

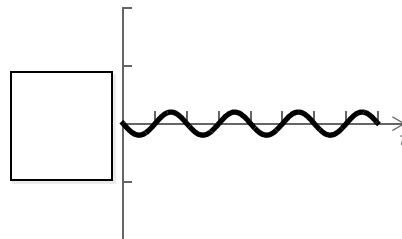
Important Notes:

PROB. Sp25-Q1.1. The figure shows six sinusoids plotted with the same scale for the time axes, and with the same scale for the vertical axes, both unspecified.

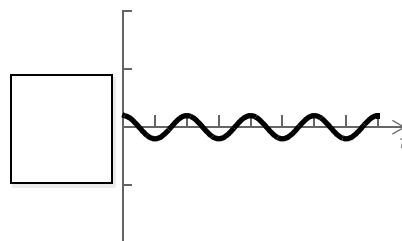
Match each equation below to its plot.

(Write a letter from {A ... F} into each answer box.)

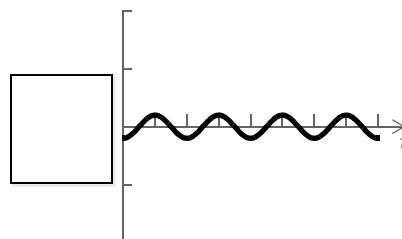
(A) $x(t) = \cos(0.25\pi t) + \sin(0.25\pi t)$



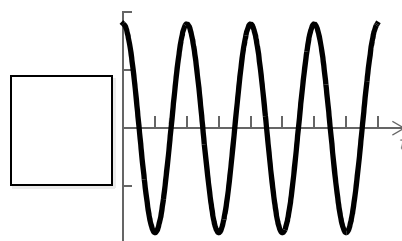
(B) $x(t) = \cos(0.25\pi(t-1)) + \sin(0.25\pi(t-1))$



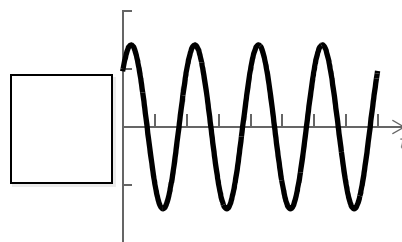
(C) $x(t) = \cos(0.25\pi t) + 0.8\sin(0.25\pi(t-6))$



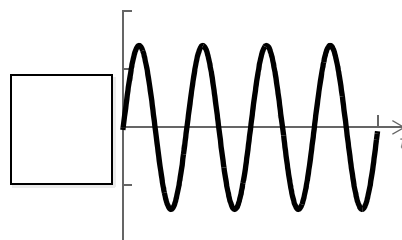
(D) $x(t) = \cos(0.25\pi(t-6)) + 0.8\sin(0.25\pi t)$



(E) $x(t) = 0.8\cos(0.25\pi t) - \sin(0.25\pi(t-6))$



(F) $x(t) = \cos(0.25\pi t) + 0.8\sin(0.25\pi(t-2))$



PROB. Sp25-Q1.2.

Define two complex numbers X and Z in terms of an unspecified positive integer N according to:

$$X = \sum_{k=1}^{13} e^{j2\pi k/N} \quad \text{and} \quad Z = \sum_{k=0}^{14} e^{j2\pi k/N}.$$

(a) Express both in *polar* form: If $N = 2$ then $X =$ and $Z =$.

(b) Express both in *polar* form: If $N = 3$ then $X =$ and $Z =$.

(c) If $X = Z$ then it must be that the positive integer N is one of two values. Specify them both:

Either $N =$,

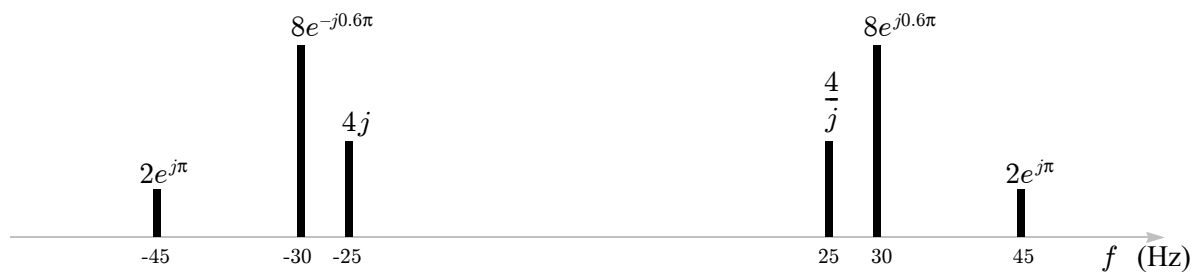
or $N =$.

