GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2014
Quiz #1

June 11, 2014

NAME: ____________ ______________ GT username: ______________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section (otherwise you lose 3 points!):

	Mon	Tue
10 – 11:45am		L02 (Moore)
12 – 1:45pm		L03 (Moore)
2:40 – 3:50pm		L04 (Davis)
4 – 5:45pm	L01 (Barry)	

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5" × 11") of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	–3	
Total		

PROB. Su14-Q1.1. (The different parts of this problem are unrelated. All answers are real numbers.)

(a) Solve $\frac{3+j}{re^{j\theta}} = 2j + 1$ for $r =$ [ ] ≥ 0 and $\theta =$ [ ] $\in (-\pi, \pi]$.

(b) Solve $je^{j0.1\pi} - 2 = \frac{1}{x - jy}$ for $x =$ [ ] and $y =$ [ ].

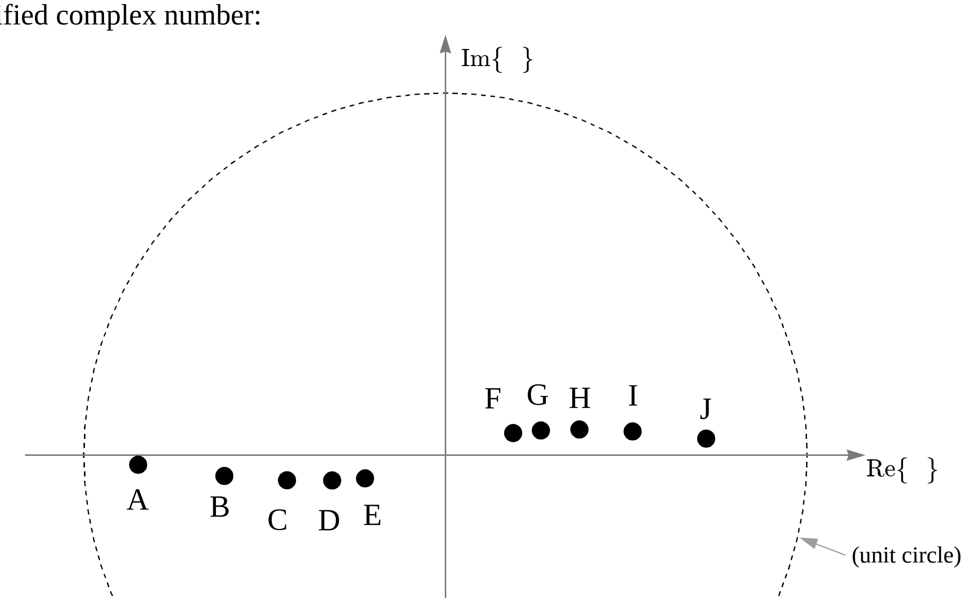
(c) Solve $\sum_{k=-1}^{19} e^{jk0.1\pi} = re^{j\theta}$ where $r =$ [ ] ≥ 0, $\theta =$ [ ] $\in (-\pi, \pi]$.

(d) Solve $z + \frac{1}{z^*} = 2e^{-j0.15\pi}$ for the unknown complex number $z = re^{j\theta}$:

$r =$ [ ] ≥ 0,

$\theta =$ [ ] $\in (-\pi, \pi]$.

