GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2017
Quiz #1

June 14, 2017

NAME: ______________________________

Important Notes:

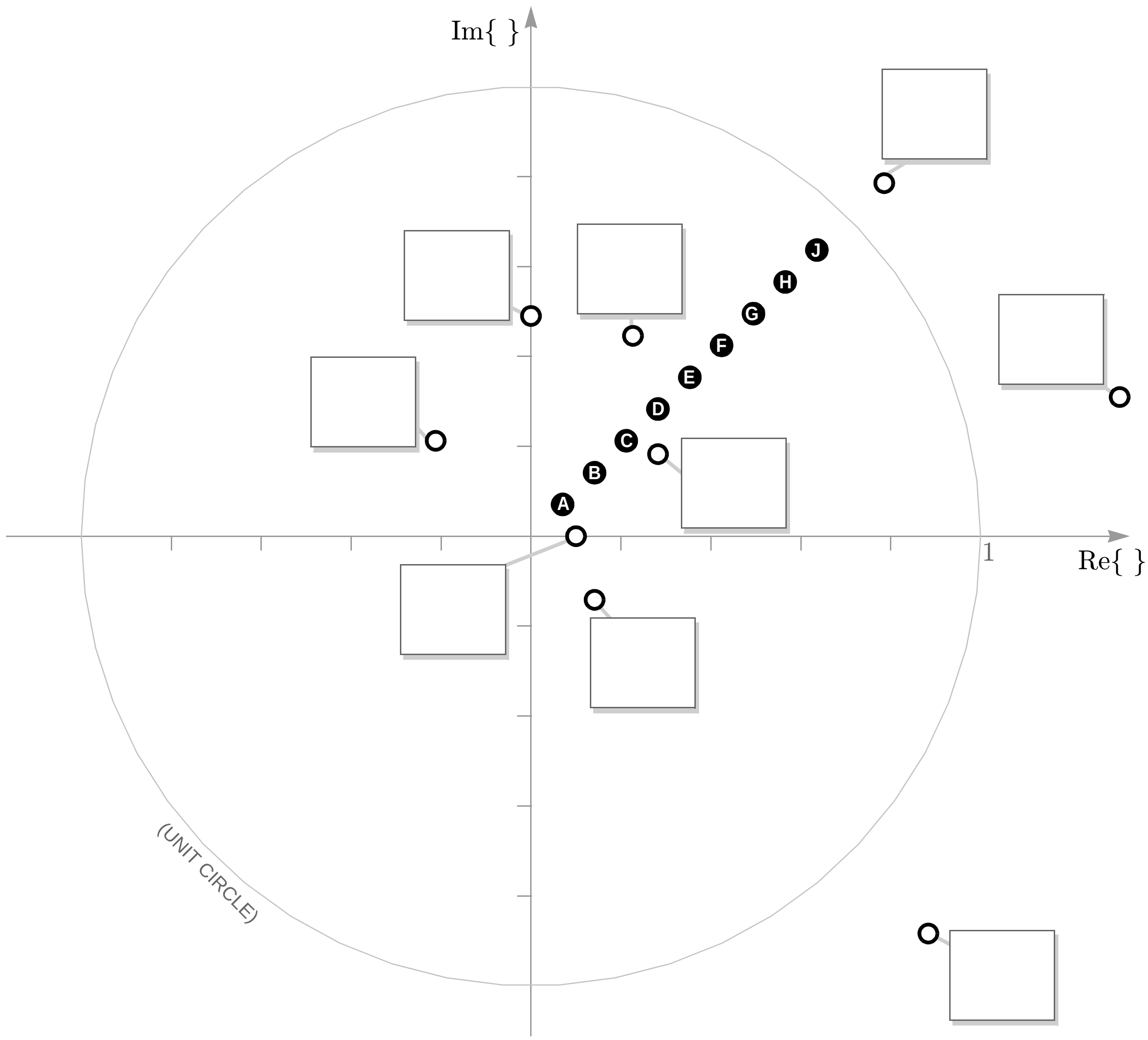
- DO NOT unstaple the test.
- One two-sided page (8.5” × 11”) of hand-written notes permitted.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

