GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 12-FEB-16 COURSE: ECE 2026A,B

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L00:Tues-9:30am (Yang) L01:Mon-3:00pm (Odom)

L03:Mon-4:30pm (Odom) L05:Tues-12:00pm (Yang)

L06:Thur-12:00pm (Zajic) L07:Tues-1:30pm (Stüber)

L08:Thur-1:30pm (Zajic) L09:Tues-3:00pm (Stüber)

L10:Thur-3:00pm (Yeredor) L12:Thur-4:30pm (Yeredor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	32	
2	32	
3	33	
Rec	3	
Total	100	

Problem Q1.1:

Each part of this problem is independent of the others.

(a) (8 points) All complex numbers have N distinct Nth roots. Find five distinct 5th roots of j, and plot them in the complex plane.

(b) (8 points) Consider complex number $z = 6 + j3$. Multiply z by its complex conjugate. Express this product in polar form.

(c) (8 points) Use the inverse Euler relations to express $\sin(\alpha x)\sin(\beta x)$ as a sum of complex exponentials.

(d) (8 points) Find a complex numerical value for:

$$z = \sum_{k=0}^{8} \mathrm{e}^{j(2\pi k/8+\pi/3)}$$

Problem Q1.2:

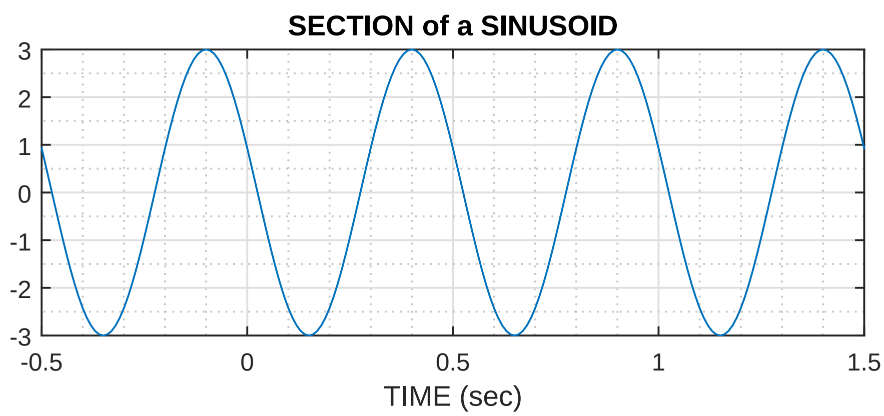
All parts are mutually unrelated.

(a) (8 pts) Express $-3\cos(7\pi t + \pi/3) + 2\cos(7\pi t - \pi/3)$ as $A\cos(\omega_0 t + \phi)$.

$A =$_________ $\phi =$ _____________ (in radians) $\omega_0 =$______________ (in radians/second)

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

(b) (8 pts) Consider the sinusoidal signal plotted below.



This signal may be written in the form $A\cos(\omega t + \phi)$. Find A, ω, and ϕ.

$A =$______________ $\omega =$_______________ $\phi =$_____________

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

(c) (8 pts) The following MATLAB code generates a signal and makes a plot. Draw a sketch of the plot that will be done by MATLAB. Determine the amplitude (A), phase (ϕ), and period of the sinusoid and label the period on your plot.

```
Fo = 25;
td=.01;
dt = 0.0001;
tt = -.04 : dt : .04;
Z = 10+10*j;
xx = real( Z*exp( j*2*pi*Fo*(tt + td) ) );
%
plot( tt, xx ),   grid
title( 'Plot' ),    xlabel('TIME   (sec)')
```

(d) (8 pts) For $P = 2e^{-j\pi/8}$, express $\Re e\{Pe^{-j10\pi(t+.3)}\}$ in standard *cosine* form, i.e., as $A\cos(\omega t+\phi)$.

Problem Q1.3:

(a) (8 pts) Consider the complex integral:

$$z = \int_0^{t_0} e^{j(100\pi t + \pi/11)} dt$$

Find the smallest value of $t_0 > 0$ such that $z = 0$. (You should not have to perform an integral.)

