GEORGIA INSTITUTE OF TECHNOLOGY SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Spring 2025 Quiz #1

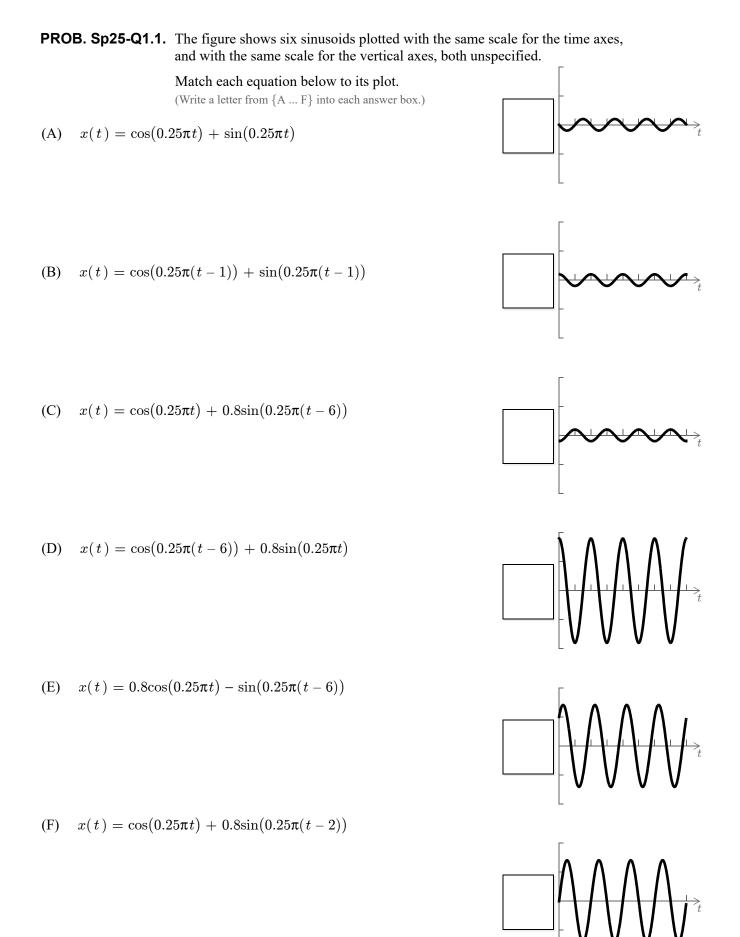
February 7, 2025

NAME:		GT username:				
_	(FIRST)	(LAST)			(e.g.,	gtxyz123)
	Circle yo	our recitation section:	L01 (Daniela)	L05 (Chun-Wei)	L07 (Chun-Wei)	L09 (Daniela)
			L02 (Greg)	L06 (Kennedy)	L08 (Kennedy)	L10 (Greg)

Important Notes:

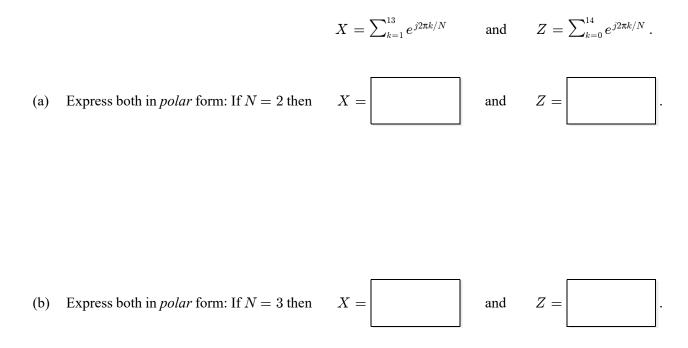
- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5" × 11") of hand-written notes.
- · Calculators are allowed, but no other electronics (no smartphones/readers/watches/tablets/laptops/etc).
- · JUSTIFY your reasoning with at least a few words to receive partial credit.
- Express all angles as a fraction of π . For example, write 0.1 π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the provided answer boxes.
- Do not write on the backs of any pages.

Problem	Value	Score Earned
1	30	
2	35	
3	35	
Total		

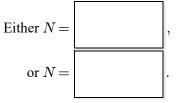


PROB. Sp25-Q1.2.

Define two complex numbers X and Z in terms of an unspecified positive integer N according to:



(c) If X = Z then it must be that the positive integer N is one of two values. Specify them both:



PROB. Sp25-Q1.3.

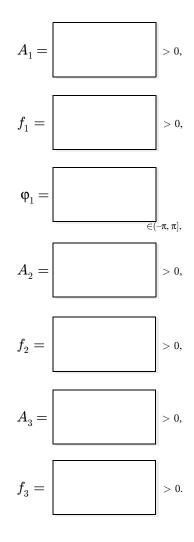
If the spectrum for:

$$x(t) = A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 (t - 0.1)) + A_3 \cos(2\pi f_3 (t - 0.01))$$

is:



then the unspecified constants (in standard form) must be:



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	Circle yo	ur recitation section:	L01 (Daniela)	L05 (Chun-Wei)	L07 (C	hun-Wei)	L09 (Daniela)
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PROB. Sp25-Q1.1. The figure shows six sinusoids plotted with the same scale for the time axes, and with the same scale for the vertical axes, both unspecified.

Match each equation below to its plot.

(Write a letter from $\{A \dots F\}$ into each answer box.)