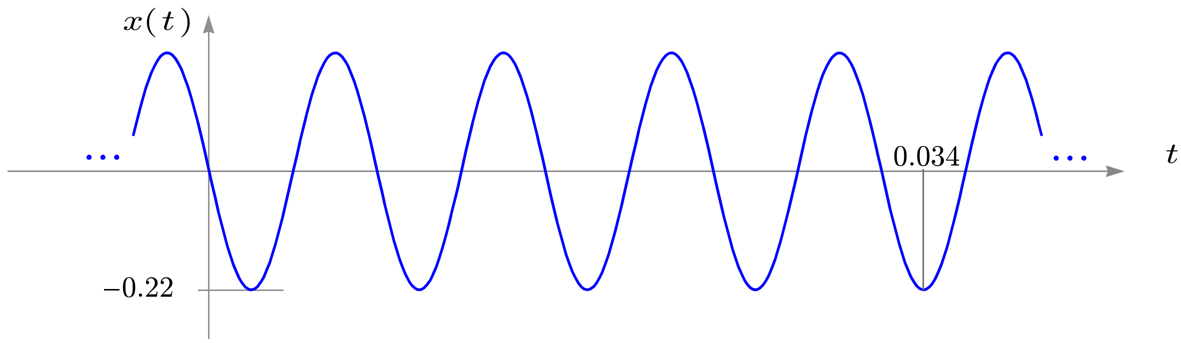
(e) Shown below are the locations of $\{z, z^2, z^3, ..., z^{10}\}$ in the complex plane, where z is an unspecified complex number:



Identify these locations by writing a letter (A, B, C, ... or J) in each answer box below.

z	z^2	z^3	z^4	z^5	z^6	z^7	z^8	z^9	z^{10}

PROB. Su14-Q1.2. The signal $x(t) = A\cos(\omega_0 t + \theta)$ is shown below. As illustrated below, it is zero at time $t = 0$, and it achieves a minimum value at time $t = 0.034$:



(a) Specify numeric values for the following parameters:

$A =$ [ ] ≥ 0,

$\omega_0 =$ [ ] rad/s,

$\theta =$ [ ] $\in (-\pi, \pi]$.

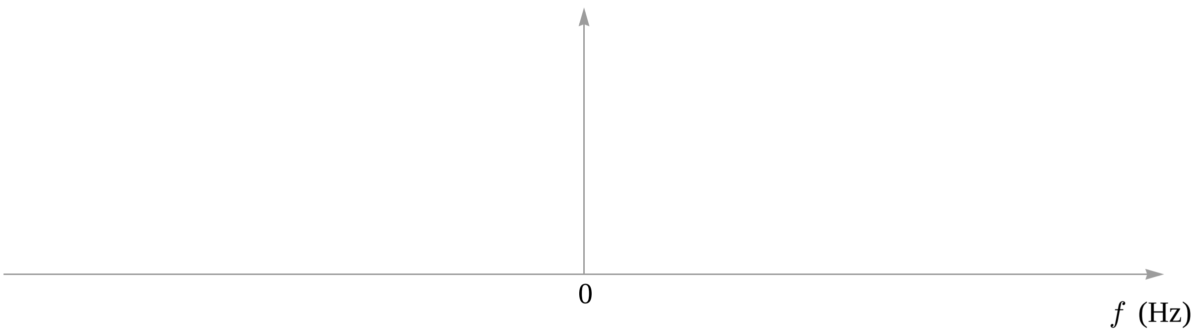
(b) Find a value for t_0 **in the range** $0.996 < t_0 < 1.004$ so that the signal $x(t) = A\cos(\omega_0 t + \theta)$ shown above can be written as the delayed sinusoid $x(t) = A\cos(\omega_0(t - t_0))$:

0.996 seconds $< t_0 =$ [ ] $<$ 1.004 seconds

PROB. Su14-Q1.3. All parts of this problem pertain to the signal:

$$x(t) = 8 + 8\cos(82\pi t + 0.2\pi) + 8\sin(102.5\pi t).$$

(a) Sketch the spectrum for $x(t)$ in the space below.
(For each line be careful to label both its frequency and its complex amplitude.)



(b) The signal $x(t)$ is periodic with: fundamental frequency $f_0 =$ ______ Hz,

and fundamental period $T_0 =$ ______ secs.

Hint: The remaining two parts below can be solved without performing any integration.

(c) Evaluate the integral (where T_0 is the fundamental period):

$$\int_0^{T_0} x(t)dt = \boxed{}.$$

(d) Evaluate the integral (where T_0 is the fundamental period):

$$\int_0^{T_0} x(t)e^{-j8\pi t/T_0}dt = \boxed{}.$$

PROB. Su14-Q1.4. The two parts of this problem are unrelated.

