a

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 30-JAN-14 COURSE: ECE 2026A,B

NAME: _____ STUDENT #: _____

LAST, FIRST

2 points	2 points	2 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L00:Tue-9:30am (Zhang) L01:Mon-3:00pm (Casinovi)

L03:Mon-4:30pm (Casinovi) L05:Tue-12:00pm (Zhang)

L06:Thu-12:00pm (Walkenhorst) L07:Tue-1:30pm (Chang)

L08:Thu-1:30pm (Walkenhorst) L09:Tue-3:00pm (Zajic)

L10:Thu-3:00pm (Fekri) L12:Thu-4:30pm (Fekri)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	18	
2	14	
3	16	
Rec	2	
Total	50	

Problem Q1.1:
Sinusoids

Each part of this problem is independent of the others.

(a) (4 pts) A sinusoid is generated and plotted by the following MATLAB code:

```
tt = -0.1:(1/1e3):0.6;
xx = 2 + sqrt(3)*cos(pi*(tt + 0.03)/0.03); plot( xx (10:160) );
```

How many cycles do you see in the plot? (your answer may be fractional)

how many samples are one period? $\Rightarrow 30 \cdot 2 = 60$

$\left(\pi \cdot \frac{30.2}{1e3} / 0.03\right) = 2\pi$

The plot command shows 150 samples* so

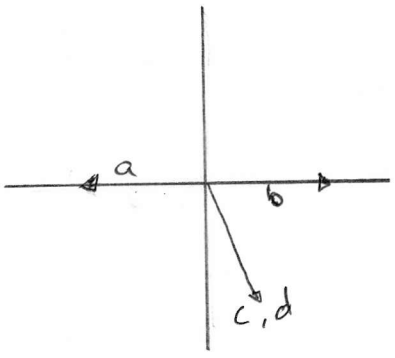
$\frac{150}{60} = \frac{5}{2} =$ **2.5 cycles**

*actually 151 cycles

(b) (4 pts) Let $A\cos(\omega t + \phi) = \underset{a}{\cos(4t+\pi)} - \underset{b}{\cos(4t-\pi)} + \underset{c}{\sin(4t+\pi/6)} + \underset{d}{\cos(4(t-\pi/12))}$.
Find A, ω, and ϕ.

$A =$ 2 $\quad \omega_0 =$ 4 radians/second $\quad \phi =$ $-\pi/3$ radians

Be sure that A is a positive, real number. Also ensure that $-\pi < \phi \leq \pi$.



$a \Leftrightarrow e^{j\pi}$

$b \Leftrightarrow e^{j0}$

$c = e^{j\pi/6} e^{-j\pi/2}$

$= e^{-j\pi/3}$

$d = e^{-j\pi/3}$

$\sin\theta = \cos(\theta - \pi/2)$

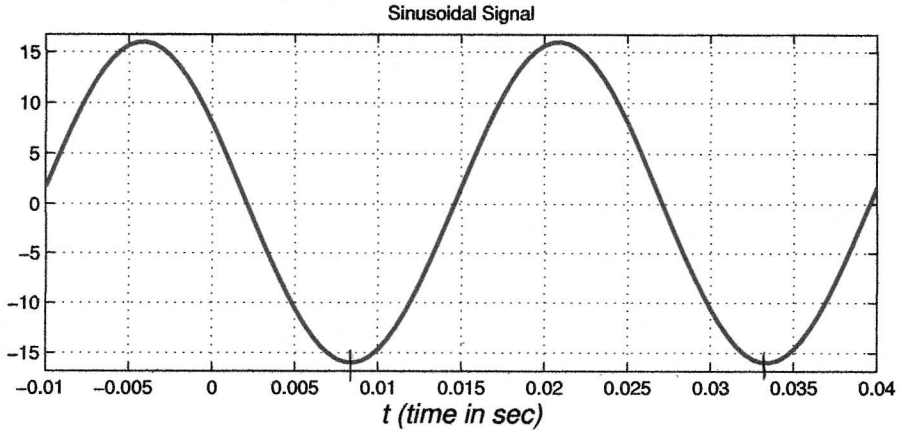
$a + b + c + d =$ **$2e^{-j\pi/3}$**

(c) (4 pts) For $X = 5e^{j\pi/7}$, express $\Re e\{Xe^{-j10\pi t}\}$ in standard *cosine* form, i.e., as $A\cos(\omega t + \phi)$.

$x(t) = 5\cos(-10\pi t + \pi/7)$

$\boxed{x(t) = 5\cos(10\pi t - \pi/7)}$

(d) (6 pts) Consider the sinusoidal signal plotted below.



This signal may be written in the form $A\cos(\omega t + \phi)$. Find A, ω, and ϕ.

