$ Hz	$f_s = 27$ Hz	$f_s = 27$ Hz
$f_s = 28$ Hz	$f_s = 28$ Hz	$f_s = 28$ Hz	$f_s = 28$ Hz
$f_s = 29$ Hz	$f_s = 29$ Hz	$f_s = 29$ Hz	$f_s = 29$ Hz
$f_s = 30$ Hz	$f_s = 30$ Hz	$f_s = 30$ Hz	$f_s = 30$ Hz

*There may be none or more than one, circle all that apply.

(f) Justify your answers in the space below.

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2018
Quiz #1

June 13, 2018

NAME: ANSWER KEY (FIRST) (LAST) GT username: _____ (e.g., gtxyz123)

To avoid losing 3 points, circle your recitation section:

10:05 – 11:55am	L01 (Beck)
12:30 – 2:20pm	L02 (Beck)
2:35 – 4:25pm	L03 (Bhattacharjea)

Important Notes:

- Do not unstaple the test.
- One two-sided page (8.5" × 11") of hand-written notes permitted.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

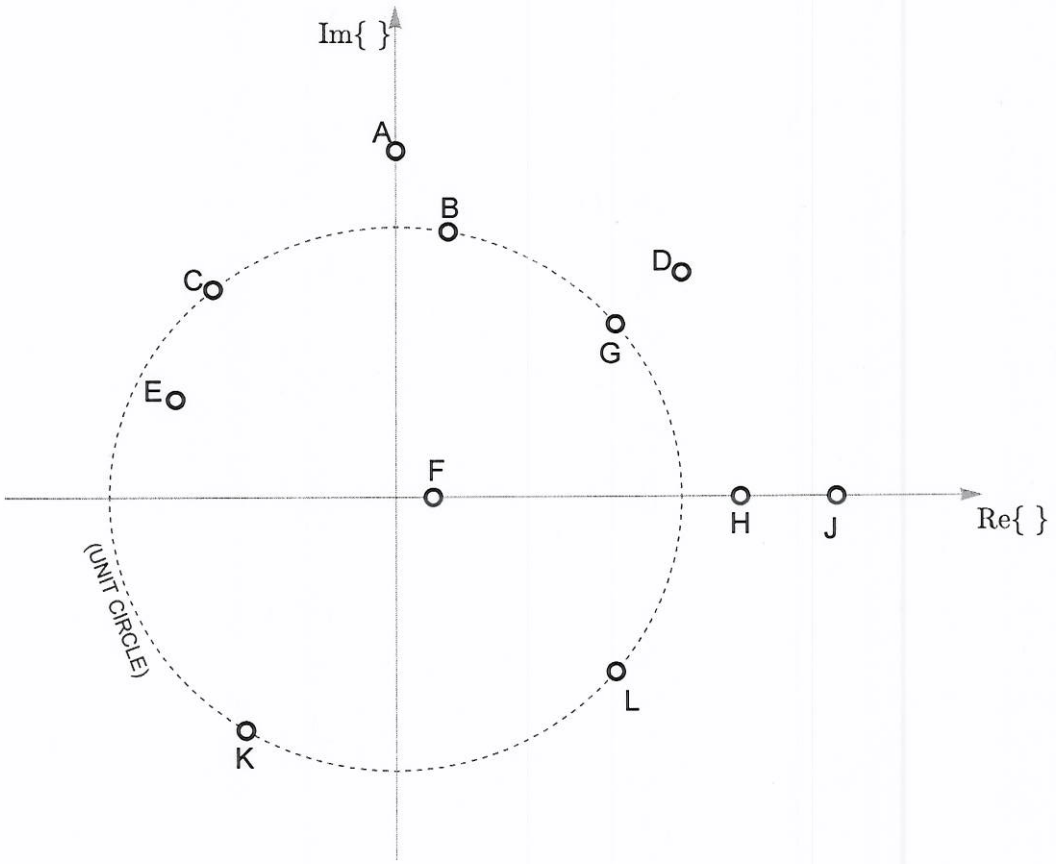
$z = 2^j = e^{j \ln 2} = e^{j 0.69} = e^{j 0.22\pi}$ (40°)

PROB. Su18-Q1.1. Let $z = 2^j$. Here is a list of twelve numbers that are all dependent on z:

G	C	L	J	A	E
z	jz	z^*	$z + z^*$	$z - z^*$	$j - z$

F	L	D	B	H	K
$\mathrm{Re}\{z\} - \mathrm{Im}\{z\}$	$\frac{1}{z}$	$\frac{z}{\mathrm{Re}\{z\}}$	$\frac{z}{z^*}$	$\frac{\mathrm{Re}\{z\}}{\mathrm{Im}\{z\}}$	z^6

Shown below are the locations of these twelve numbers in the complex plane. Match each number above to its corresponding location in the complex plane below; indicate your answer by writing a letter from {A, B, C, ... L} in each answer box above. (A letter may be used more than once.)



