GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2016
Quiz #1

June 15, 2016

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

To avoid losing 3 points, circle your recitation section:

	Tue
10 – 11:45am	L01 (Stuber)
12 – 1:45pm	L02 (Stuber)
2 – 3:45pm	L03 (Zhang)
4 – 5:45pm	L04 (Zhang)

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	–3	
Total		

PROB. Su16-Q1.1. (The different parts of this problem are unrelated. All answers are real numbers.)

(a) Solve $re^{j\theta} = \sum_{k=1}^{2026} j^k$ for $r =$ ☐ ≥ 0 and $\theta =$ ☐ $\in (-\pi, \pi]$.

(b) Solve $(1.5 + j)e^{j\theta} = re^{j\theta} + 1$ for $r =$ ☐ ≥ 0 and $\theta =$ ☐ $\in (-\pi, \pi]$.

(c) Solve $x + jy = \dfrac{e^{j\pi/3}}{x + jy}$ for $x =$ ☐ and $y =$ ☐.

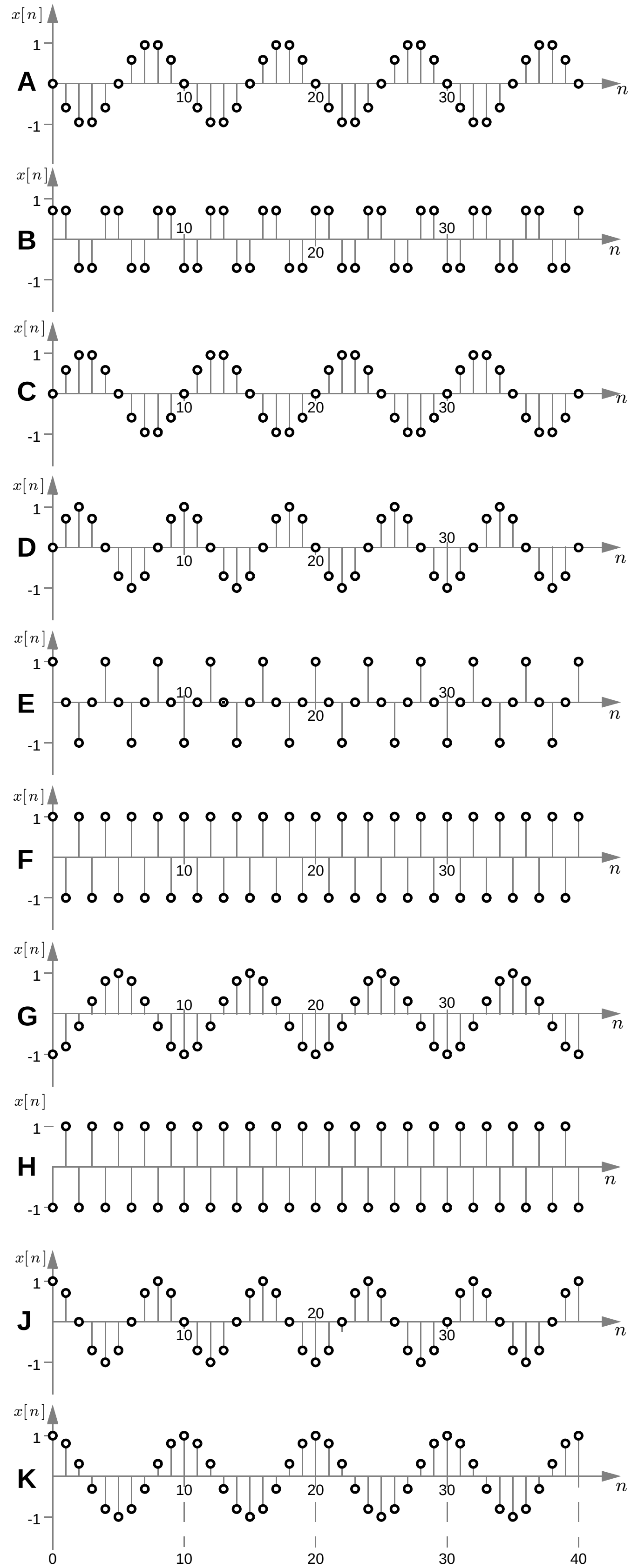
(d) Solve $\dfrac{1 + j}{r - j} = e^{j\theta}$ for $r =$ ☐ ≥ 0 and $\theta =$ ☐ $\in (-\pi, \pi]$.