(a) Consider the equation below, in which two sinusoids are added to yield a third:

$$2\cos(\omega_0 t + \theta) + A\sin(\omega_0 t + 0.2\pi) = A\cos(122\pi t).$$

Solve this equation for the unknown parameters A, ω_0, and θ:

$A =$ [ ] ≥ 0,

$\omega_0 =$ [ ] rad/s,

$\theta =$ [ ] $\in (-\pi, \pi]$.

(b) Consider the equation below, in which the sum of N sinusoids is zero:

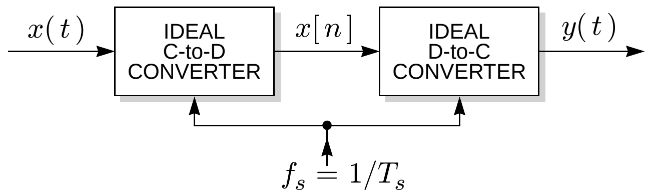
$$\sum_{k=1}^{N} \cos(2026\pi t + 0.02\pi k) = 0.$$

There are multiple values of N for which this equation is true.
Specify three different possible values, all of which are positive:

$N =$ [ ], or $N =$ [ ], or $N =$ [ ].

PROB. Su14-Q1.5.

Suppose that a continuous-time signal $x(t)$ is sampled with sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:



The input signal being sampled is the sum of two sinusoids:

$$x(t) = \cos(80\pi t) + \cos(160\pi t).$$

(a) Find a value for the sampling rate f_s so that the output $y(t)$ is periodic with fundamental frequency $f_0 = 40$ Hz:

$f_s =$ [ ] samples/s.

(b) Find the **largest** possible value for the sampling rate f_s so that the output $y(t)$ is periodic with fundamental frequency $f_0 = 20$ Hz:

$f_s =$ [ ] samples/s.

(c) Find the **largest** possible value for the sampling rate f_s so that the output is a constant, $y(t) = A$ for all time t:

$f_s =$ [ ] samples/s.

The corresponding constant is $y(t) = A =$ [ ].

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No/Wrong Rec	–3	
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PROB. Su14-Q1.1. (The different parts of this problem are unrelated. All answers are real numbers.)

(a) Solve $\frac{3+j}{re^{j\theta}} = 2j + 1$ for $r = \boxed{\sqrt{2}} \geq 0$ and $\theta = \boxed{-0.25\pi} \in (-\pi, \pi]$.

(b) Solve $je^{j0.1\pi} - 2 = \frac{1}{x - jy}$ for $x = \boxed{-0.37}$ and $y = \boxed{0.15}$.

(c) Solve $\sum_{k=-1}^{19} e^{jk0.1\pi} = re^{j\theta}$ where $r = \boxed{1} \geq 0$, $\theta = \boxed{-0.1\pi} \in (-\pi, \pi]$.

Only the $k = -1$ term; the other terms sum to zero

(d) Solve $z + \frac{1}{z^*} = 2e^{-j0.15\pi}$ for the unknown complex number $z = re^{j\theta}$:

$re^{j\theta} + \frac{1}{re^{-j\theta}} = (r + \frac{1}{r})e^{j\theta}$

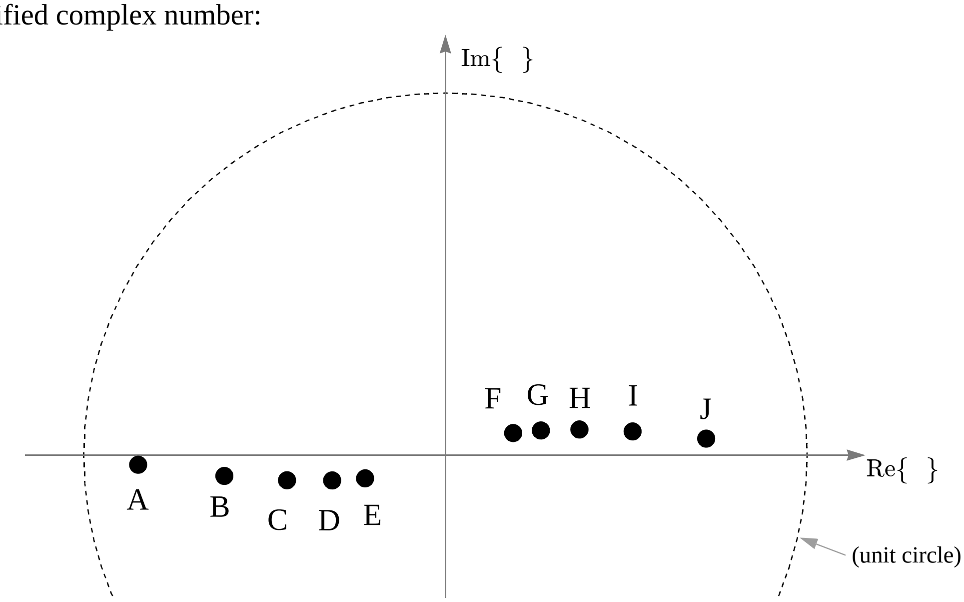
equate angles $\Rightarrow \theta = -0.15\pi$

equate magnitudes $\Rightarrow r + 1/r = 2 \Rightarrow r = 1.$

$r = \boxed{1} \geq 0,$

$\theta = \boxed{-0.15\pi} \in (-\pi, \pi].$

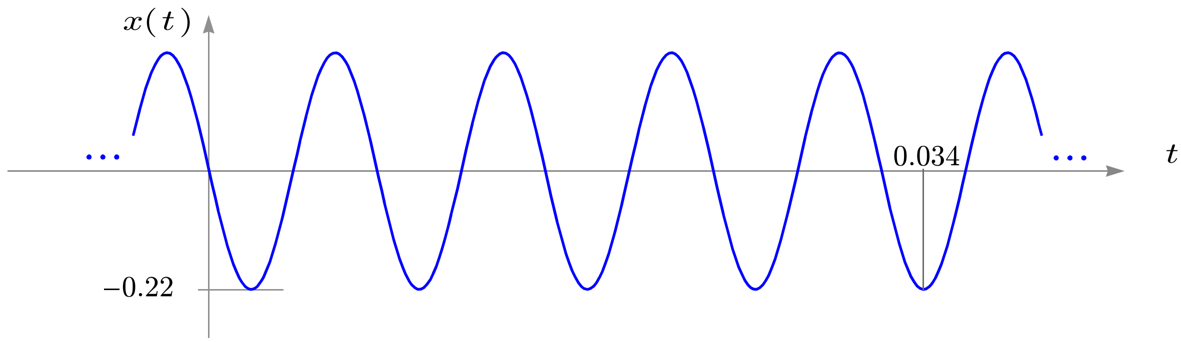
(e) Shown below are the locations of $\{z, z^2, z^3, ..., z^{10}\}$ in the complex plane, where z is an unspecified complex number:



Identify these locations by writing a letter (A, B, C, ... or J) in each answer box below.

A	J	B	I	C	H	D	G	E	F
z	z^2	z^3	z^4	z^5	z^6	z^7	z^8	z^9	z^{10}

PROB. Su14-Q1.2. The signal $x(t) = A\cos(\omega_0 t + \theta)$ is shown below. As illustrated below, it is zero at time $t = 0$, and it achieves a minimum value at time $t = 0.034$:



(a) Specify numeric values for the following parameters:

$A =$ 0.22 ≥ 0,

$\omega_0 =$ 250π rad/s,

$\theta =$ $\pi/2$ $\in (-\pi, \pi]$.

