GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 9-Feb-18 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________

LAST, FIRST (ex: gtz123a)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

			L06:Thur-Noon (Fekri)
			L08:Thurs-1:30pm (Fekri)
L01:M-3pm (Valenta)	L09:Tues-3pm (Rohling)	L02:W-3pm (Yang)	L10:Thur-3pm (Marenco)
L03:M-4:30pm (Valenta)	L11:Tues-4:30pm (Rohling)	L04:W-4:30pm (Yang)	L12:Thur-4:30pm (Marenco)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	34	
2	33	
3	33	
No/Wrong Rec	-3	

PROBLEM sp-18-Q.1.1:

The sum of two sinusoids is another sinusoid:

$$A\cos(\omega t + \varphi) = \cos(1.5\pi t - 5\pi/4) + 1.6\cos(1.5\pi(t-7))$$

(a) Determine the numerical values of A and φ, as well as ω (give the correct units).

$A =$ 1.139

$\varphi =$ -0.72π rad

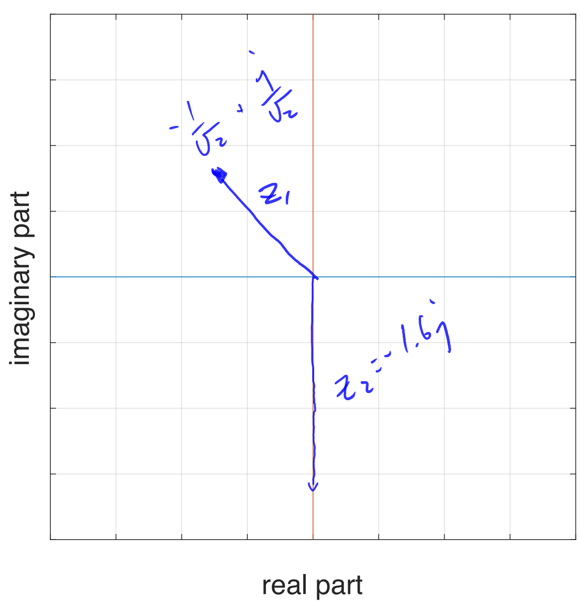
$\omega =$ 1.5π rad/s

$$z_1 = e^{-j5\pi/4} = \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$z_2 = 1.6\,e^{-j\frac{21\pi}{2}} = 1.6\,e^{-j\pi/2}$$

$$= -1.6j$$

(b) Make a complex plane plot to illustrate how complex amplitudes (phasors) were combined to solve part (a). Show a vector plot of the two complex amplitudes whose values are given by the sinusoids on the **right** hand side of the equal sign. *Use an appropriate scale on the grid below.*



(c) The signal $x(t)$ is defined by complex exponentials and complex amplitudes:

$$x(t) = 4e^{j\pi/4}e^{-j25\pi t} + 4e^{-j\pi/4}e^{j25\pi t} + 55e^{j\pi}$$

Write the formula for $x(t)$ as a sum of real-valued sinusoids.

$$x(t) = 8\left(\frac{e^{j(25\pi t - \pi/4)} + e^{-j(25\pi t - \pi/4)}}{2}\right) - 55$$

$$= 8\cos(25\pi t - \pi/4) - 55$$

PROBLEM sp-18-Q.1.2:

Each part of this problem is independent of the others.

(a) One or more of the following are solutions to the equation $z^n - A = 0$ for $n > 1$ and $A > 0$.

(i)	$z = A^{3/n} e^{j6\pi/n}$	(ii)	$z = A^{1/n} e^{-j2\pi/n}$	(iii)	$z = A^{1/n}$
(iv)	$z = A^{-1/n}$	(v)	$z = A^{2/n} e^{j4\pi/n}$	(vi)	$z = A^{1/n} e^{j4\pi/n}$

Circle each correct solution (there may be more than one).

$$Z = \left(A e^{j2\pi k}\right)^{1/n} \quad \text{for integer } k$$

$$= A^{1/n} e^{j2\pi k/n}$$

(b) Evaluate the following sum and express it in the form $Ae^{j\theta}$.

$$z = \sum_{k=0}^{10} e^{j(2\pi k/12 + \pi^{-1})}$$

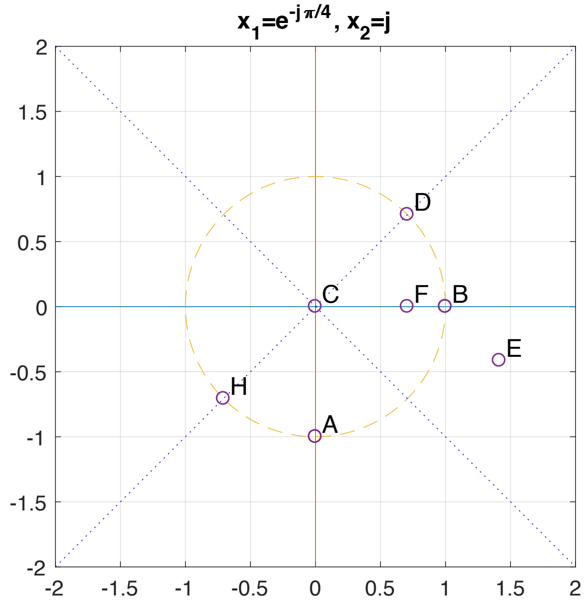
$$Z = e^{j/\pi}\left(\overset{0}{\cancel{\sum_{k=0}^{11} e^{j2\pi k/12}}} - e^{j2\pi \cdot 11/12} \right)$$

