GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 1

DATE: 10-June-15 COURSE: ECE-2026

NAME: ______________________________ GTID: ______________
LAST, FIRST ex: gtaburDEll

Circle your correct **recitation section** number - failing to do so will cost you 3 points

L01(MW) 4-545: BARRY	L02(TTH) 10-1145: ZHANG	L03 (TTH) 12-145: ZHANG

- Write your name on the front page ONLY. **<u>DO NOT unstaple the test</u>**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}''\times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK TO RECEIVE CREDIT**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. **<u>Circle</u>** your answers, or write them in the **<u>boxes/spaces</u>** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π instead of 1.257)**
- **ALL RADIAN ANSWERS SHOULD BE IN THE RANGE (-π, π].**

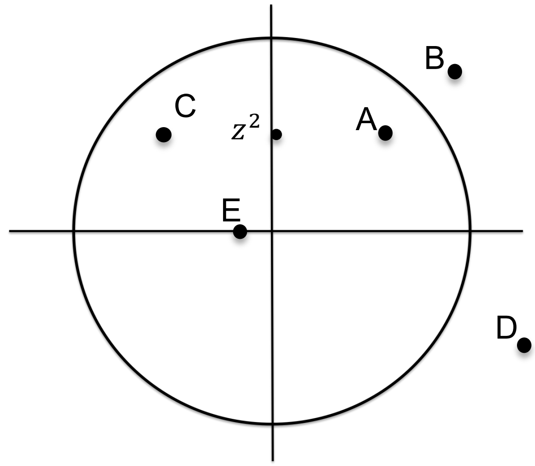
Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

PROBLEM 1:

Parts a, b, and c can be solved independently of each other.

(a) Consider a complex number defined as $z = re^{j\theta}$ (where $\theta \in (0, \pi]$). The location of z^2 is shown on the complex plane below (where the circle represents the unit circle of radius 1). Consider the following lettered operations on the complex number z (A-E). Place the appropriate letter on the provided complex plane in the approximate location that it should be. (They do not have to be exact, but they should still be correct relative to the position of z^2). (10 points)

A.	z
B.	$1/z^*$
C.	$z^* e^{-j\pi}$
D.	$z^* + e^{j2\pi}$
E.	z^4



(b) Let $A\cos(\omega_0 t + \varphi) = 3\cos\left(20\pi t + \frac{\pi}{6}\right) + 2\cos\left(20\pi t - \frac{\pi}{3}\right) + 5\sin(20\pi t)$. Find $A, \omega_0, \text{and } \varphi$. (5 points)

Use phasor addition

$A =$ __6.35__________ $\omega_0 =$ __20π__________ $\varphi =$ __ -0.3082π_____

(c) Solve the following equation for r and θ.

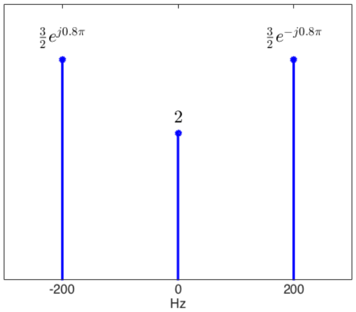
$$\sum_{k=-16}^{16} e^{j\frac{\pi}{16}k} = re^{j\theta}$$

$$\sum_{k=-16}^{16} e^{j\frac{2\pi}{32}k} = \sum_{k=-16}^{15} e^{j\frac{2\pi}{32}k} + e^{j\frac{2\pi}{32}16} = 0 + e^{j\pi} = e^{j\pi}$$

$r =$ _____1__________ $\theta =$ _____π__________

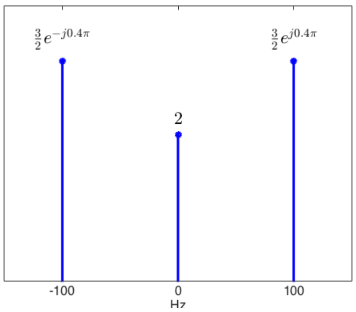
PROBLEM 2:
Match the spectrums to the appropriate sinusoidal plots. Write your answers **in the boxes** provided.