(b) (8 pts) Consider the following equation by using complex amplitudes, i.e., phasors,

$$\sin(60\pi t + \pi/6) \quad = \quad A\cos(60\pi(t - t_0)) + 2\cos(60\pi t - \pi/3)$$

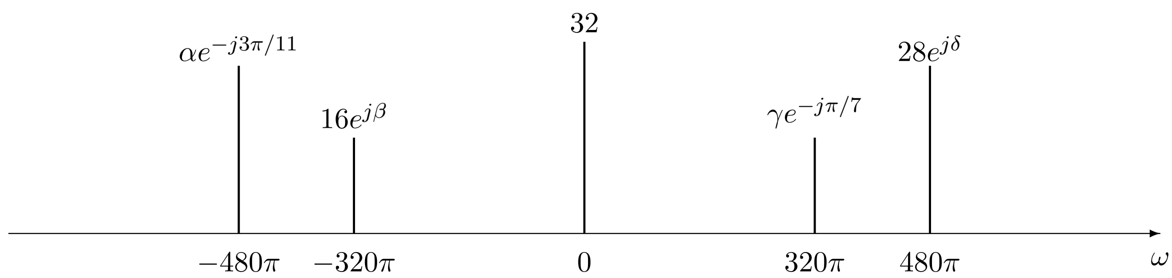
where the unknown amplitude A is positive, and the unknown delay (t_0) lies between $(-1/60, 1/60)$. Solve for A and t_0.

Express your answers in decimal form.

$A =$______________. $t_0 =$_______________.

(c) (8 points) Sketch the spectrum of $\Re e\{3e^{j7\pi(t-1/5)}\}$ in radians/sec.

(d) (9 points) The signal $x(t)$ is real and has the two-sided spectrum representation shown below:



Solve for $\alpha, \beta, \gamma, \delta$ and then write an equation for the signal $x(t)$. Make sure to express $x(t)$ as a real-valued signal. Following our conventions, write your answer using $\cos(\cdot)$ functions and decimal values. Your answer should not contain α, β, γ, or δ.

Version A Solution

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 12-FEB-16 COURSE: ECE 2026A,B

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L00:Tues-9:30am (Yang) L01:Mon-3:00pm (Odom)

L03:Mon-4:30pm (Odom) L05:Tues-12:00pm (Yang)

L06:Thur-12:00pm (Zajic) L07:Tues-1:30pm (Stüber)

L08:Thur-1:30pm (Zajic) L09:Tues-3:00pm (Stüber)

L10:Thur-3:00pm (Yeredor) L12:Thur-4:30pm (Yeredor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	32	
2	32	
3	33	
Rec	3	
Total	100	

Problem Q1.1:

Each part of this problem is independent of the others.

(a) (8 points) All complex numbers have N distinct Nth roots. Find five distinct 5th roots of j, and plot them in the complex plane.
Answer:

$$\begin{aligned} j &= e^{j(\pi/2+k2\pi)} \\ j^{1/5} &= e^{(j\pi/2+k2\pi)/5} \\ &= e^{j\pi/10}; e^{j5\pi/10}; e^{j9\pi/10}; e^{j13\pi/10}; e^{j17\pi/10} \end{aligned}$$

(b) (8 points) Consider complex number $z = 6 + j3$. Multiply z by its complex conjugate. Express this product in polar form.

Answer: $zz^* = 6^2 + 3^2 = 45$ This is in polar form with an angle of 0.

(c) (8 points) Use the inverse Euler relations to express $\sin(\alpha x)\sin(\beta x)$ as a sum of complex exponentials.

Answer:

$$\begin{aligned} & (e^{j\beta x} - e^{-j\beta x})(e^{j\alpha x} - e^{-j\alpha x})/(-4) \\ = & (e^{j(\beta-\alpha)x} + e^{-j(\beta-\alpha)x} - e^{j(\beta+\alpha)x} - e^{-j(\beta+\alpha)x})/4 \end{aligned}$$

(d) (8 points) Find a complex numerical value for:

$$z = \sum_{k=0}^{8} \mathrm{e}^{j(2\pi k/8+\pi/3)}$$

Answer: Since $\sum_{k=1}^{8} \mathrm{e}^{j(2\pi k/8+\pi/3)} = 0$ we only have the term with $k = 0$ left $\longrightarrow z = e^{j\pi/3}$

Problem Q1.2:

All parts are mutually unrelated.