$$= e^{j\left(11\pi/6 + \pi^{-1} - \pi\right)}$$

$$\boxed{Z = e^{j\left(5\pi/6 + \pi^{-1}\right)}}$$

(c) Let $z_1 = e^{-j\pi/4}$ and $z_2 = j$. Find the corresponding point in the plot designated by a capital letter for each expression in the table below. (Note that a point may correspond to multiple expressions.)

$z = z_1 z_2$	D
$z = \Re e(z_1 z_1^*)$	B
$z = z_1 z_1$	A
$z = 2z_1 + z_2$	E
$z = \lvert z_1 z_2 \rvert$	B
$z = z_1/z_2$	H
$z = z_1^* z_2^*/z_1$	B
$z = z_2^*$	A
$z = \Re e(z_1 z_2)$	F



PROBLEM sp-18-Q.1.3:

Two questions about sinusoids, $A\cos(\omega t + \varphi)$.

(a) The following MATLAB code makes a plot of a sinusoid:

```
tt = 0:0.0001:1;
znum = 2*exp(j*5*pi*tt)-2*j*exp(j*5*pi*tt);
zden = exp(-j*5*pi*tt)+j*exp(-j*5*pi*tt);
xx = real(znum./zden);
plot(tt,xx), grid on, shg
```

Determine the mathematical formula by giving numerical values for A, φ, and ω (in rad/s).

$A = 2$

$\varphi = -\pi/2$

$\omega = 10\pi$

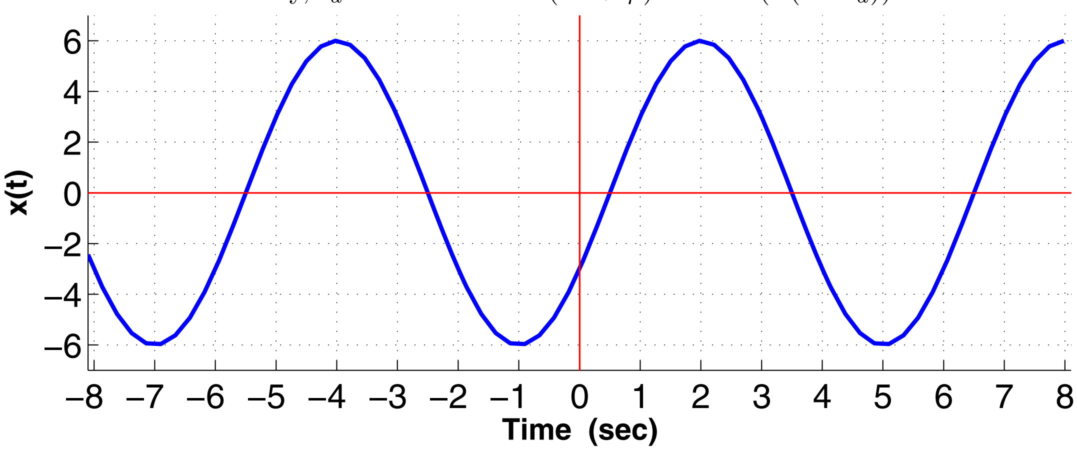
$$\frac{2e^{j5\pi t} - 2je^{j5\pi t}}{e^{-j5\pi t} + je^{-j5\pi t}} \cdot \frac{e^{j5\pi t} - je^{j5\pi t}}{e^{j5\pi t} - je^{j5\pi t}}$$

$$= \frac{2e^{j10\pi t} - 2je^{j10\pi t} - 2je^{j10\pi t} - 2e^{j10\pi t}}{1 + \cancel{j - j} + 1}$$

$$= -2je^{j10\pi t}$$

$$= 2e^{j(10\pi t - \pi/2)}$$

(b) For the sinusoid plotted below, determine its amplitude, phase, and frequency (in rad/s). Also determine the delay, t_d such that $A\cos(\omega t + \varphi) = A\cos(\omega(t - t_d))$.



$T = 6 \Rightarrow \omega = 2\pi/T$

$\varphi = -\omega t_d = -\frac{2\pi}{3}$

$A = 6$

$\varphi = -\frac{2\pi}{3}$ rad

$\omega = \pi/3$ rad/s

$t_d = 2$ s