GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Spring 2023
Quiz #1

February 10, 2023

NAME: _____ _____ GT username: _____
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section:

L01 (Chen)	L07 (Davenport)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Duan)	L10 (Chen)	

Important Notes:

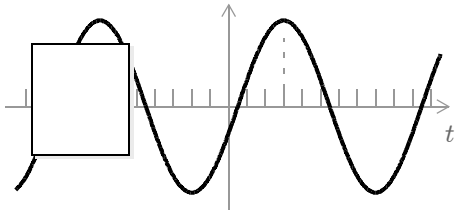
- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5" × 11") of hand-written notes.
- Calculators are allowed, but no other electronics (no smartphones/readers/watches/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	35	
2	30	
3	35	
Total		

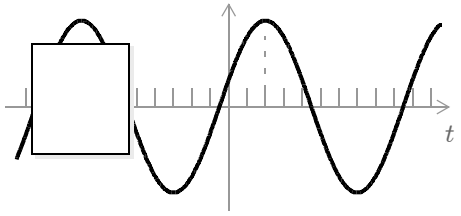
PROB. Sp23-Q1.1.

Consider the seven sinusoids shown in the figure; they all have the same amplitudes and frequencies, they differ only in their phases. The axes are not labeled.

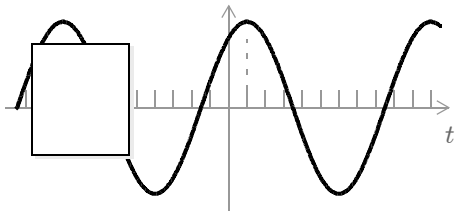
Match each equation below to its corresponding plot. Indicate answers by writing a letter (from {A ... G}) in each answer box:



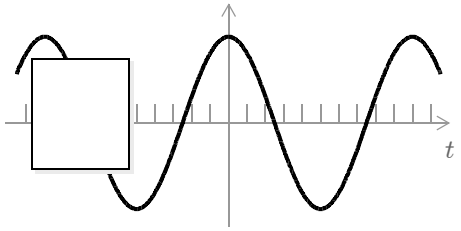
(A) $x(t) = \cos(\pi(t - 2.6))$



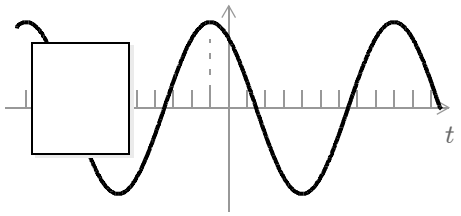
(B) $x(t) = 0.5e^{j(\pi t + 0.2\pi)} + 0.5e^{-j(\pi t + 0.2\pi)}$



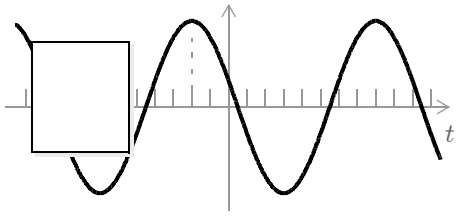
(C) $x(t) = \mathrm{Re}\{Xe^{j\pi t}\}$, where $X = \cos(0.6\pi) + j\sin(0.6\pi)$



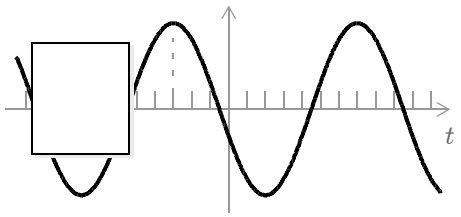
(D) $x(t) = 2\cos^2(0.5\pi t) - 1$



(E) $x(t) = \sin(\pi t + 0.9\pi)$



(F) $x(t) = -\cos(1.4\pi - \pi t)$



(G) $x(t) = -\sin(\pi t + 1.3\pi)$

PROB. Sp23-Q1.2.

Solve each equation below for the unknowns $A > 0$ and $\varphi \in (-\pi, \pi]$. (The different parts are unrelated.)

(a) $\sqrt{96}\left(\cos(\frac{\pi}{6}) + \sqrt{3}\, j\sin(\frac{\pi}{6})\right) = Ae^{j\varphi}$:

$A =$ [ ] > 0,

$\varphi =$ [ ] $\in (-\pi, \pi]$,

(b) $\cos(7\pi t + \varphi) = A\cos(7\pi t) - \frac{1}{2}\sin(7\pi t)$, for all t:

$A =$ [ ] > 0,

$\varphi =$ [ ] $\in (-\pi, \pi]$,

(c) $\sum_{k=1}^{17} \cos\left(\frac{\pi}{8}(t-k)\right) = A\cos(\frac{\pi}{8}t + \varphi)$, for all t:

$A =$ [ ] > 0,

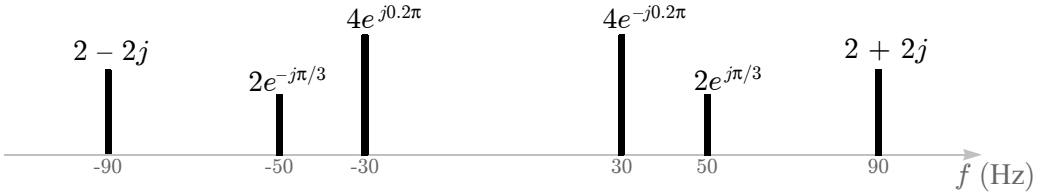
$\varphi =$ [ ] $\in (-\pi, \pi]$,

PROB. Sp23-Q1.3.