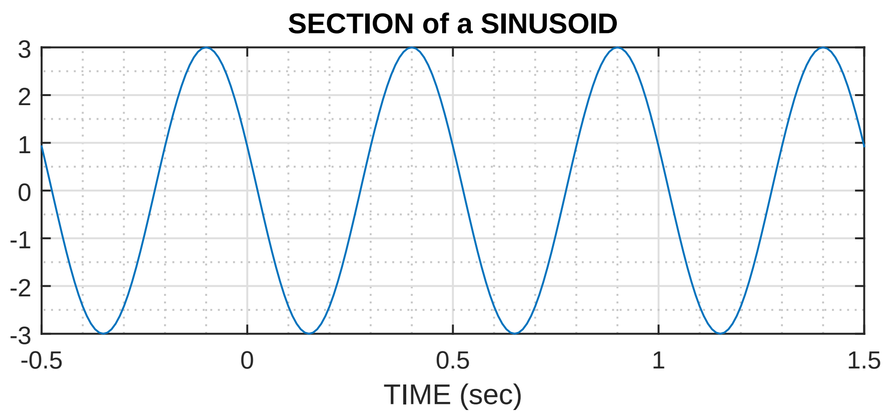
(a) (8 pts) Express $-3\cos(7\pi t + \pi/3) + 2\cos(7\pi t - \pi/3)$ as $A\cos(\omega_0 t + \phi)$.

$A =$_________ $\phi =$ _____________ (in radians) $\omega_0 =$______________ (in radians/second)

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

Answer: $A = 4.36$; $\phi = 1.69$; $\omega_0 = 7\pi$

(b) (8 pts) Consider the sinusoidal signal plotted below.



This signal may be written in the form $A\cos(\omega t + \phi)$. Find A, ω, and ϕ.

$A =$______________ $\omega =$_______________ $\phi =$_____________

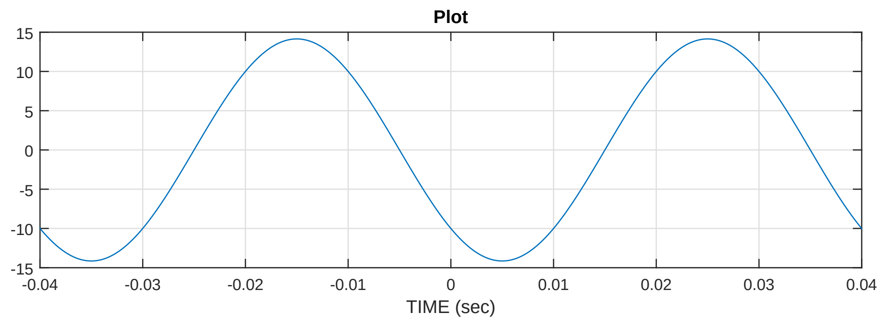
Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

Answer: $A = 3$; $\omega = 4\pi$; $\phi = 2\pi/5 = 1.26$

(c) (8 pts) The following MATLAB code generates a signal and makes a plot. Draw a sketch of the plot that will be done by MATLAB. Determine the amplitude (A), phase (ϕ), and period of the sinusoid and label the period on your plot.

```
Fo = 25;
td=.01;
dt = 0.0001;
tt = -.04 : dt : .04;
Z = 10+10*j;
xx = real( Z*exp( j*2*pi*Fo*(tt + td) ) );
plot( tt, xx ), grid
title( Plot ), xlabel(TIME (sec))
```

Answer:



(d) (8 pts) For $P = 2e^{-j\pi/8}$, express $\Re e\{Pe^{-j10\pi(t+.3)}\}$ in standard *cosine* form, i.e., as $A\cos(\omega t+\phi)$.