(e) Solve $\sin(t) + \cos(t-1) = A\cos(t + \varphi)$ for $A =$ ☐ ≥ 0 and $\varphi =$ ☐ $\in (-\pi, \pi]$.

PROB. Su16-Q1.2.

Consider the discrete-time sinusoidal signals shown in the stem plots on the right, labeled A, B, C, ... K.

Match each stem plot to its corresponding equation below by writing the appropriate letter (from A ... K) into each answer box.



$x[n] = \cos(4.25\pi n + 3.5\pi)$

$x[n] = \cos(2.5\pi n - 0.25\pi)$

$x[n] = \cos(2026.2\pi n - 0.5\pi)$

$x[n] = \cos(3\pi n + \pi)$

$x[n] = \cos(2.25\pi n)$

$x[n] = \cos(7.8\pi n - 0.5\pi)$

$x[n] = \cos(4.2\pi n)$

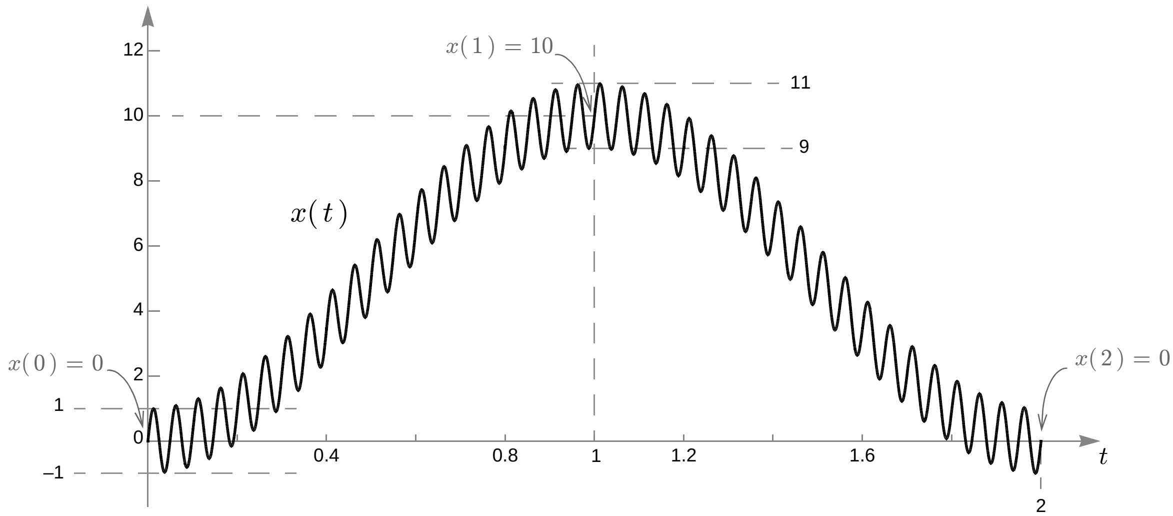
$x[n] = \cos(1.5\pi n)$

$x[n] = \cos(5\pi n)$

$x[n] = \cos(0.2\pi n - 5\pi)$

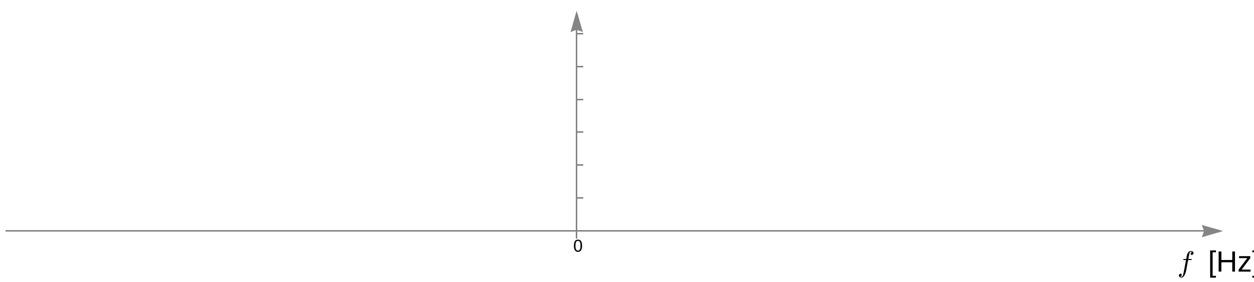
PROB. Su16-Q1.3.

Shown below is one full period of a *periodic* signal $x(t)$ that satisfies $x(t) = x(t+2)$, $x(0) = 0$ and $x(1) = 10$:



(a) The fundamental frequency is $f_0 =$ ☐ Hz, and the DC component is $a_0 =$ ☐.

(b) In the space below, sketch as accurately as possible the two-sided spectrum for $x(t)$.
(*Hint:* There are a total of five lines.) *Label* the *frequency* (Hz) and *amplitude* (polar) for each line.

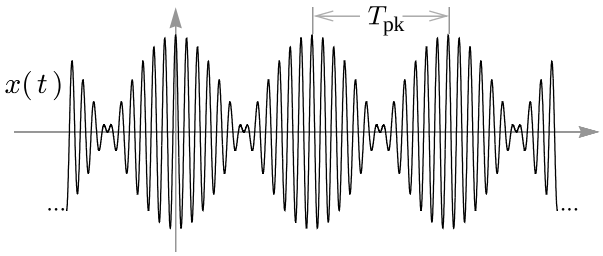


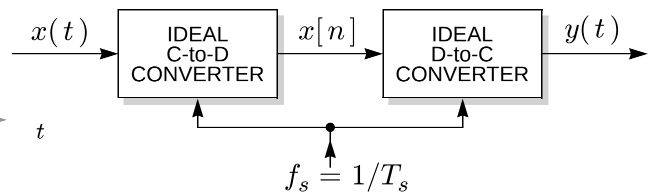