PROB. Su18-Q1.2.

$f_0 = 1000m$ where $m \in \{0, 1, 2, \ldots, 15\}$

Here is a list of 16 possible values for a frequency parameter f_0:

$f_0 = 0$ kHz	$f_0 = 4$ kHz	$f_0 = 8$ kHz	$f_0 = 12$ kHz
$f_0 = 1$ kHz	$f_0 = 5$ kHz	$f_0 = 9$ kHz	$f_0 = 13$ kHz
$f_0 = 2$ kHz	$f_0 = 6$ kHz	$f_0 = 10$ kHz	$f_0 = 14$ kHz
$f_0 = 3$ kHz	$f_0 = 7$ kHz	$f_0 = 11$ kHz	$f_0 = 15$ kHz

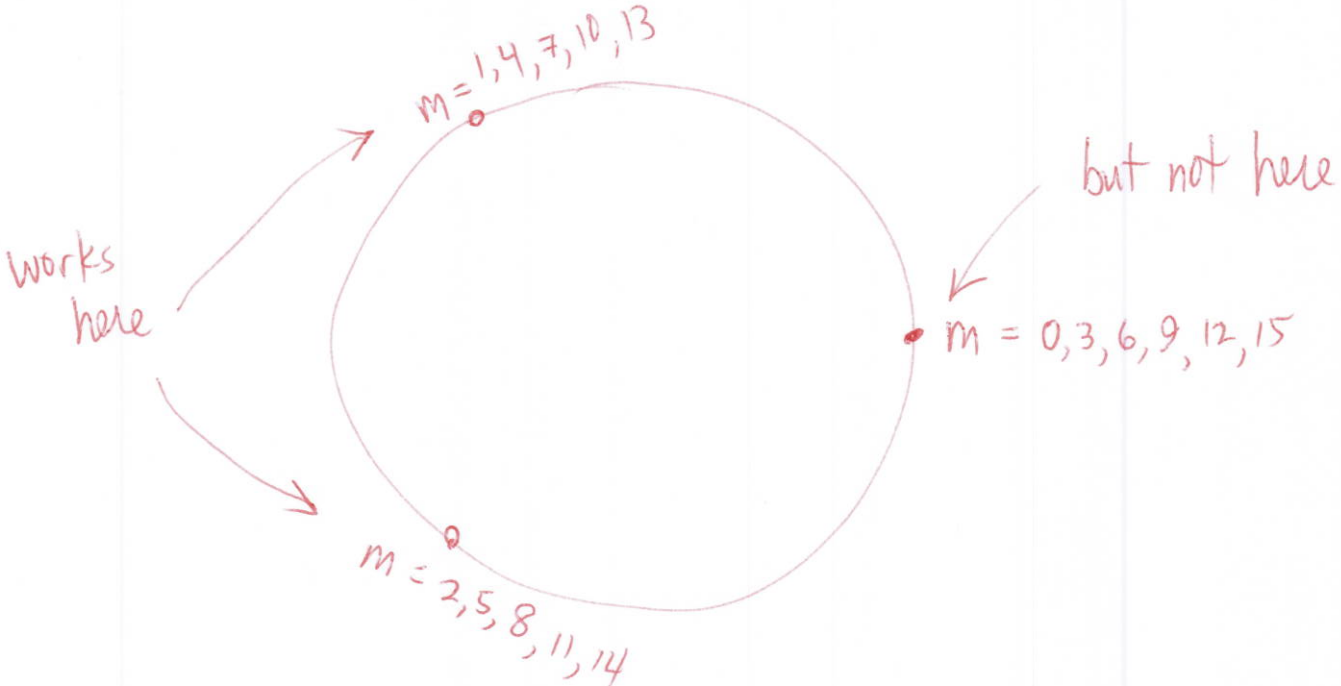
(a) Circle *all* of the values (if any) in the above list for which the following sum of sinusoids is zero (there may not be any, or there may be more than one):

$$\cos(2\pi f_0 t) + \cos(2\pi f_0 (t - \tfrac{1}{3000})) + \cos(2\pi f_0 (t - \tfrac{2}{3000})) = 0, \qquad \text{for all time } t.$$

(b) Explain.