PROB. Su17-Q1.1. Shown below are 18 complex numbers in the complex plane. The first nine are "starting point" numbers labeled $\{A, B, C, D, E, F, G, H, J\}$. The remaining nine are "ending point" numbers that are derived by doing the following to each of the starting points (in a scrambled order):

1.	2.	3.	4.	5.	6.	7.	8.	9.
z^*	$\lvert z \rvert$	$e^{j0.1\pi}z$	$z - 0.1j$	jz	$1 + e^{-j0.1\pi}z$	$1/z$	$1/z^*$	z^2

Match both the operation and the starting point to each ending point by writing both a number from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and a letter from $\{A, B, C, D, E, F, G, H, J\}$ in each answer box. For example, write "8-J" to indicate that the number results from operation 8 applied to number J.



PROB. Su17-Q1.2.

Consider a real-valued signal $x(t)$ whose spectrum is shown below:

(a) Specify numeric values for all of the following constants:

$b =$ ☐ $c =$ ☐ $d =$ ☐

(b) The signal $x(t)$ is periodic with fundamental frequency $f_0 =$ ☐ Hz.

(c) In the Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_0 t}$, identify numeric values for the following FS coefficients:

$a_0 =$ ☐ $a_1 =$ ☐ $a_2 =$ ☐ $a_3 =$ ☐ $a_4 =$ ☐ $a_5 =$ ☐

(d) If $y(t) = x(t) + A\cos(2\pi f_1 t + \theta)$ is periodic with fundamental frequency $f_0 = 24$ Hz, then it must be that

$A =$ ☐ $f_1 =$ ☐ Hz $\theta =$ ☐

PROB. Su17-Q1.3.

Consider the equation below, in which two sinusoids are added to yield a third:

$$4\cos(2\pi f_0 t + \theta) + A\cos(2026\pi t + 0.3\pi) = \cos(2\pi f_0 t + \theta).$$

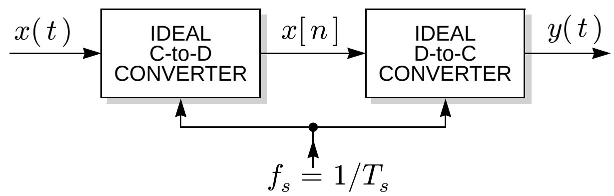
Solve this equation for the unknown parameters A, f_0, and θ:

$A =$ [ ] ≥ 0,

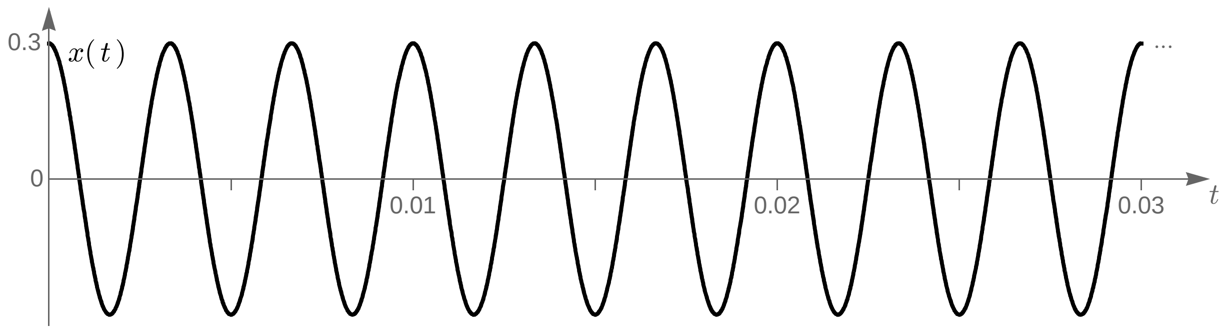
$f_0 =$ [ ] Hz,

$\theta =$ [ ] $\in (-\pi, \pi]$.

PROB. Su17-Q1.4.