F



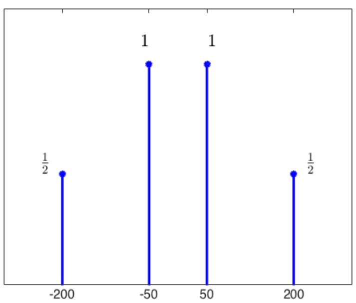
$T_0 = 0.005$
$t_d = -\frac{-0.8\pi}{400\pi}$
$= 0.002$

D



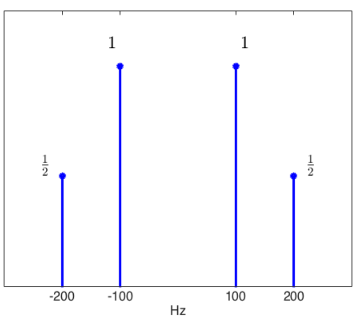
$T_0 = 0.01$
$t_d = -\frac{0.4\pi}{200\pi}$
$= -.002$

A



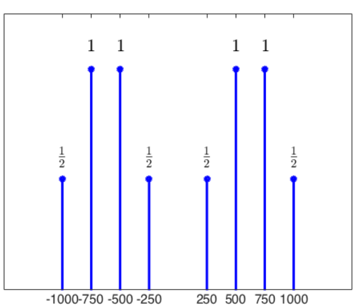
$f_0 = 50$
$T_0 = 0.02$

C



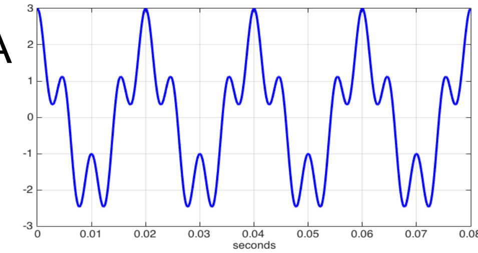
$f_0 = 100$
$T_0 = 0.01$

E

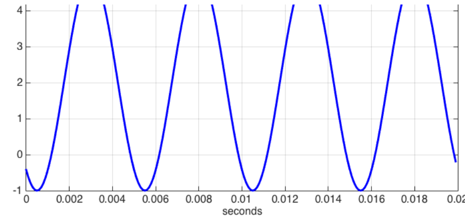


$f_0 = 250$
$T_0 = 0.004$

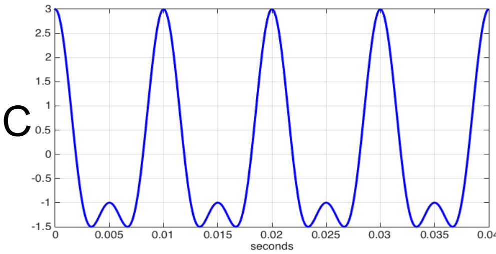
A



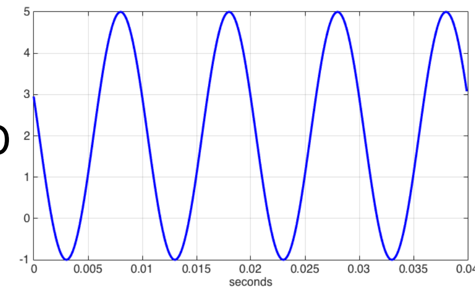
B



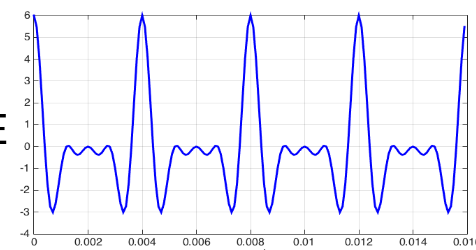
C



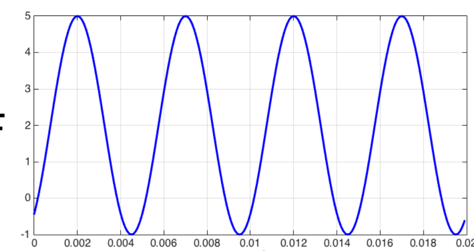
D



E



F



PROBLEM 3:
Assume that the Fourier series coefficients for the representation of $x(t)$ are given by the integral:

$$a_k = \frac{1}{2}\int_0^1 (1+t)\, e^{-j\pi kt}\, dt$$

Put your answers in the boxes provided.

(a) Determine the fundamental period T_0 of the signal $x(t)$. (4 points)

Can be seen from the equation format.

2

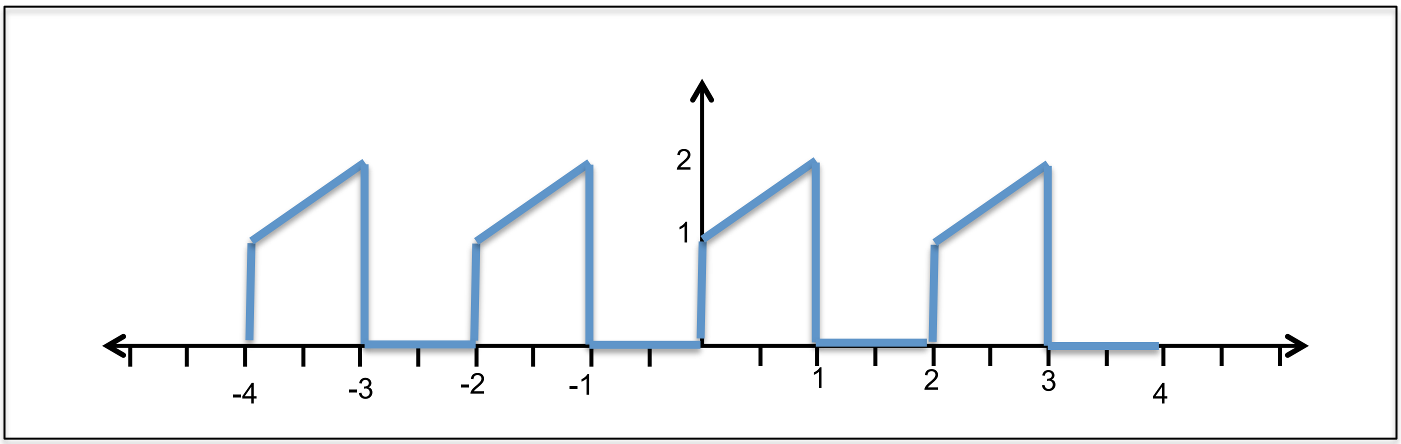
(b) Determine the DC value of $x(t)$. Give your answer as a number. (4 points)

Can be solved with direct integration or realizing that this is the area of a triangle on top of a 1x1 square. The area is therefore:

$$\frac{1}{2} * [(area\ of\ 1x1\ square) + \ triangle] = \frac{1}{2}\left[(1*1) + \frac{1}{2}(1*1)\right] = \frac{3}{4}$$

3/4

(c) In the plot area below, draw a plot of $x(t)$ over the range of $-4 \le t \le 4$ seconds. Label your plot carefully. (8 points)



(d) Assume that a new signal is created from $x(t)$ as follows:

$$y(t) = 10x(t - 0.5)$$

Determine the fundamental period T_0 of $y(t)$ and the DC value of $y(t)$. (4 points)

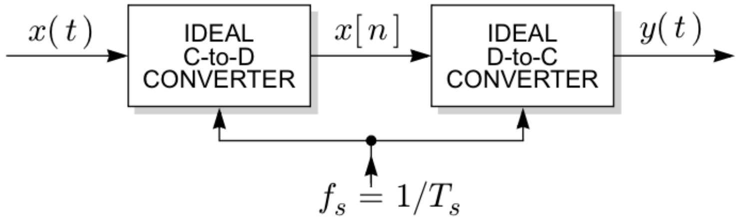
Scaling and shifting does not affect the period. The DC value would be scaled by 10.