Answer:

$$\Re e\{2e^{-j\pi/8}e^{-j10\pi(t+.3)}\} =$$
$$\Re e\{2e^{-j\pi/8}e^{-j10\pi t-j3\pi)}\} =$$
$$\Re e\{2e^{-j10\pi t-j3\pi-j\pi/8}\} =$$
$$\Re e\{2e^{-j10\pi t+j7\pi/8}\} =$$
$$\Re e\{2e^{-j(10\pi t-7\pi/8)}\} =$$
$$2\cos(10\pi t - 7\pi/8)$$

Problem Q1.3:

(a) (8 pts) Consider the complex integral:

$$z = \int_0^{t_0} e^{j(100\pi t + \pi/11)} dt$$

Find the smallest value of $t_0 > 0$ such that $z = 0$. (You should not have to perform an integral.)

Answer: Any complex sinusoid integrates to zero over one period. Therefore make $t_0 = 1/50 = .02$

(b) (8 pts) Consider the following equation by using complex amplitudes, i.e., phasors,

$$\sin(60\pi t + \pi/6) \quad = \quad A\cos(60\pi(t - t_0)) + 2\cos(60\pi t - \pi/3)$$

where the unknown amplitude A is positive, and the unknown delay (t_0) lies between $(-1/60, 1/60)$. Solve for A and t_0.

Express your answers in decimal form.

$A =$______________. $t_0 =$________________.

Answer:

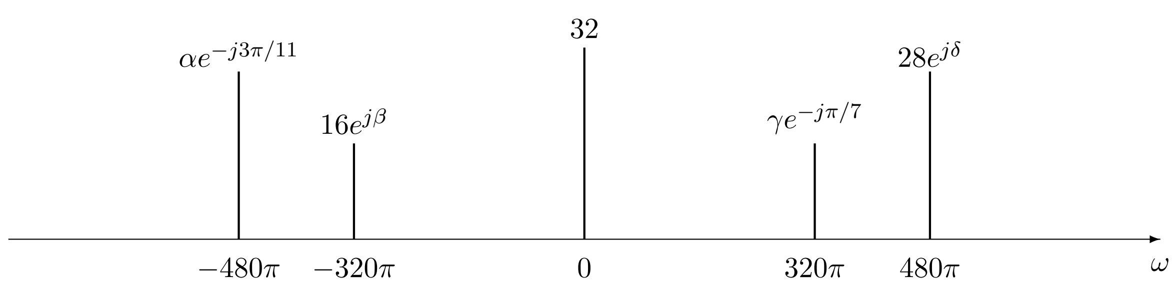
$$e^{j(\pi/6-\pi/2)} = Ae^{-j60\pi t_0} + 2e^{-j\pi/3}$$
$$-e^{-j\pi/3} = Ae^{-j60\pi t_0}$$
$$e^{j2\pi/3} = Ae^{-j60\pi t_0}$$
$$A = 1$$
$$2\pi/3 = -60\pi t_0$$
$$t_0 = 1/90 = 0.01111$$

(c) (8 points) Sketch the spectrum of $\Re e\{3e^{j7\pi(t-1/5)}\}$ in radians/sec.

Answer:

$$\begin{aligned} & \Re e\{3e^{j7\pi(t-1/5)}\} \\ = & \Re e\{3e^{j7\pi t - j7\pi/5}\} \\ = & \Re e\{3e^{j7\pi t + j3\pi/5}\} \\ = & 3\cos(7\pi t + 3\pi/5) \end{aligned}$$

(d) (9 points) The signal $x(t)$ is real and has the two-sided spectrum representation shown below:



Solve for $\alpha, \beta, \gamma, \delta$ and then write an equation for the signal $x(t)$. Make sure to express $x(t)$ as a real-valued signal. Following our conventions, write your answer using $\cos(\cdot)$ functions and decimal values. Your answer should not contain α, β, γ, or δ.

Answer: $\alpha = 28$; $\beta = \pi/7$; $\gamma = 16$; $\delta = 3\pi/11$

$x(t) = 32 + 32\cos(320\pi t - \pi/7) + 56\cos(480\pi t + 3\pi/11)$