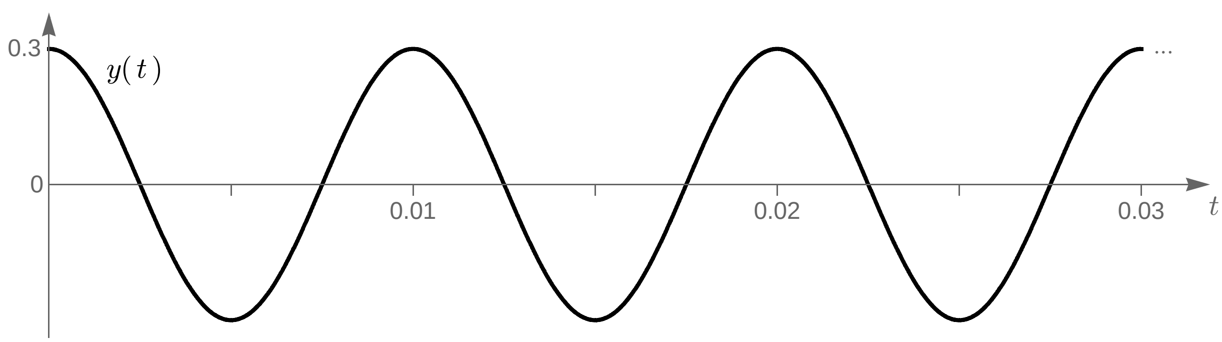
Consider a system which samples the input $x(t)$ shown below with sampling rate f_s, and then feeds the samples immediately to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$:



(a) In order for the reconstructed signal $y(t)$ to be equal to the original input $x(t)$, we need

$f_s >$ [ ] samples/s.

(b) In order for the reconstructed signal $y(t)$ to be the signal shown below,



the sampling rate must be either $f_s =$ [ ] or $f_s =$ [ ] samples/s.

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SOLUTIONS

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PROB. Su17-Q1.1. Shown below are 18 complex numbers in the complex plane. The first nine are "starting point" numbers labeled $\{$A, B, C, D, E, F, G, H, J$\}$. The remaining nine are "ending point" numbers that are derived by doing the following to each of the starting points (in a scrambled order):

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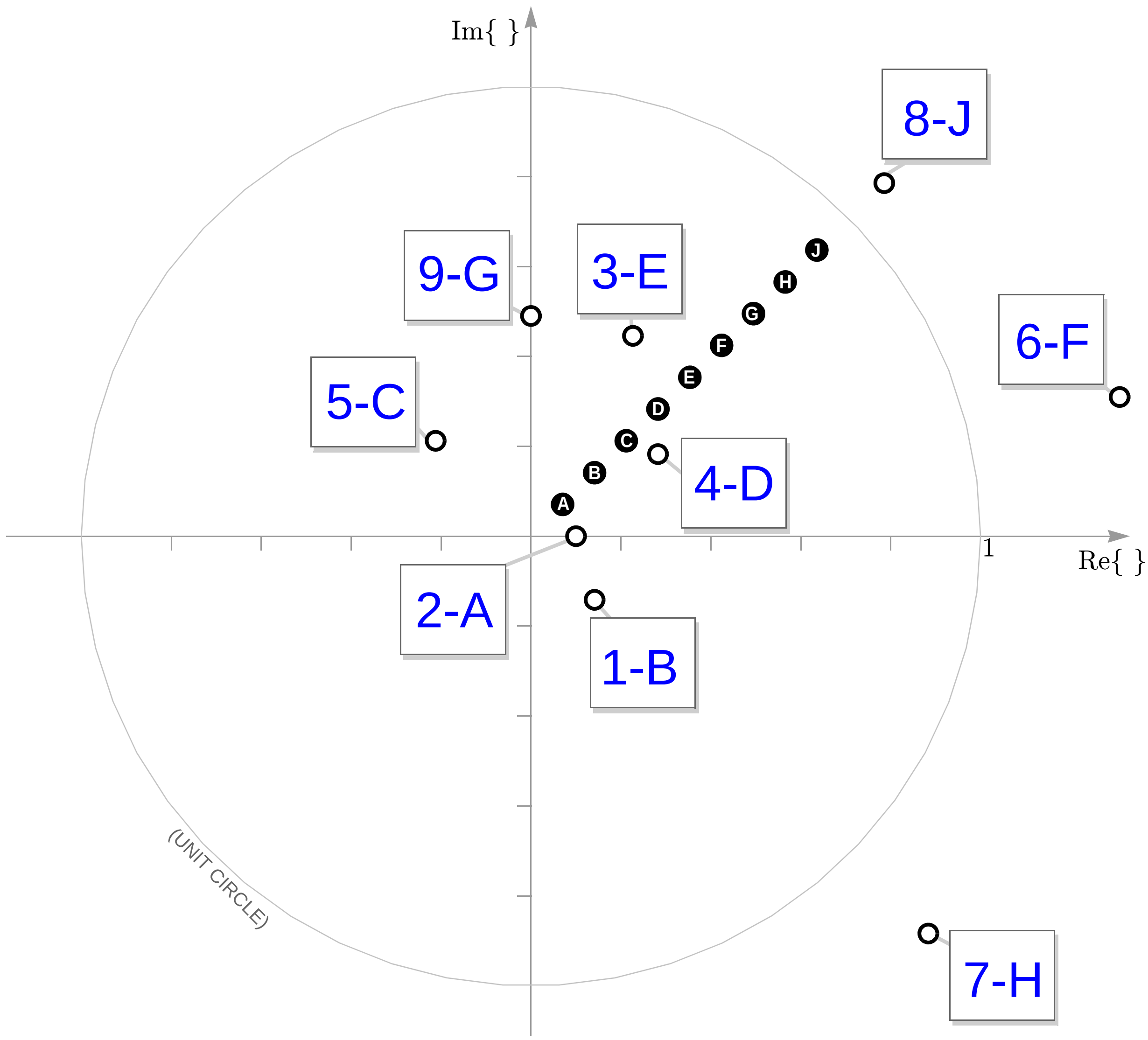
z^*	$\lvert z \rvert$	$e^{j0.1\pi}z$	$z - 0.1j$	jz	$1 + e^{-j0.1\pi}z$	$1/z$	$1/z^*$	z^2