Observe 4.25 cycles in 0.034 seconds
$\Rightarrow T_0 = 0.034/4.25 = 0.008$
$\Rightarrow f_0 = 1/T_0 = 125$ Hz
$\Rightarrow \omega_0 = 2\pi f_0 = 250\pi$ rad/s

(b) Find a value for t_0 **in the range** $0.996 < t_0 < 1.004$ so that the signal $x(t) = A\cos(\omega_0 t + \theta)$ shown above can be written as the delayed sinusoid $x(t) = A\cos(\omega_0(t - t_0))$:

0.996 seconds $< t_0 =$ 0.998 $<$ 1.004 seconds

The peak closest to $t = 0$ is at time $t_p = -T_0/4 \Rightarrow$ we can write $x(t) = A\cos(\omega_0(t - t_p))$.

Of course, being periodic we can further delay by any multiple of T_0 with no change $\Rightarrow$ we can also write $x(t) = A\cos(\omega_0(t - t_0))$, where:

$$t_0 = -T_0/4 + mT_0 \qquad \text{for any integer } m.$$

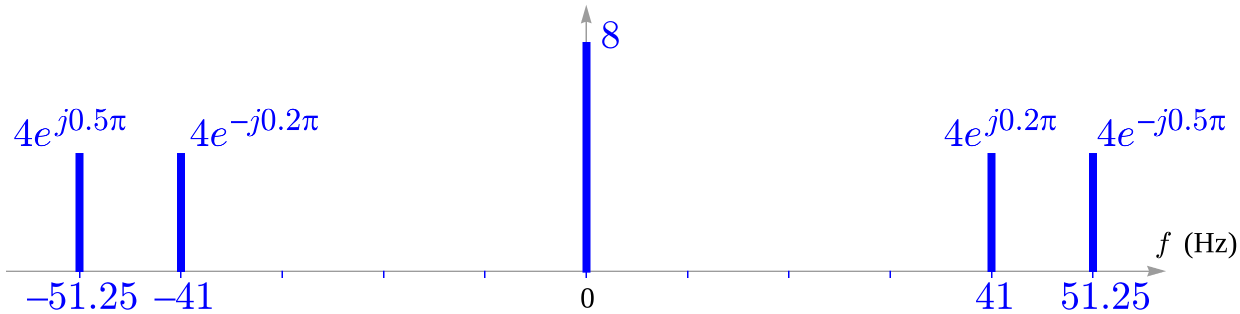
Choosing $m = 125$ puts this in the desired range:

$$t_0 = -T_0/4 + 125T_0$$
$$= -0.002 + 1 \qquad = 0.998.$$

PROB. Su14-Q1.3. All parts of this problem pertain to the signal:

$$x(t) = 8 + 8\cos(82\pi t + 0.2\pi) + 8\sin(102.5\pi t).$$

(a) Sketch the spectrum for $x(t)$ in the space below.
(For each line be careful to label both its frequency and its complex amplitude.)



(b) The signal $x(t)$ is periodic with: fundamental frequency $f_0 =$ 10.25 Hz,

and fundamental period $T_0 =$ 0.0976 secs.

$$\omega_0 = \gcd(82\pi, 102.5\pi) = 0.5\pi\gcd(164, 205) = 0.5\pi(41) = 20.5\pi$$

$$\Rightarrow f_0 = \omega_0/(2\pi) = 10.25 \text{ Hz}$$

$$\Rightarrow T_0 = 1/f_0 = 0.0976 \text{ seconds}$$

Hint: The remaining two parts below can be solved without performing any integration.

(c) Evaluate the integral (where T_0 is the fundamental period):

$$\int_0^{T_0} x(t)dt = \boxed{0.78}.$$

This is almost the formula for a_0, which by inspection is 8, except there is a $1/T_0$ factor missing

$\Rightarrow$ the integral is: $T_0 a_0 = (0.0976)(8) = 0.78.$

(d) Evaluate the integral (where T_0 is the fundamental period):

$$\int_0^{T_0} x(t)e^{-j8\pi t/T_0}dt = \boxed{0.39e^{j0.2\pi}}.$$

This is almost the formula for a_4, which by inspection is $4e^{j0.2\pi}$, except there is a $1/T_0$ factor missing

$\Rightarrow$ the integral is: $T_0 a_4 = (0.0976)(4e^{j0.2\pi}) = 0.39e^{j0.2\pi}.$

PROB. Su14-Q1.4. The two parts of this problem are unrelated.