(A)
$$x(t) = \cos(0.25\pi t) + \sin(0.25\pi t)$$

$$1 - j = \sqrt{2} e^{-j0.25\pi}$$

(B)
$$x(t) = \cos(0.25\pi(t-1)) + \sin(0.25\pi(t-1))$$

$$e^{-j0.25\pi} + e^{-j0.75\pi} = \sqrt{2}e^{-j0.5\pi}$$

(C)
$$x(t) = \cos(0.25\pi t) + 0.8\sin(0.25\pi (t-6))$$

 $1 + 0.8e^{-j2\pi} = 1.8$

(D)
$$x(t) = \cos(0.25\pi(t-6)) + 0.8\sin(0.25\pi t)$$

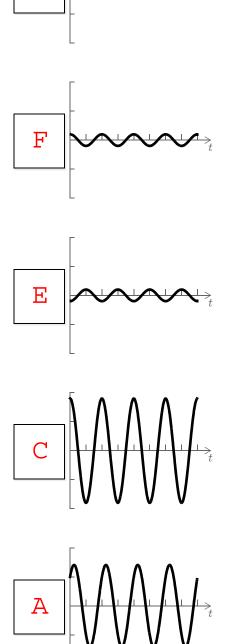
 $e^{-j1.5\pi} + 0.8e^{-j0.5\pi} = 0.2e^{j0.5\pi}$

(E)
$$x(t) = 0.8\cos(0.25\pi t) - \sin(0.25\pi (t-6))$$

 $0.8 - e^{-j2\pi} = -0.2$

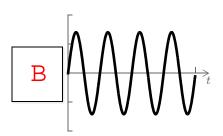
(F)
$$x(t) = \cos(0.25\pi t) + 0.8\sin(0.25\pi (t-2))$$

 $1 + 0.8e^{-j\pi} = 0.2$



 \rightarrow_t

D



PROB. Sp25-Q1.2.

Define two complex numbers X and Z in terms of an unspecified positive integer N according to:

$$X = \sum_{k=1}^{13} e^{j2\pi k/N} \quad \text{and} \quad Z = \sum_{k=0}^{14} e^{j2\pi k/N}.$$
(a) Express both in *polar* form: If $N = 2$ then $X = -1 = e^{j\pi}$ and $Z = 1$.

$$X = -1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 = -1$$

$$Z = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 = 1$$

(b) Express both in *polar* form: If
$$N = 3$$
 then $X = \begin{bmatrix} e^{j2\pi/3} \\ 0 \end{bmatrix}$ and $Z = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Roots of unity sum to zero

$$\Rightarrow \text{ sum of any } N = 3 \text{ consecutive powers of } e^{j2\pi/3} \text{ is zero} \\\Rightarrow X = \sum_{k=1}^{12} e^{j2\pi k/3} + e^{j2\pi 13/3} = 4(0) + e^{j26\pi/3} = e^{j2\pi/3}, \\Z = \sum_{k=0}^{14} e^{j2\pi k/3} = 5(0) = 0.$$

(c) If X = Z then it must be that the positive integer N is one of two values. Specify them both:

Either
$$N = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$
,
 $0 = Z - X$
 $= \sum_{k=0}^{14} - \sum_{k=1}^{13}$ or $N = \begin{bmatrix} 28 \end{bmatrix}$.
 $= 1 + e^{j2\pi(14)/N}$

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$$\Rightarrow \frac{28\pi}{N} = \text{odd multiple of } \pi$$
$$= m_{\text{odd}}\pi \text{ for some odd } m_{\text{odd}}$$
$$\Rightarrow N = \frac{28}{m_{\text{odd}}} \in \{4, 28\}$$
(Only the odd integers $m = 7$ and $m = 1$

result in a positive integer N)

PROB. Sp25-Q1.3.

If the spectrum for:

$$x(t) = A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 (t - 0.1)) + A_3 \cos(2\pi f_3 (t - 0.01))$$

is:



then the unspecified constants (in standard form) must be:

• Phase change when sinusoid delay is 0.01 seconds:

 $\begin{array}{ccc} \hline \hline 25 \text{ Hz} \Rightarrow -0.5\pi \\ \hline 30 \text{ Hz} \Rightarrow -0.6\pi \\ \hline 45 \text{ Hz} \Rightarrow -0.9\pi \end{array} \xrightarrow{} \begin{array}{c} \text{Only match to spectrum} \\ \Rightarrow f_3 = 25 \text{ Hz}, \\ A_3 = 8 \end{array}$

• Phase change when sinusoid delay is 0.1 seconds:

$$\begin{array}{l} 30 \ \mathrm{Hz} \Rightarrow -6\pi \\ \hline (45 \ \mathrm{Hz} \Rightarrow -9\pi) & \longrightarrow \end{array} \quad \text{Only match to spectrum} \\ \Rightarrow f_2 = 45 \ \mathrm{Hz}, \\ A_2 = 4 \end{array}$$

 $\phi_1 = 0.6\pi$

• Only 30 Hz is left \Rightarrow $f_1 = 30$ Hz, $A_1 = 16$,

$$A_{1} = \boxed{16} > 0,$$

$$f_{1} = \boxed{30 \text{ Hz}} > 0,$$

$$\phi_{1} = \boxed{0.6\pi}_{\in(-\pi, \pi],}$$

$$A_{2} = \boxed{4} > 0,$$

$$f_{2} = \boxed{45 \text{ Hz}} > 0,$$

$$A_{3} = \boxed{8} > 0,$$

$$f_{3} = \boxed{25 \text{ Hz}} > 0.$$