PROB. Sp25-Q1.3.

If the spectrum for:

$$x(t) = A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2(t - 0.1)) + A_3 \cos(2\pi f_3(t - 0.01))$$

is:



then the unspecified constants (in standard form) must be:

$$A_1 = \boxed{} > 0,$$

$$f_1 = \boxed{} > 0,$$

$$\varphi_1 = \boxed{} \in (-\pi, \pi],$$

$$A_2 = \boxed{} > 0,$$

$$f_2 = \boxed{} > 0,$$

$$A_3 = \boxed{} > 0,$$

$$f_3 = \boxed{} > 0.$$

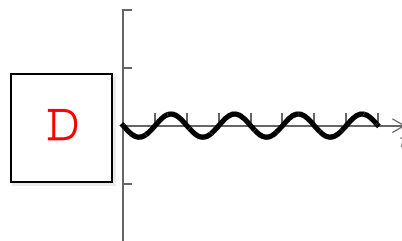
PROB. Sp25-Q1.1. The figure shows six sinusoids plotted with the same scale for the time axes, and with the same scale for the vertical axes, both unspecified.

Match each equation below to its plot.

(Write a letter from {A ... F} into each answer box.)

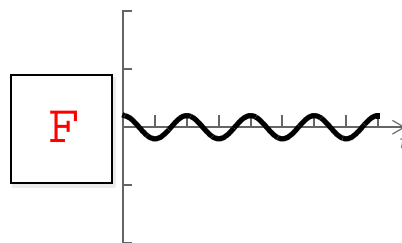
(A) $x(t) = \cos(0.25\pi t) + \sin(0.25\pi t)$

$$1 - j = \sqrt{2}e^{-j0.25\pi}$$



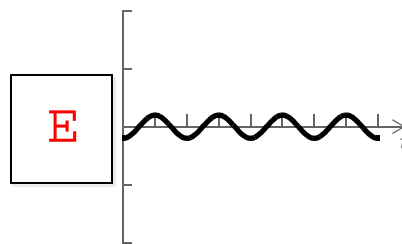
(B) $x(t) = \cos(0.25\pi(t-1)) + \sin(0.25\pi(t-1))$

$$e^{-j0.25\pi} + e^{-j0.75\pi} = \sqrt{2}e^{-j0.5\pi}$$



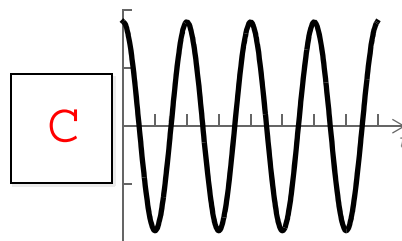
(C) $x(t) = \cos(0.25\pi t) + 0.8\sin(0.25\pi(t-6))$

$$1 + 0.8e^{-j2\pi} = 1.8$$



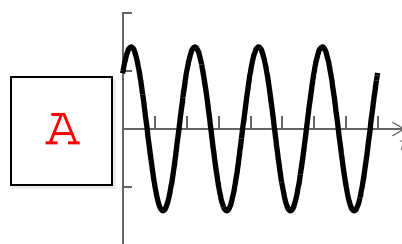
(D) $x(t) = \cos(0.25\pi(t-6)) + 0.8\sin(0.25\pi t)$

$$e^{-j1.5\pi} + 0.8e^{-j0.5\pi} = 0.2e^{j0.5\pi}$$



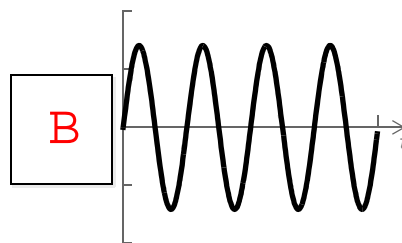
(E) $x(t) = 0.8\cos(0.25\pi t) - \sin(0.25\pi(t-6))$

$$0.8 - e^{-j2\pi} = -0.2$$



(F) $x(t) = \cos(0.25\pi t) + 0.8\sin(0.25\pi(t-2))$

$$1 + 0.8e^{-j\pi} = 0.2$$



PROB. Sp25-Q1.2.