(a) Consider the equation below, in which two sinusoids are added to yield a third:

$$2\cos(\omega_0 t + \theta) + A\sin(\omega_0 t + 0.2\pi) = A\cos(122\pi t).$$

Solve this equation for the unknown parameters A, ω_0, and θ:

$A =$ 2.203 ≥ 0,

$\omega_0 =$ 122π rad/s,

$\theta =$ 0.35π $\in (-\pi, \pi]$

Since $\sin(\omega_0 t + 0.2\pi) = \cos(\omega_0 t + 0.2\pi - 0.5\pi)$, the corresponding phasor equation is:

$$2e^{j\theta} + Ae^{-j0.3\pi} = A$$

$$\Rightarrow \; 2 + Ae^{-j\theta}e^{-j0.3\pi} = Ae^{-j\theta}$$

$$\Rightarrow \; Ae^{-j\theta} = \frac{2}{1 - e^{-j0.3\pi}} = 2.203e^{-j0.35\pi}$$

(b) Consider the equation below, in which the sum of N sinusoids is zero:

$$\sum_{k=1}^{N} \cos(2026\pi t + 0.02\pi k) = 0.$$

There are multiple values of N for which this equation is true.
Specify three different possible values, all of which are positive:

$N =$ 100, or $N =$ 200, or $N =$ 300.

The corresponding phasor equation is:

$$\sum_{k=1}^{N} e^{j0.02\pi k} = 0, \quad \text{or}$$

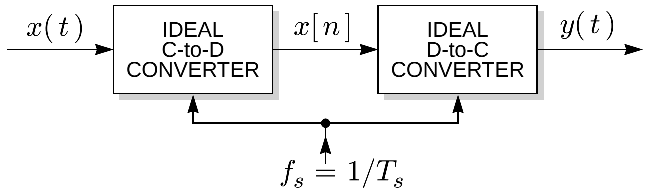
$$\sum_{k=1}^{N} e^{j2\pi k/100} = 0.$$

This is the sum of N of the 100^{th} roots of unity

$\Rightarrow$ it will be zero whenever N is a multiple of 100.

PROB. Su14-Q1.5.

Suppose that a continuous-time signal $x(t)$ is sampled with sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:



The input signal being sampled is the sum of two sinusoids:

$$x(t) = \cos(80\pi t) + \cos(160\pi t).$$

(a) Find a value for the sampling rate f_s so that the output $y(t)$ is periodic with fundamental frequency $f_0 = 40$ Hz:

$f_s =$ 160^+ (anything > 160) samples/s.

The input is already periodic like this, so we need only choose sampling rate to avoid aliasing. Any f_s bigger than $2f_{\max} = 160$ Hz will do.

(b) Find the **largest** possible value for the sampling rate f_s so that the output $y(t)$ is periodic with fundamental frequency $f_0 = 20$ Hz:

$f_s =$ 140 samples/s.

The lower-frequency sinusoid is 40 Hz, which is a multiple of 20 Hz. The output will be periodic with fundamental 20 Hz whenever the higher-frequency sinusoid aliases to a frequency that is a multiple of 20 Hz, but **not** a multiple of 40 Hz. The only possibility is 60 Hz. The 80 Hz sinusoid will alias to 60 Hz when the sample rate is 140 Hz.

(c) Find the **largest** possible value for the sampling rate f_s so that the output is a constant, $y(t) = A$ for all time t:

$f_s =$ 40 samples/s.

Substituting n/f_s for t yields:

$$x[n] = \cos(80\pi n/f_s) + \cos(160\pi n/f_s)$$

At $n = 0$ this reduces to $x[0] = 1 + 1 = 2$. To get this for any n, the arguments of both $\cos(\,\cdot\,)$ must be integer multiples of 2π; the largest f_s that does this is $f_s = 40$ Hz.

The corresponding constant is $y(t) = A =$ 2 .