GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ-1

DATE: 09-Feb-24 COURSE: ECE-2026

NAME: ______________________________ gt Account: ________________

LAST, FIRST (ex: gburdell21)

Instructions: READ and SIGN your name below

failing to write your name, gtAccount, and sign below may result in a 5% penalty

- You have **50 minutes** (students with accommodations will have the appropriate time extension automatically added to their time) to complete this exam.
- Graphics calculators are permitted. One sheet of paper is permitted. You can write on both sides hand-written notes.
- WRITE ANY RADIAN ANSWERS AS A MULTIPLE OF π. (i.e., write 0.4286π or $3\pi/7$ instead of 1.3464). ALL RADIAN ANSWERS MUST BE IN THE RANGE$(-\pi, \pi]$ FOR CREDIT.

The Academic Honor Code will be strictly enforced. Forgeries and plagiarism are violations of the Georgia Tech honor code and will be referred to the Dean of Students for disciplinary action. You are not to discuss exam content or to share any written, electronic, or any other form of exam information with anyone during or after the exam until the solutions have been posted. By submitting this exam, you affirm that you have neither given nor received inappropriate assistance during this exam. I have read the instructions above and affirm that I will abide by the guidelines provided.

____

Sign your name on the line above

PROBLEM SP-24-Q.1.1:

The following questions are not related to each other. Answer each question independently of the other questions.

(a) (5 points) Solve the following equation by finding the complex number $z = re^{j\theta}$. Make sure that $r \geq 0$ is a non-negative real number and $-\pi < \theta \leq \pi$ is expressed as a multiple of π (i.e., write 0.4286π or $3\pi/7$ instead of 1.3464).

$$\sum_{k=-1}^{19} e^{jk0.1\pi} = re^{j\theta}$$

Write your answer clearly within the answer box.

$r =$ __________

$\theta =$ __________

(b) (4 points) Solve the following equation by finding the real numbers x and y that make up the complex number $z = x + jy$. Make sure to express x and y as real numbers.

$$je^{j0.1\pi} - 2 = \frac{1}{x - jy}$$

Write your answer clearly within the answer box.

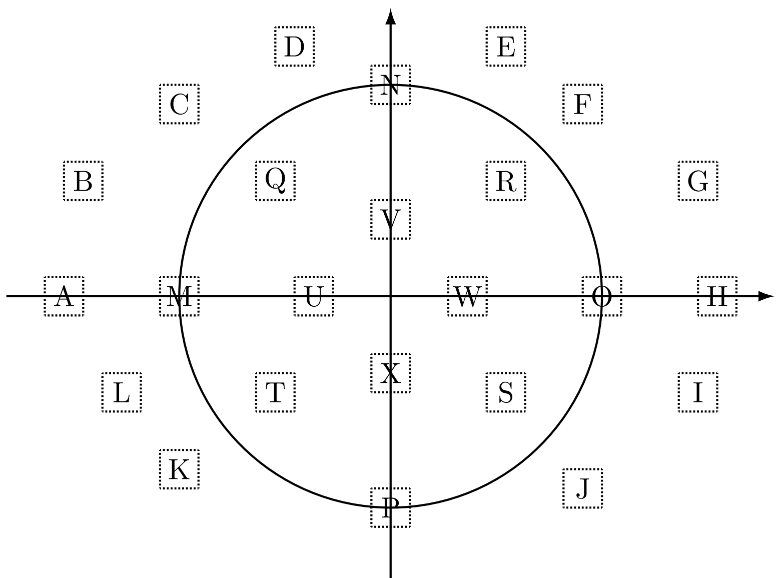
$x =$ ____________

$y =$ ____________

(c) (24 points) Consider a complex number, $z = re^{j\theta}$, that has the following properties:

- $\sum_{k=0}^{\infty} r^k < \infty$
- $\cos(\theta) > 0$, $\sin(\theta) > 0$, and $\tan(\theta) = 1$.

A complex plane is represented below with the circle representing a radius of $r = 1$. Each of the dashed squares on the plot represents a complex number. There are 24 complex numbers on the complex plane. Listed below are eight complex numbers that are expressed in terms of the complex number (defined above), z. Note that one of the depicted complex numbers on the figure is z itself. The task is to determine where on the plot is each of these eight complex numbers.



Write your answer clearly within the answer box. For every complex number write ONE of the corresponding letters, (A) - (X) shown within the boxes on the complex-plane plot.