(c) In the Fourier series expansion $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 kt}$, the number of nonzero a_k coefficients is ☐.

(d) In the Fourier series expansion $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 kt}$, the k=1 coefficient is $a_1 =$ ☐.

PROB. Su16-Q1.4.

Suppose that the *product* of sinusoids $x(t) = \cos(4\pi t)\cos(100\pi t)$ sketched below is sampled with sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$:





(a) The time between peaks of the input $x(t)$, as indicated above, is $T_{pk} =$ ☐ seconds.

(b) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ ☐ Hz.

(c) For what values of the sampling rate f_s will the output $y(t)$ be the same as the input $x(t)$?

$f_s >$ ☐ samples/s.

(d) If the output is a *single* sinusoid of the form $y(t) = A\cos(2\pi f_1 t + \varphi)$, where the frequency satisfies $f_1 > 25$ Hz, then it must be that:

$f_s =$ ☐ samples/s

$A =$ ☐ ≥ 0

$f_1 =$ ☐ Hz

$\varphi =$ ☐ radians.

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PROB. Su16-Q1.1. **(The different parts of problem are unrelated. All answers are real numbers.)**

(a) **Solve** $re^{j\theta} = \sum_{k=1}^{2026} j^k$ **for** $r =$ $\boxed{\sqrt{2}}$ ≥ 0 **and** $\theta =$ $\boxed{+0.75\pi}$ $\in (-\pi, \pi]$.