$A =$ 16 $\omega_0 =$ 80π radians/second $\phi =$ $\pi/3$ radians

Be sure that A is a positive, real number. Also ensure that $-\pi < \phi \leq \pi$.

$T = 0.025 \Rightarrow f = 40$

$t_m \approx -0.004$

$\phi = -\omega t_m = -(80\pi)(-0.004) = 0.32\pi$

(exact answer is $\pi/3$)

Problem Q1.2:
Complex Numbers

Each part of this problem is independent of the others.

(a) (6 pts) One or more of the following are solutions to the equation $z^n - A = 0$ for $n > 0$ and $A > 0$.

(1) $z = A^{3/n} e^{j6\pi/n}$	(2) $z = A^{1/n} e^{-j\pi n}$	(3) $z = 0$
(4) $z = A^{1/n}$	(5) $z = A^{1/n} e^{-j4\pi/n}$	(6) $z = A^{1/n} e^{j3\pi n}$

Circle each correct solution (there may be more than one).

(b) (4 pts) Use the inverse Euler relations to find the value of $x = \sin(-j\pi/3)$. Show your work (do not just plug into a calculator). Please make note of the j inside the argument.

$$\sin(-j\pi/3) = \frac{1}{2j}\left(e^{j(-j\pi/3)} - e^{-j(-j\pi/3)}\right) = \frac{1}{2j}\left(e^{\pi/3} - e^{-\pi/3}\right)$$

$$= j\left(\frac{e^{-\pi/3} - e^{\pi/3}}{2}\right) = \boxed{-1.2494j}$$

(c) (4 pts) Evaluate the following sum and express it in the form $Ae^{j\theta}$.

$$z = \sum_{k=0}^{10} e^{j(2\pi k/12 + \pi^{-1})}$$

$$z = e^{j\pi} \sum_{k=0}^{10} e^{j(2\pi k/12)}$$

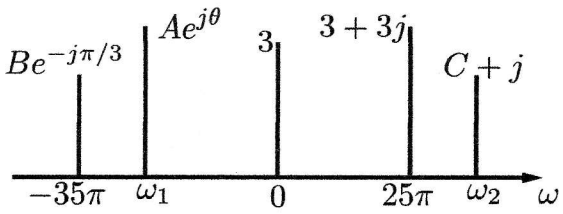
$\frac{1}{12}$ roots of unity.
summing 12 of them = 0

$$z = e^{j\pi}\left(\sum_{k=0}^{11} e^{j\left(\frac{2\pi k}{12}\right)} - e^{j\frac{2\pi \cdot 11}{12}}\right)$$

$$\boxed{z = -e^{j\left(\frac{11\pi}{6} + \frac{1}{\pi}\right)}}$$

Problem Q1.3:
Spectra

Suppose a real-valued signal x(t) has the following two-sided spectrum:



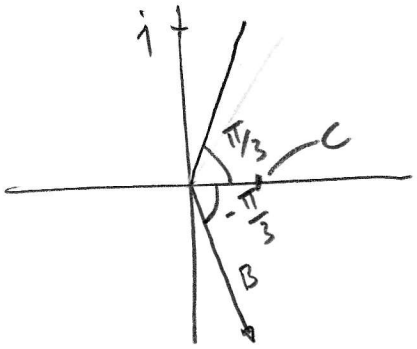
The unknown variables A, B, C, θ, ω_1, and ω_2 are all real, satisfying $A > 0$, $B > 0$, and $-\pi < \theta \leq \pi$. Although these variables are not given explicitly, they can be determined uniquely from the spectrum plot.

(a) (12 pts) Find A, B, C, θ, ω_1, and ω_2

$A = 3\sqrt{2}$ $\quad B = \frac{2}{\sqrt{3}}$ $\quad C = 1/\sqrt{3}$

$\theta = -\pi/4$ $\quad \omega_1 = -25\pi$ $\quad \omega_2 = 35\pi$

$3+3j = 3\sqrt{2}\, e^{j\pi/4}$ $\Rightarrow Ae^{j\theta} = 3\sqrt{2}\, e^{-j\pi/4}$



$B\sin\frac{\pi}{3} = 1 \quad B = \frac{2}{\sqrt{3}}$

$C = B\cos\frac{\pi}{3} = \frac{2}{\sqrt{3}} \cdot \frac{1}{2} = \frac{1}{\sqrt{3}}$

(b) (4 pts) Write an expression for $x(t)$ involving only real numbers and cosine functions. Any phase angles should be between $-\pi$ and π.

$x(t) = 3 + 6\sqrt{2}\cos(25\pi t + \pi/4) + \frac{4}{\sqrt{3}}\cos(35\pi t + \pi/3)$