$z^2 =$ ____________

$1/z =$ ____________

$jz =$ ____________

$1/z^* =$ ____________

$zz^* =$ ____________

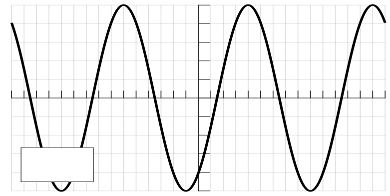
$z + 1 =$ ____________

$z + j =$ ____________

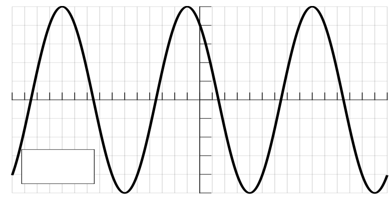
$z/z^* =$ ____________

PROBLEM SP-24-Q.1.2:

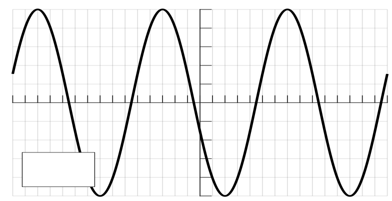
(35 points) Seven sinusoids are plotted versus time. They all have the same amplitudes and frequencies. They differ only in their phases. The axes are not labeled.
Match each equation below to its corresponding plot. Indicate answers by writing a letter (A, B, C, D, E, F, or G) in each answer box.



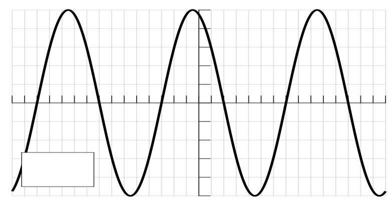
(A) $x(t) = \cos(2\pi(t - 0.4))$



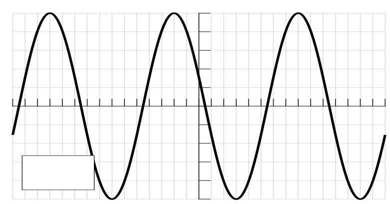
(B) $x(t) = 0.5e^{j(2\pi t + 0.1\pi)} + 0.5e^{-j(2\pi t + 0.1\pi)}$



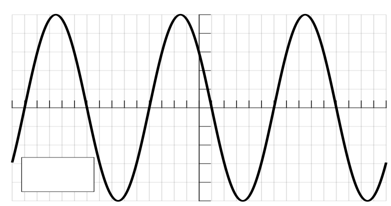
(C) $x(t) = Re\{Xe^{j2\pi t}\}$,
where $X = \cos(0.3\pi) + j\sin(0.3\pi)$



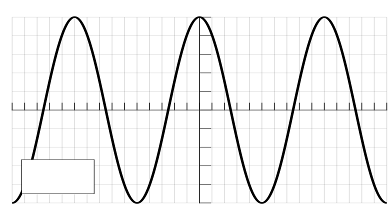
(D) $x(t) = \sin(2\pi t + 0.7\pi)$



(E) $x(t) = Re\left\{\frac{d}{dt}g(t)\right\}$,
where $g(t) = \frac{1}{2\pi}e^{j0.1\pi}e^{j2\pi t}$



(F) $x(t) = -\sin(2\pi t + 1.9\pi)$

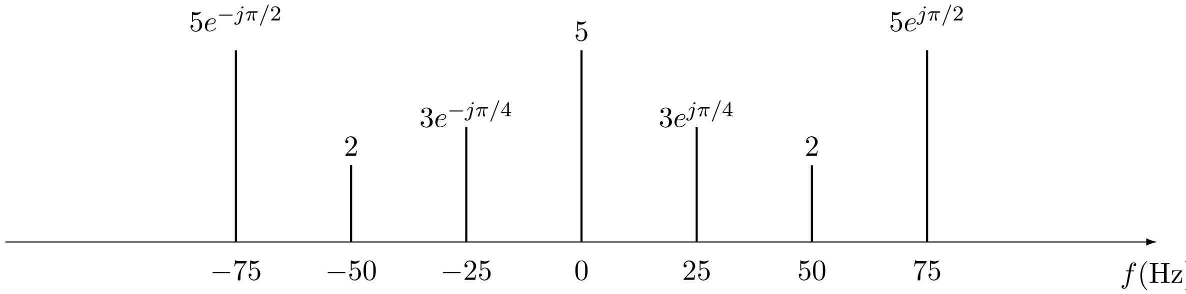


(G) $x(t) = \frac{1}{\sqrt{2}}(\cos(2\pi t + 0.25\pi) + \cos(2\pi t - 0.25\pi))$

PROBLEM SP-24-Q.1.3:

[32 points] The two-sided spectrum representation of the signal $x(t)$ is shown below.

$$x(t) = A + B\cos(50\pi t + \phi_1) + C\cos(\omega_2 t) + D\cos(\omega_3(t - t_d))$$



Find the below values and make sure that the phases are expressed between $(-\pi\ ,\ \pi]$ and as multiples of π, , the amplitudes are all real numbers and positive, and all frequencies as multiples of π. Write 0.4286π or $3\pi/7$ instead of 1.3464.