$j = e^{j2\pi/4}$ is a 4-th root of unity

$\Rightarrow$ the first $(506)(4) = 2024$ terms sum to zero.

Including the last two terms yields $re^{j\theta} = 0 + j + j^2 = -1 + j = \sqrt{2}\, e^{+j0.75\pi}$

(b) **Solve** $(1.5 + j)e^{j\theta} = re^{j\theta} + 1$ **for** $r =$ $\boxed{1.5}$ ≥ 0 **and** $\theta =$ $\boxed{-0.5\pi}$ $\in (-\pi, \pi]$.

Dividing both sides by $e^{j\theta}$:

$1.5 + j = r + e^{-j\theta}$

Taking the imaginary part of both sides:

$\Rightarrow 1 = -\sin(\theta)$

$\Rightarrow \theta = -0.5\pi$

With $\theta = -0.5\pi$, this equation reduces to:

$1.5 + j = r + j$

$\Rightarrow r = 1.5$

(c) **Solve** $x + jy = \dfrac{e^{j\pi/3}}{x + jy}$ **for** $x =$ $\boxed{\pm\frac{\sqrt{3}}{2}}$ **and** $y =$ $\boxed{\pm 0.5}$.

$\approx \pm 0.866$

(signs must match)

Multiplying both sides by $z = x + jy$:

$\Rightarrow z^2 = e^{j\pi/3}$

Euler

$\Rightarrow z = \pm e^{j\pi/6} = \pm(\cos(\pi/6) + j\sin(\pi/6)) = \pm(\frac{\sqrt{3}}{2} + 0.5j)$

(d) **Solve** $\dfrac{1 + j}{r - j} = e^{j\theta}$ **for** $r =$ $\boxed{1}$ ≥ 0 **and** $\theta =$ $\boxed{0.5\pi}$ $\in (-\pi, \pi]$.

Taking squared magnitude of both sides:

$\Rightarrow \left|\dfrac{1 + j}{r - j}\right|^2 = \dfrac{2}{r^2 + 1} = |e^{j\theta}|^2 = 1$

$\Rightarrow r^2 = 1 \quad \Rightarrow r = 1$ (to satisfy ≥ 0)

Substituting $r = 1$ yields:

$\dfrac{1 + j}{1 - j} = \dfrac{\sqrt{2}e^{j0.25\pi}}{\sqrt{2}e^{-j0.25\pi}} = e^{j0.5\pi} = e^{j\theta} \quad \Rightarrow \theta = 0.5\pi$

(e) **Solve** $\sin(t) + \cos(t-1) = A\cos(t + \varphi)$ **for** $A =$ $\boxed{1.92}$ ≥ 0 **and** $\varphi =$ $\boxed{-0.41\pi}$ $\in (-\pi, \pi]$.

Complex ampltiude of $\sin(t) = \cos(t - 0.5\pi)$ is $e^{-j0.5\pi}$

Complex ampltiude of $\cos(t-1)$ is e^{-j}

Phasor addition rule $\Rightarrow Ae^{j\varphi} = e^{-j0.5\pi} + e^{-j} = 1.92e^{-j0.41\pi}$

PROB. Su16-Q1.2.

Consider the discrete-time sinusoidal signals shown in the stem plots on the right, labeled A, B, C, ... K.

Match each stem plot to its corresponding equation below by writing the appropriate letter (from A ... K) into each answer box.

1. Simplify equations by reducing freq to $\pm\pi$ range
2. Use period N to determine which of four $\hat{\omega}$ values.
3. Use value at time zero to distinguish phase.