Define two complex numbers X and Z in terms of an unspecified positive integer N according to:

$$X = \sum_{k=1}^{13} e^{j2\pi k/N} \quad \text{and} \quad Z = \sum_{k=0}^{14} e^{j2\pi k/N}.$$

(a) Express both in *polar* form: If $N = 2$ then $X = \boxed{-1 = e^{j\pi}}$ and $Z = \boxed{1}$.

$$X = -1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 = -1$$

$$Z = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 = 1$$

(b) Express both in *polar* form: If $N = 3$ then $X = \boxed{e^{j2\pi/3}}$ and $Z = \boxed{0}$.

Roots of unity sum to zero

\Rightarrow sum of any $N = 3$ consecutive powers of $e^{j2\pi/3}$ is zero

$$\Rightarrow X = \sum_{k=1}^{12} e^{j2\pi k/3} + e^{j2\pi 13/3} = 4(0) + e^{j2\pi/3} = e^{j2\pi/3},$$

$$Z = \sum_{k=0}^{14} e^{j2\pi k/3} = 5(0) = 0.$$

(c) If $X = Z$ then it must be that the positive integer N is one of two values. Specify them both:

\swarrow
 $0 = Z - X$

$$= \sum_{k=0}^{14} - \sum_{k=1}^{13}$$

$$= 1 + e^{j2\pi(14)/N}$$

Either $N = \boxed{4}$,

or $N = \boxed{28}$.

$$\Rightarrow \frac{28\pi}{N} = \text{odd multiple of } \pi$$

$$= m_{\text{odd}}\pi \text{ for some odd } m_{\text{odd}}$$

$$\Rightarrow N = \frac{28}{m_{\text{odd}}} \in \{4, 28\}$$

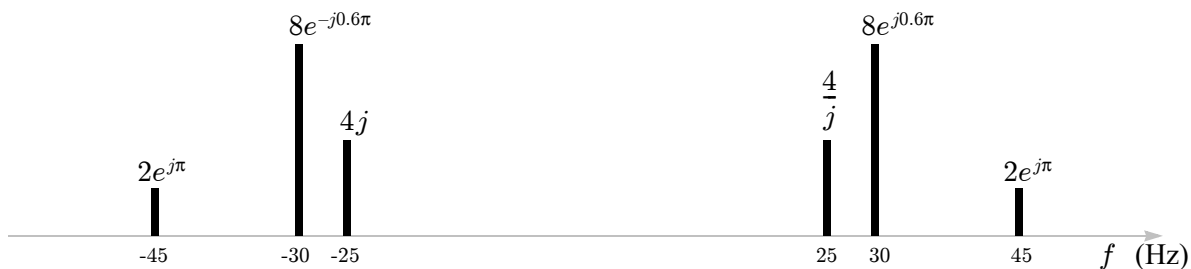
(Only the odd integers $m = 7$ and $m = 1$ result in a positive integer N)

PROB. Sp25-Q1.3.

If the spectrum for:

$$x(t) = A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2(t - 0.1)) + A_3 \cos(2\pi f_3(t - 0.01))$$

is:



then the unspecified constants (in standard form) must be:

$$A_1 = \boxed{16} > 0,$$

$$f_1 = \boxed{30 \text{ Hz}} > 0,$$

$$\varphi_1 = \boxed{0.6\pi} \in (-\pi, \pi],$$

$$A_2 = \boxed{4} > 0,$$

$$f_2 = \boxed{45 \text{ Hz}} > 0,$$

$$A_3 = \boxed{8} > 0,$$

$$f_3 = \boxed{25 \text{ Hz}} > 0.$$

- Phase change when sinusoid delay is 0.01 seconds:

$$\boxed{25 \text{ Hz} \Rightarrow -0.5\pi} \longrightarrow \text{Only match to spectrum}$$

$$30 \text{ Hz} \Rightarrow -0.6\pi \Rightarrow f_3 = 25 \text{ Hz},$$

$$45 \text{ Hz} \Rightarrow -0.9\pi \quad A_3 = 8$$

- Phase change when sinusoid delay is 0.1 seconds:

$$30 \text{ Hz} \Rightarrow -6\pi$$

$$\boxed{45 \text{ Hz} \Rightarrow -9\pi} \longrightarrow \text{Only match to spectrum}$$

$$\Rightarrow f_2 = 45 \text{ Hz},$$

$$A_2 = 4$$

- Only 30 Hz is left \Rightarrow $f_1 = 30 \text{ Hz},$
 $A_1 = 16,$
 $\varphi_1 = 0.6\pi$