If the spectrum for:

$$x(t) \;=\; A_1\cos(2\pi f_1 t + \varphi) \;\;+\;\; A_2\cos(2\pi f_2(t - \frac{1}{300})) \;\;+\;\; \mathrm{Re}\{(a + jb)e^{j100\pi t}\}$$

is:



then the unspecified constants (in standard form) must be:

$A_1 =$ [ ] $> 0,$

$f_1 =$ [ ] $> 0,$

$\varphi =$ [ ] $\in(-\pi, \pi],$

$A_2 =$ [ ] $> 0,$

$f_2 =$ [ ] $> 0,$

$a =$ [ ] ,

$b =$ [ ] .

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PROB. Sp23-Q1.1.

Consider the seven sinusoids shown in the figure; they all have the same amplitudes and frequencies, they differ only in their phases. The axes are not labeled.

Match each equation below to its corresponding plot. Indicate answers by writing a letter (from {A ... G}) in each answer box:

(A) $x(t) = \cos(\pi(t - 2.6))$
$= \cos(\pi(t - 0.6))$
$= \cos(\pi(t - 0.3T))$
TIME OF PEAK

(B) $x(t) = 0.5e^{j(\pi t + 0.2\pi)} + 0.5e^{-j(\pi t + 0.2\pi)}$
$= \cos(\pi t + 0.2\pi)$
$= \cos(\pi(t + 0.2))$
$= \cos(\pi(t + 0.1T))$

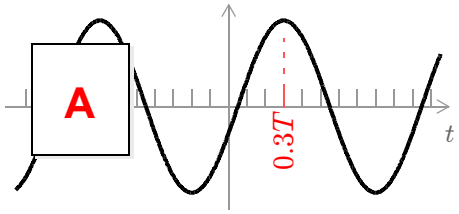
(C) $x(t) = \mathrm{Re}\{Xe^{j\pi t}\}$, where $X = \cos(0.6\pi) + j\sin(0.6\pi)$
$= \cos(\pi t + 0.6\pi)$
$= \cos(\pi(t + 0.6))$
$= \cos(\pi(t + 0.3T))$

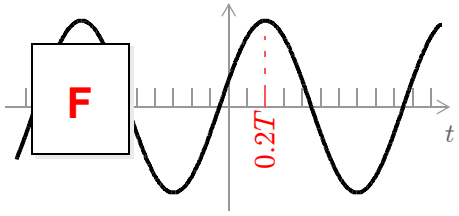
(D) $x(t) = 2\cos^2(0.5\pi t) - 1$
$= \cos(\pi t)$

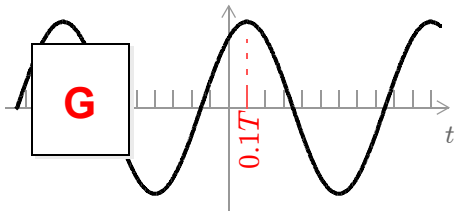
(E) $x(t) = \sin(\pi t + 0.9\pi)$
$= \cos(\pi t + 0.4\pi)$
$= \cos(\pi(t + 0.4))$
$= \cos(\pi(t + 0.2T))$

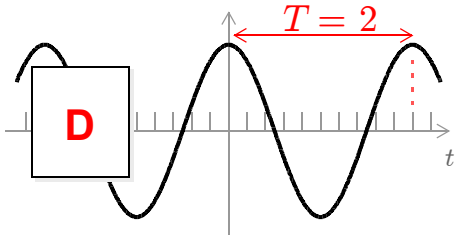
(F) $x(t) = -\cos(1.4\pi - \pi t)$
$= \cos(\pi t - 0.4\pi)$
$= \cos(\pi(t - 0.4))$
$= \cos(\pi(t - 0.2T))$

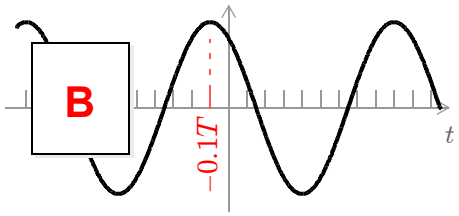
(G) $x(t) = -\sin(\pi t + 1.3\pi)$
$= \cos(\pi t + 1.8\pi)$ $= \cos(\pi t - 0.2\pi)$
$= \cos(\pi(t - 0.2))$
$= \cos(\pi(t - 0.1T))$