D $x[n] = \cos(4.25\pi n + 3.5\pi)$
$= \cos(0.25\pi n - 0.5\pi)$

B $x[n] = \cos(2.5\pi n - 0.25\pi)$
$= \cos(0.5\pi n - 0.25\pi)$

C $x[n] = \cos(2026.2\pi n - 0.5\pi)$
$= \cos(0.2\pi n - 0.5\pi)$

H $x[n] = \cos(3\pi n + \pi)$
$= \cos(\pi n + \pi)$

J $x[n] = \cos(2.25\pi n)$
$= \cos(0.25\pi n)$

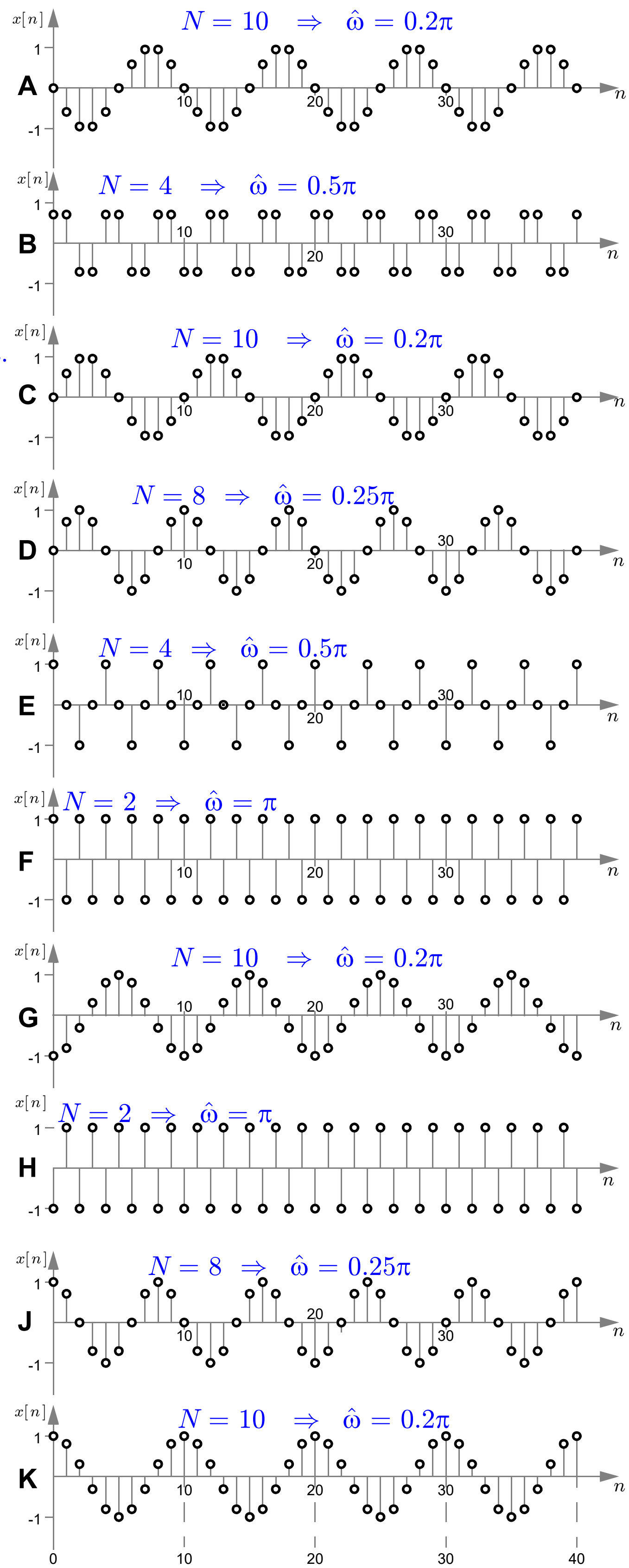
A $x[n] = \cos(7.8\pi n - 0.5\pi)$
$= \cos(0.2\pi n + 0.5\pi)$