T_0: 2 seconds

DC: 15/2

PROBLEM 4:

Suppose that a continuous-time signal $x(t)$ is sampled with sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$ as shown below:



The input signal is defined as:

$$x(t) = 3 + \cos(150\pi t + \varphi) + \cos(900\pi t + \theta)$$

(a) What is the Nyquist Rate for $x(t)$? (4 points)

2 * max frequency = 2*450 = 900 Hz

900 Hz

(b) Find the value for f_s such that: $y(t) = 3 + \cos(150\pi t + \varphi) + \cos(100\pi t - \theta)$ (8 points)

We need the copy @ 450Hz to fold to 50Hz (note the change in phase). However, we can not alias the component at 75Hz. We can accomplish this with $f_s = 500$Hz

500 Hz

(c) Find the **largest** possible value for f_s such that: $y(t) = A$ (i.e., it is a constant) (8 points)

Note that the x(t) is periodic with $f_0 = 75$Hz. To get $y(t) = A$, we need to alias BOTH sinusoids to DC. Since they are both integer multiples of 75Hz, setting $f_s = 75$Hz would alias both signals to DC. (NOTE: Any larger and the component at 75Hz would not alias DC. There are smaller values that would work however. For example, setting $f_s = 75/k$ (where *k* is an integer) would also results in both signals being aliased to DC. However, 75 is the **largest** (for *k*=1) as requested by the problem.

75 Hz

PROBLEM 5:
Suppose the following MATLAB code is run:

```
tt = 0:(1/2000):3;
xx = cos(400*pi*tt.^2 + 200*pi*tt + pi/3);
```

If a box is provided, write your answer in the box.

(a) How many samples are contained in the vector xx? (4 points)

2000*3+1=6001 (we add one because 0 is included).

6001

(b) Find the equation for the instantaneous frequency, $f_i(t)$ (in Hz). (6 points)

$$\frac{1}{2\pi}\frac{d\psi(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left(400\pi t^2 + 200\pi t + \frac{\pi}{3}\right) = \frac{1}{2\pi}(800\pi t + 200\pi) = 400t + 100\ (Hz)$$

$f_i(t) = 400t + 100\ (Hz)$

(c) Will the signal xx alias at any point during the interval from 0 to 3 seconds? If NO, explain why it will NOT alias. If YES, explain why it WILL alias and find the time (in seconds) at which point it will alias. (Circle one and then provide your explanation) (YES OR NO) (5 points)

From 0 to 3 seconds, the instantaneous frequency varies from 100 Hz to 1300 Hz. The sampling rate is 2000 Hz meaning every frequency over 1000 Hz will begin to alias. To find the point where aliasing beings we solve:

$$400t + 100 = 1000 \Rightarrow t = \frac{9}{4}\text{ seconds}$$

(d) Assuming that the time vector is redefined to be a very long time length (e.g., tt=0:1/2000:10000000), specify the **lowest** and the **highest** *unique* frequency component that will be present in the signal vector xx (including aliasing) if the spectrogram was plotted in MATLAB. (Remember to explain/show your reasoning.) (5 points)

x(t) is a linear chirp increasing from 100 Hz. At some point, it will cross integer multiples of 2000 which will alias to 0 Hz. The Nyquist Frequency for a sampling rate of 2000 Hz is 1000 Hz which is the highest frequency that can be represented without aliasing. All other frequencies past the time determined in part c will simply cycle through frequencies between 0 and 1000 Hz.

lowest frequency:
0 Hz

highest frequency:
1000 Hz