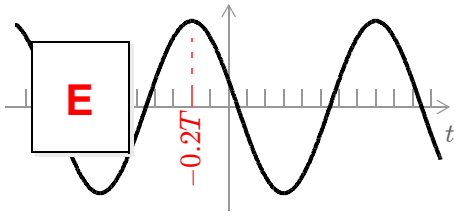


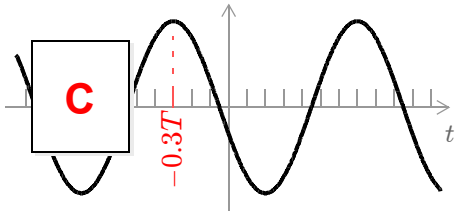












PROB. Sp23-Q1.2.

Solve each equation below for the unknowns $A > 0$ and $\varphi \in (-\pi, \pi]$. (The different parts are unrelated.)

(a) $\sqrt{96}\left(\cos(\frac{\pi}{6}) + \sqrt{3}\, j\sin(\frac{\pi}{6})\right) = Ae^{j\varphi}$:

$$= \sqrt{96}\left(\frac{\sqrt{3}}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{72}\left(1 + j\right)$$

$$= \sqrt{144}\, e^{j0.25\pi}$$

$$= 12e^{j0.25\pi}$$

$A =$ [12] > 0,

$\varphi =$ [0.25π] $\in (-\pi, \pi]$,

(b) $\cos(7\pi t + \varphi) = A\cos(7\pi t) - \frac{1}{2}\sin(7\pi t)$, for all t:

The corresponding phasor equation:

$$e^{j\varphi} = A + 0.5j$$

Equate imaginary parts $\Rightarrow \varphi = \pi/6$

Equate real parts $\Rightarrow A = \frac{\sqrt{3}}{2}$

$A =$ [$\frac{\sqrt{3}}{2} \approx 0.866$] > 0,

$\varphi =$ [$\frac{\pi}{6}$] $\in (-\pi, \pi]$,

(c) $\sum_{k=1}^{17} \cos\left(\frac{\pi}{8}(t-k)\right) = A\cos(\frac{\pi}{8}t + \varphi)$, for all t:

The corresponding phasor equation:

$$\sum_{k=1}^{17} e^{-j2\pi k/16} = Ae^{j\varphi}$$

The last 16 terms on the left sum to zero

$$\Rightarrow Ae^{j\varphi} = e^{-j2\pi(1)/16}$$

$$= e^{-j\pi/8}$$

$A =$ [1] > 0,

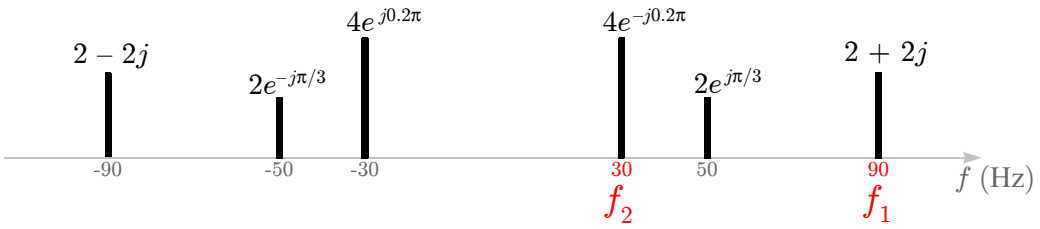
$\varphi =$ [$\frac{-\pi}{8}$] $\in (-\pi, \pi]$,

PROB. Sp23-Q1.3.

If the spectrum for:

$$x(t) = A_1\cos(2\pi f_1 t + \varphi) + A_2\cos(2\pi f_2(t - \tfrac{1}{300})) + \mathrm{Re}\{(a + jb)e^{j100\pi t}\}$$

is:



then the unspecified constants (in standard form) must be:

- Delay by $\frac{1}{300}$ results in phase shift $\frac{2\pi f_2}{300}$, only match is $f_2 = 30$ Hz.

$A_1 =$ $4\sqrt{2} \approx 5.66$ > 0,

$f_1 =$ 90 > 0,

- A_2 comes from coefficient at f_2.

$\varphi =$ 0.25π $\in(-\pi, \pi]$,

- Last term is clearly 50 Hz $\Rightarrow f_1 = 90$ Hz.

$A_2 =$ 8 > 0,

- A_1 and φ come from coefficient at f_1.

$f_2 =$ 30 > 0,

- Equate $a + jb$ to twice coefficient at 50 Hz:

$$\Rightarrow a + jb = 4e^{j\pi/3} = 2 + j2\sqrt{3}$$

$a =$ 2,

$b =$ $2\sqrt{3} \approx 3.46$.