K $x[n] = \cos(4.2\pi n)$
$= \cos(0.2\pi n)$

E $x[n] = \cos(1.5\pi n)$
$= \cos(0.5\pi n)$

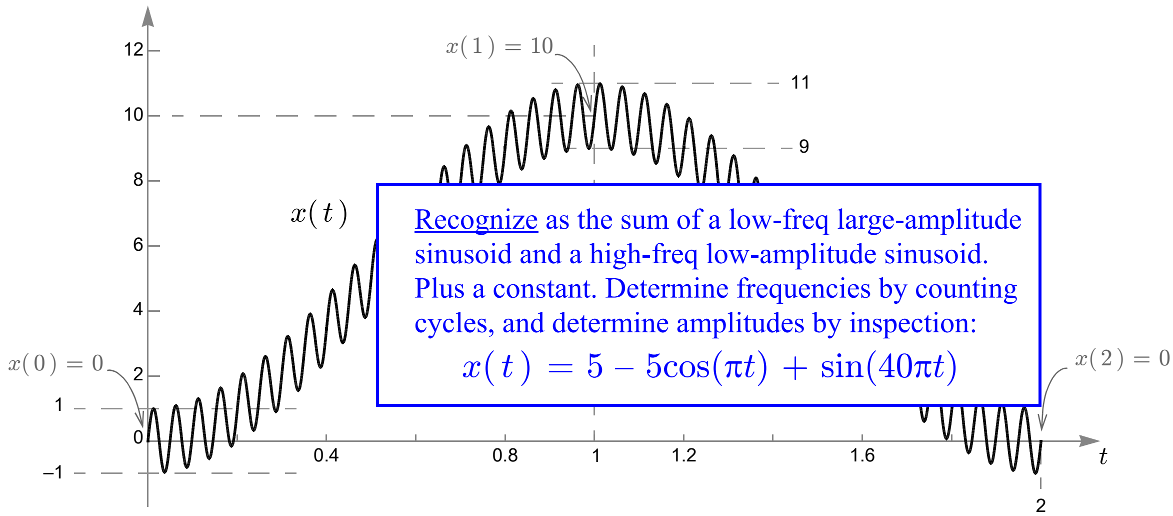
F $x[n] = \cos(5\pi n)$
$= \cos(\pi n)$

G $x[n] = \cos(0.2\pi n - 5\pi)$
$= \cos(0.2\pi n - \pi)$



PROB. Su16-Q1.3.

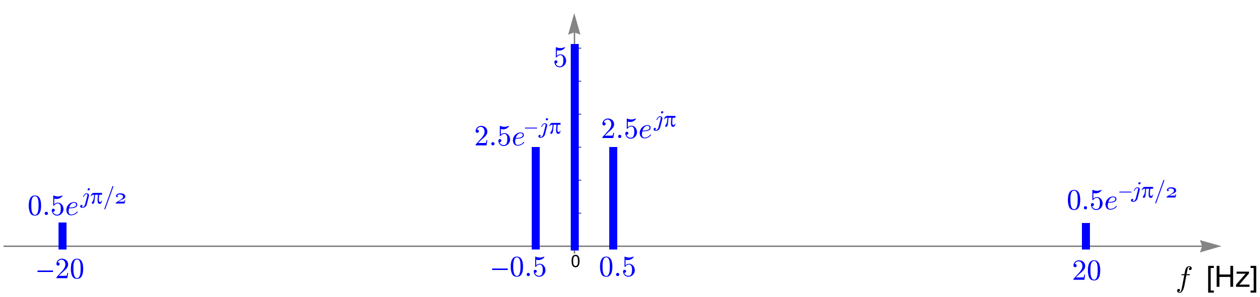
Shown below is one full period of a *periodic* signal $x(t)$ that satisfies $x(t) = x(t+2)$, $x(0) = 0$ and $x(1) = 10$:



(a) **The fundamental frequency is $f_0 =$** 0.5 **Hz, and the DC component is $a_0 =$** 5 **.**

We are told $x(t) = x(t+2)$

(b) **In the space below, sketch as accurately as possible the two-sided spectrum for $x(t)$. (*Hint:* There are a total of five lines.) *Label* the *frequency* (Hz) and *amplitude* for each line.**