$A =$ ____________

$B =$ ____________

$C =$ ____________

$D =$ ____________

$\phi_1 =$ ____________

$\omega_2 =$ ____________

$\omega_3 =$ ____________

$t_d =$ ____________

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$$\sum_{k=-1}^{19} e^{jk0.1\pi} = re^{j\theta}$$

Write your answer clearly within the answer box.

$r = 1$ __________

$\theta = -0.1\pi$ __________

$$\sum_{k=-1}^{19} e^{jk0.1\pi} = re^{j\theta} = \sum_{k=-0}^{19+1-1} e^{jk0.1\pi} + e^{-j0.1\pi} = e^{-j0.1\pi}$$

(b) (4 points) Solve the following equation by finding the real numbers x and y that make up the complex number $z = x + jy$. Make sure to express x and y as real numbers.

$$je^{j0.1\pi} - 2 = \frac{1}{x - jy}$$

Write your answer clearly within the answer box.

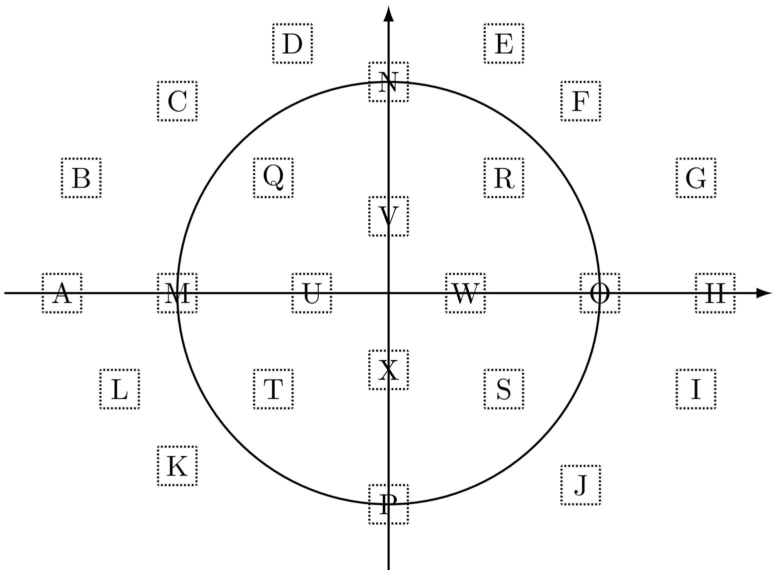
$x = -0.3703$ ____________

$y = 0.1525$ ____________

(c) (24 points) Consider a complex number, $z = re^{j\theta}$, that has the following properties:

- $\sum_{k=0}^{\infty} r^k < \infty$
- $\cos(\theta) > 0$, $\sin(\theta) > 0$, and $\tan(\theta) = 1$.

A complex plane is represented below with the circle representing a radius of $r = 1$. Each of the dashed squares on the plot represents a complex number. There are 24 complex numbers on the complex plane. Listed below are eight complex numbers that are expressed in terms of the complex number (defined above), z. Note that one of the depicted complex numbers on the figure is z itself. The task is to determine where on the plot is each of these eight complex numbers.



Write your answer clearly within the answer box. For every complex number write ONE of the corresponding letters, (A) - (X) shown within the boxes on the complex-plane plot.

$z^2 = V$ __________

$1/z = J$ __________

$jz = Q$ __________

$1/z^* = F$ __________

$zz^* = W$ __________

$z + 1 = G$ __________

$z + j = E$ __________

$z/z^* = N$ __________

$|r| < 1$ and $\theta = 0.25\pi$; therefore, z is R.

$z^2 = r^2 e^{j0.5\pi}$

$z^{-1} = \frac{1}{r} e^{-j0.25\pi}$

$jz = re^{j0.75\pi}$

$\frac{1}{z^*} = \frac{1}{r} e^{j0.25\pi}$

$zz^* = r^2$

$z + 1$ adds 1 to the real part

$z + j$ adds 1 to the imaginary part

$\frac{z}{z^*} = e^{j0.5\pi}$

PROBLEM SP-24-Q.1.2:

(35 points) Seven sinusoids are plotted versus time. They all have the same amplitudes and frequencies. They differ only in their phases. The axes are not labeled.
Match each equation below to its corresponding plot. Indicate answers by writing a letter (A, B, C, D, E, F, or G) in each answer box.