corresponding phasor equation:

$$1 + e^{-j\theta} + e^{-j2\theta} = 0 \quad \text{where } \theta = \frac{2\pi f_0}{3000} = \frac{2\pi}{3} m$$



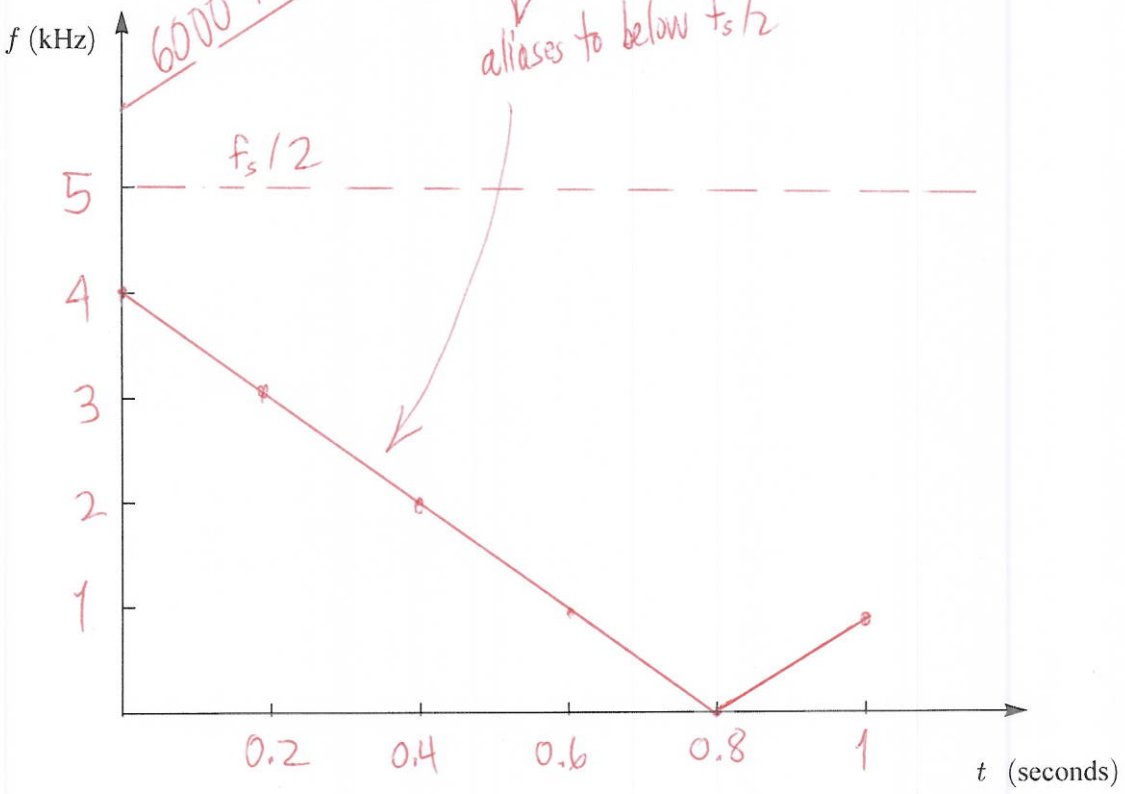
$\frac{\psi(t)}{2\pi} = At - \frac{1}{2}Bt^2 \Rightarrow f_i(t) = \frac{\psi'(t)}{2\pi} = A - Bt$
$= 6000 + 5000t$

PROB. Su18-Q1.3. The following MATLAB code generates a spectrogram plot:

```
fs = 10000;
t = 0:1/fs:1;
A = 6000; B = -5000;
x = cos(2*pi*A*t - pi*B*(t.^2));
plotspec(x,fs);
% spectrogram(x,512,[],[],fs,'yaxis');
```

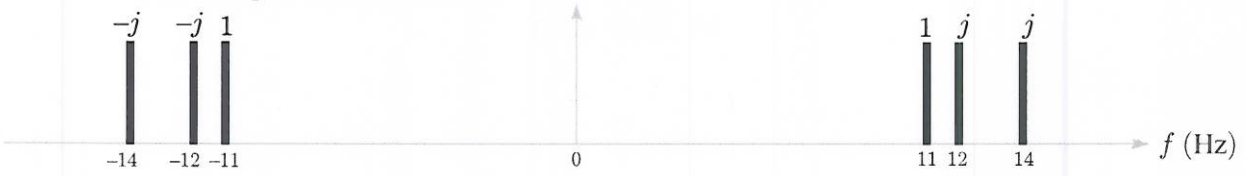
(Both the plotspec and spectrogram command would generate the same picture.)