(c) **In the Fourier series expansion $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 kt}$, the <u>number of nonzero</u> a_k's is** 5 **.**

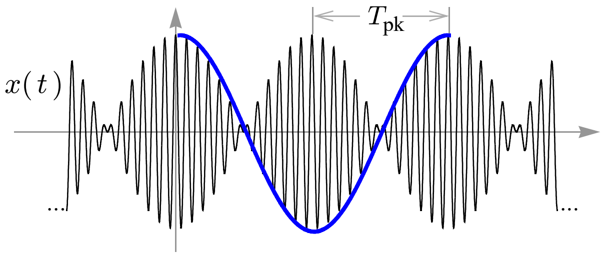
One for each spectral line.

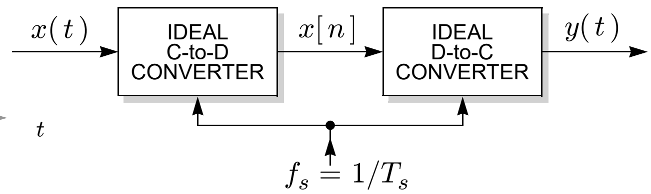
(d) **In the Fourier series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 kt}$, the k=1 coefficient is $a_1 =$** $2.5e^{j\pi}$ **.**

The complex amplitude of the line at the fundamental frequency $f_0 = 0.5$ Hz is a_1. We can read the answer off of the spectrum.

PROB. Su16-Q1.4.

Suppose that the *product* of sinusoids $x(t) = \cos(4\pi t)\cos(100\pi t)$ sketched below is sampled with sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$:





(a) **The time between peaks of the input $x(t)$, as indicated above, is $T_{pk} =$** 0.25 **seconds.**

One period of the slow sinusoid $\cos(4\pi t)$ is overlaid in the above figure.
The slow sinusoid has period 0.5 s. The time between peaks is thus half of that, $T_{pk} = 0.25$.

(b) **The input $x(t)$ is periodic with fundamental frequency $f_0 =$** 4 **Hz.**

One approach: By inspection, fundamental period is $T_0 = 0.25 \Rightarrow f_0 = 4$ Hz.

Another: Via trig identity, $x(t) = 0.5\cos(96\pi t) + 0.5\cos(104\pi t)$.
In other words, the sum of a 48-Hz and 52-Hz sinusoid.
The fundamental freq is thus $f_0 = \gcd(48, 52) = 4$ Hz.

(c) **For what values of the sampling rate f_s will the output $y(t)$ be the same as the input $x(t)$?**

$f_s >$ 104 **samples/s.**

Sampling theorem: $f_s > 2f_{max} = 2(52) = 104$ Hz

(d) **If the output is a *single* sinusoid of the form $y(t) = A\cos(2\pi f_1 t + \varphi)$, where the frequency satisfies $f_1 > 25$ Hz, then it must be that:**

$f_s =$ 100 **samples/s**

$A =$ 1 ≥ 0

$f_1 =$ 48 **Hz**

$\varphi =$ 0 **radians.**

Input is $x(t) = 0.5\cos(96\pi t) + 0.5\cos(104\pi t)$.
We need aliasing to get a single sinusoid output.
In particular, we need the 52-Hz sinusoid to alias down to align with the 48-Hz sinusoid.

This is 4 Hz of aliasing. So we must sample too slow by 4 Hz:

$\Rightarrow f_s = 104 - 4 = 100$ Hz.

$$\Rightarrow x[n] = 0.5\cos(96\pi n/100) + 0.5\cos(104\pi n/100)$$
$$= 0.5\cos(96\pi n/100) + 0.5\cos(-96\pi n/100)$$
$$= \cos(96\pi n/100).$$

$$\Rightarrow y(t) = x[100t] = \cos(96\pi t).$$