(A) $x(t) = \cos(2\pi(t - 0.4))$
$x(t) = \cos(2\pi(t - 0.4T))$

(B) $x(t) = 0.5e^{j(2\pi t + 0.1\pi)} + 0.5e^{-j(2\pi t + 0.1\pi)}$
$x(t) = \cos(2\pi(t + 0.05T))$

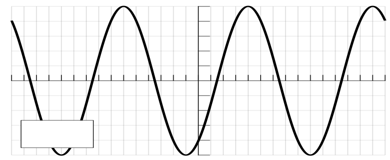
(C) $x(t) = Re\{Xe^{j2\pi t}\}$,
where $X = \cos(0.3\pi) + j\sin(0.3\pi)$
$x(t) = \cos(2\pi(t + 0.15T))$

(D) $x(t) = \sin(2\pi t + 0.7\pi)$
$x(t) = \cos(2\pi(t + 0.1T))$

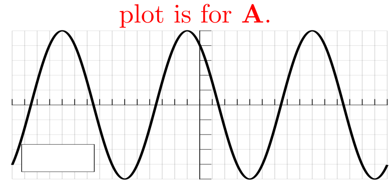
(E) $x(t) = Re\left\{\frac{d}{dt}g(t)\right\}$,
where $g(t) = \frac{1}{2\pi}e^{j0.1\pi}e^{j2\pi t}$
$x(t) = \cos(2\pi(t + 0.3T))$

(F) $x(t) = -\sin(2\pi t + 1.9\pi)$
$x(t) = \cos(2\pi(t + 0.2T))$

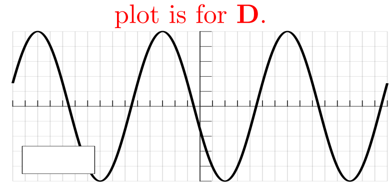
(G) $x(t) = \frac{1}{\sqrt{2}}(\cos(2\pi t + 0.25\pi) + \cos(2\pi t - 0.25\pi))$
$x(t) = \cos(2\pi t)$



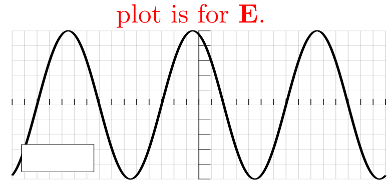
From the plot, the peak is at $0.4T$. Thus, this plot is for **A**.



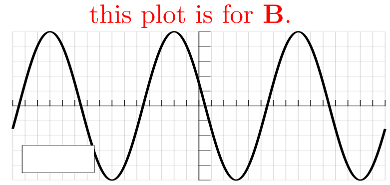
From the plot, the peak is at $-0.1T$. Thus, this plot is for **D**.



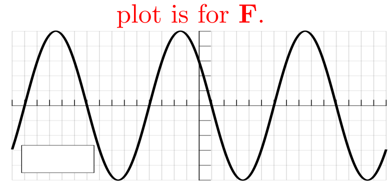
From the plot, the peak is at $-0.3T$. Thus, this plot is for **E**.



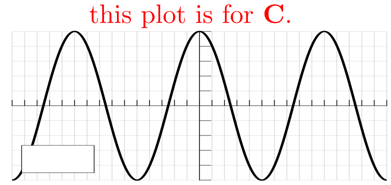
From the plot, the peak is at $-0.05T$. Thus, this plot is for **B**.



From the plot, the peak is at $-0.2T$. Thus, this plot is for **F**.



From the plot, the peak is at $-0.15T$. Thus, this plot is for **C**.



From the plot, the peak is at 0. Thus, this plot is for **G**.

PROBLEM SP-24-Q.1.3:

[32 points] The two-sided spectrum representation of the signal $x(t)$ is shown below.

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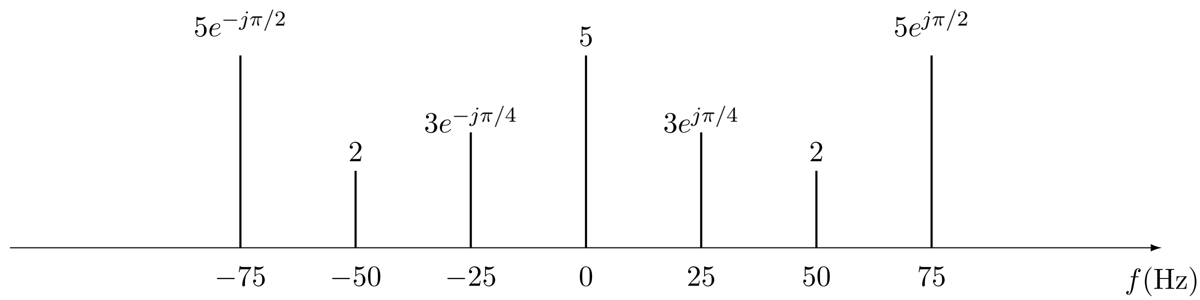

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$A = 5$ __________

$B = 6$ __________

$C = 4$ __________

$D = 10$ __________

$\phi_1 = 0.25\pi$ __________

$\omega_2 = 100\pi$ __________

$\omega_3 = 150\pi$ __________

$t_d = -0.0033$ or $\frac{-1}{300}$ __________