Predict what the resulting plot will look like by carefully sketching it in the space below. Be sure to label both the x axis (in units of seconds) and the y axis (in units of kHz)!

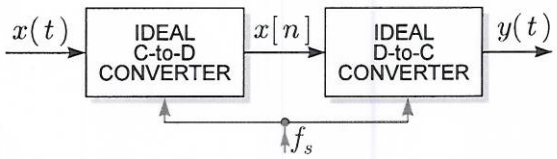


PROB. Su18-Q1.4.

Consider the spectrum shown below:



Suppose we sample the signal $x(t)$ having the above spectrum with sampling rate f_s, and then feed the samples to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output $y(t)$, as shown here:



(a) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ [1] Hz.

gcd (11, 12, 14)

(b) For which* of the sampling rates listed below will the output $y(t)$ be the **same** as the input $x(t)$?

(c) For which* of the sampling rates listed below will the output $y(t)$ be a **constant**, independent of t?

(d) For which* of the sampling rates listed below will the output $y(t)$ be a **single** sinusoid, *i.e.*, of the form $y(t) = A\cos(\omega_0 t + \varphi)$ for some parameters $A > 0$, $\omega_0 > 0$, and φ?

(e) For which* of the sampling rates listed below will the output $y(t)$ be the **sum of two** sinusoids, *i.e.*, of the form $y(t) = A_1\cos(\omega_1 t + \varphi_1) + A_2\cos(\omega_2 t + \varphi_2)$ for $A_1 > 0$, $A_2 > 0$, and $\omega_1 \neq \omega_2$?

NO ALIAS

answers for part (b)	CONST answers for part (c)	ONE answers for part (d)	answers for part (e)
$f_s = 15$ Hz	$f_s = 15$ Hz	$f_s = 15$ Hz	$f_s = 15$ Hz
$f_s = 16$ Hz	$f_s = 16$ Hz	$f_s = 16$ Hz	$f_s = 16$ Hz
$f_s = 17$ Hz	$f_s = 17$ Hz	$f_s = 17$ Hz	$f_s = 17$ Hz
$f_s = 18$ Hz	$f_s = 18$ Hz	$f_s = 18$ Hz	$f_s = 18$ Hz
$f_s = 19$ Hz	$f_s = 19$ Hz	$f_s = 19$ Hz	$f_s = 19$ Hz
$f_s = 20$ Hz	$f_s = 20$ Hz	$f_s = 20$ Hz	$f_s = 20$ Hz
$f_s = 21$ Hz	$f_s = 21$ Hz	$f_s = 21$ Hz	$f_s = 21$ Hz
$f_s = 22$ Hz	$f_s = 22$ Hz	$f_s = 22$ Hz	$f_s = 22$ Hz
$f_s = 23$ Hz	$f_s = 23$ Hz	$f_s = 23$ Hz	$f_s = 23$ Hz
$f_s = 24$ Hz	$f_s = 24$ Hz	$f_s = 24$ Hz	$f_s = 24$ Hz
$f_s = 25$ Hz	$f_s = 25$ Hz	$f_s = 25$ Hz	$f_s = 25$ Hz
$f_s = 26$ Hz	$f_s = 26$ Hz	$f_s = 26$ Hz	$f_s = 26$ Hz
$f_s = 27$ Hz	$f_s = 27$ Hz	$f_s = 27$ Hz	$f_s = 27$ Hz
$f_s = 28$ Hz	$f_s = 28$ Hz	$f_s = 28$ Hz	$f_s = 28$ Hz
$f_s = 29$ Hz	$f_s = 29$ Hz	$f_s = 29$ Hz	$f_s = 29$ Hz
$f_s = 30$ Hz	$f_s = 30$ Hz	$f_s = 30$ Hz	$f_s = 30$ Hz

12-Hz aliases to 11-Hz

12-Hz samples to zero

14-Hz aliases to 11 Hz

14-Hz samples to zero

*There may be none or more than one, circle all that apply.

$f_s > 2f_{max} = 28$

NONE (would need $f_s = 1$ Hz or $f_s = 1/k$ for integer k)

the 14-Hz sinusoid cancels the 12-Hz sinusoid after aliasing

(f) Justify your answers in the space below.

from spectrum we can write

$$x(t) = 2\cos(22\pi t) - 2\sin(24\pi t) - 2\sin(28\pi t)$$

$$\uparrow$$

$$f_{max} = 14 \text{ Hz}$$

$\Rightarrow$ need to sample > 28 Hz to avoid aliasing