Match both the operation and the starting point to each ending point by writing both a number from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and a letter from $\{$A, B, C, D, E, F, G, H, J$\}$ in each answer box. For example, write "8-J" to indicate that the number results from operation 8 applied to number J.



PROB. Su17-Q1.2.

Consider a real-valued signal $x(t)$ whose spectrum is shown below:

(a) Specify numeric values for all of the following constants:

$b =$ -2.753 $c =$ 3.403 $d =$ -0.707

(b) The signal $x(t)$ is periodic with fundamental frequency $f_0 =$ 8 Hz.

(c) In the Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_0 t}$, identify numeric values for the following FS coefficients:

$a_0 =$ 0 $a_1 =$ 0 $a_2 =$ 0 $a_3 =$ $ce^{j0.3\pi}$ $a_4 =$ 0 $a_5 =$ $e^{-j0.25\pi}$

$3.4e^{j0.3\pi}$

$2 + 2.75j$

0.707 - 0.707j

(d) If $y(t) = x(t) + A\cos(2\pi f_1 t + \theta)$ is periodic with fundamental frequency $f_0 = 24$ Hz, then it must be that

$A =$ 2 $f_1 =$ 40 Hz $\theta =$ 0.75π

PROB. Su17-Q1.3.

Consider the equation below, in which two sinusoids are added to yield a third:

$$4\cos(2\pi f_0 t + \theta) + A\cos(2026\pi t + 0.3\pi) = \cos(2\pi f_0 t + \theta).$$

Solve this equation for the unknown parameters A, f_0, and θ:

$A =$ **3** ≥ 0,

$f_0 =$ **1013** Hz,

$\theta =$ **-0.7π** $\in (-\pi, \pi]$

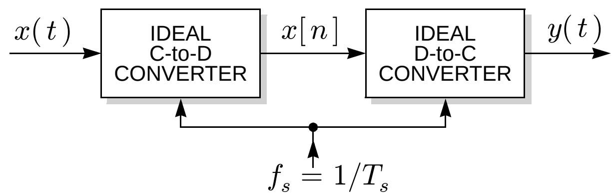
The corresponding phasor equation is:

$$4e^{j\theta} + Ae^{j0.3\pi} = e^{j\theta}$$

$$\Rightarrow \quad 4 + Ae^{-j\theta}e^{j0.3\pi} = 1$$

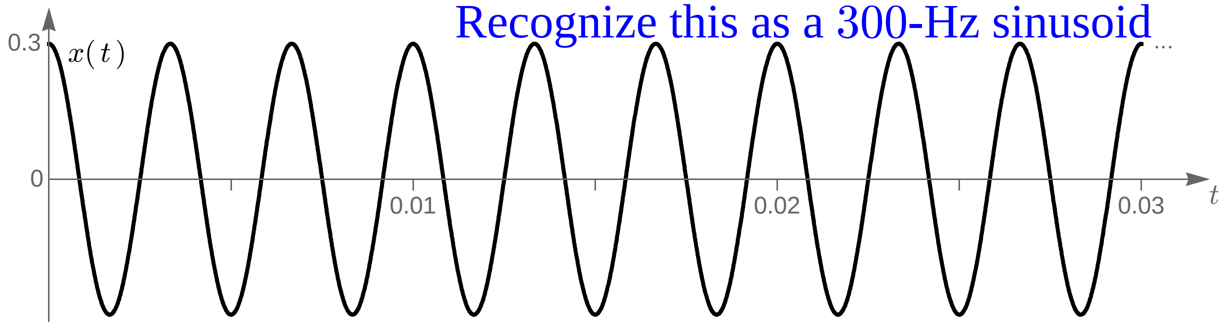
$$\Rightarrow \quad Ae^{-j\theta} = \frac{-3}{e^{j0.3\pi}} = 3e^{j0.7\pi}$$

PROB. Su17-Q1.4.



Consider a system which samples the input $x(t)$ shown below with sampling rate f_s, and then feeds the samples immediately to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$:

Recognize this as a 300-Hz sinusoid



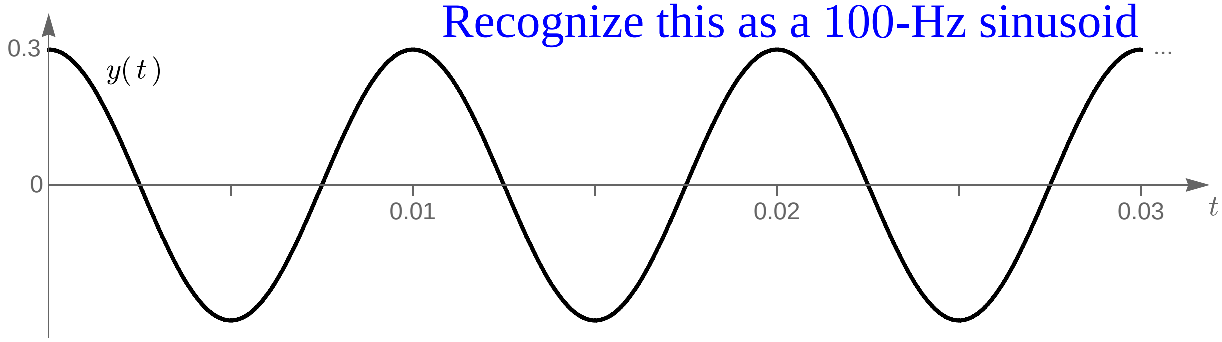
(a) In order for the reconstructed signal $y(t)$ to be equal to the original input $x(t)$, we need

Sampling theorem:
$f_s > 2f_{\max} = 2(300)$

$f_s >$ 600 samples/s.

(b) In order for the reconstructed signal $y(t)$ to be the signal shown below,

Recognize this as a 100-Hz sinusoid



the sampling rate must be either $f_s =$ 400 or $f_s =$ 200 samples/s.

Option 1: 300 Hz aliases down to 100 Hz
$\Rightarrow$ 200 Hz of shift
$\Rightarrow$ sample 200 Hz too slow $\Rightarrow$ $f_s = 600 - 200 = 400$ Hz

Option 2: 300 Hz aliases to –100 Hz
$\Rightarrow$ 400 Hz too slow $\Rightarrow$ $f_s = 600 - 400 = 200$ Hz