GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 31-JAN-14 ⇒ 03-FEB-14 COURSE: ECE 2026A,B

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01:Mon-3:00pm (Causey)	L02:Wed-3:00pm (Romberg)
L03:Mon-4:30pm (Causey)	L04:Wed-4:30pm (Romberg)
L05:Tues-Noon (Fekri)	L06:Thur-Noon (Chang)
L07:Tues-1:30pm (Fekri)	L08:Thur-1:30pm (Stüber)
L10:Thur-3:00pm (Stüber)	L12:Thur-4:30pm (Chang)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	32	
2	32	
3	33	
Rec	3	
Total	100	

Problem Q1.1:

(8 pts each) Each part of this problem is independent of the others.

(a) Let r be a complex number where $|r| = 1$ and $\angle r = \pi/3$. Find and plot one possible value for the complex vector r^j. (The answer is not unique. Bonus $+1$: find a second distinct value)

(b) Find and sketch the complex number $x = j\cos(\theta) - \sin(\theta)$ where θ represents an angle in the first quandrant.

(c) Use the inverse Euler relations to find the value of $x = \cos(-j\pi/7)$. Show your work (do not just plug into a calculator). Please make note of the j inside the argument.

(d) Find a complex numerical value for:

$$z = \sum_{k=1}^{11} \mathrm{e}^{j(2\pi k/12 + \pi^{-1})}$$

Problem Q1.2:

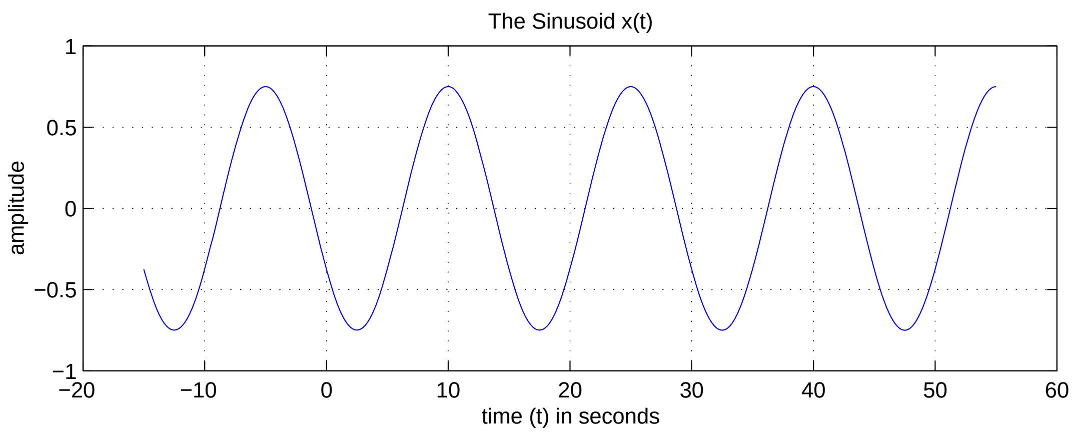
All parts are mutually unrelated.

(a) (9 pts) Express $3\cos(3\pi t + \pi/3) + 2\cos(3\pi t - \pi/3)$ as $A\cos(\omega_0 t + \phi)$.

$A =$________ $\phi =$ ____________ (in radians) $\omega_0 =$_____________ (in radians/second)

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

(b) (9 pts) Consider the sinusoidal signal plotted below.



This signal may be written in the form $A\cos(\omega t + \phi)$. Find A, ω, and ϕ.

$A =$_____________ $\omega =$______________ $\phi =$____________

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

(c) (7 pts) For $P = 3e^{-j\pi/5}$, express $\Re e\{Pe^{-j10\pi t}\}$ in standard *cosine* form, i.e., as $A\cos(\omega t + \phi)$.

(d) (7 pts) Simplify the following expression:

$$x(t) = \sum_{k=0}^{40} \cos\left(206\pi t + \frac{k}{10}2\pi\right)$$

You should be able to do this without doing any explicit calculations!

Problem Q1.3:

All parts are mutually unrelated.

(a) (9 points) Consider the signal:

$$x(t) = \cos^2(60\pi t)$$

which can be written as:

$$x(t) = A + B\cos(\alpha t)$$

Using the Euler and/or inverse Euler relations, find A, B, and α.

$A =$ __________. $B =$_____________. $\alpha =$______________.

(b) (8 pts) Find a complex-valued signal $z_1(t) = (Ae^{j\varphi})e^{j\omega t}$ such that

$$\Re e\{\tfrac{d}{dt}z_1(t)\} = 7\cos(50\pi(t - 0.01))$$

(Express your answer in decimal form where $A \geq 0$ and $-\pi < \varphi \leq \pi$ in radians)

$A =$____________. $\varphi =$_____________. $\omega =$____________.

(c) (8 pts) Consider the complex integral:

$$z = \int_{0.01}^{t_0} e^{j(100\pi t + \pi/11)}\,dt$$

Find the smallest value of $t_0 > 0.01$ such that $z = 0$. (You should not have to perform an integral.)

(d) (8 pts) Consider the following simultaneous equations by using complex amplitudes, i.e., phasors,

$$\begin{aligned} \sin(\omega_0 t + \pi/3) &= A_1 \cos(\omega_0 t + \varphi_1) + A_2 \cos(\omega_0 t + \varphi_2 - \pi/3) \\ 2\cos(\omega_0 t - \pi/7) &= A_1 \cos(\omega_0 t + \varphi_1) + A_2 \cos(\omega_0 t + \varphi_2 - \pi/3 - \pi) \end{aligned}$$

where the unknown amplitudes (A_k) are positive, and the unknown phases (φ_k) lie between $\pm\pi$. Solve for A_1 and φ_1. (Once you set the problem up correctly, the solution is straightforward.) Express your answers in decimal form.

$A_1 =$______________. $\varphi_1 =$_______________.

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QUIZ #1 Version A Solution

DATE: 31-JAN-14 ⇒ 03-FEB-14 **VERSION A Solution** **COURSE: ECE 2026A,B**

NAME: ______________________ STUDENT #: ______________
LAST, FIRST

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Problem	*Value*	*Score*
1	32	
2	32	
3	33	
Rec	3	
Total	100	

Problem Q1.1:

(8 pts each) Each part of this problem is independent of the others.

(a) Let r be a complex number where $|r| = 1$ and $\angle r = \pi/3$. Find and plot one possible value for the complex vector r^j. (The answer is not unique. Bonus +1: find a second distinct value)

$r = e^{j\pi/3}$, $r^j = e^{-\pi/3} = 0.3509$. Also $r = e^{j(\pi/3+2\pi k)}$, $r^j = e^{-\pi/3-2\pi k}$. For $k = -1$, $r^j = 187.91$.

(b) Find and sketch the complex number $x = j\cos(\theta) - \sin(\theta)$ where θ represents an angle in the first quandrant.

$x = je^{j\theta} = e^{j(\theta+\pi/2)}$. Therefore x is in the second quadrant with length 1 and angle $\theta + \pi/2$.

(c) Use the inverse Euler relations to find the value of $x = \cos(-j\pi/7)$. Show your work (do not just plug into a calculator). Please make note of the j inside the argument.

$\cos(\alpha) = \frac{e^{j\alpha}+e^{-j\alpha}}{2}$. $\cos(j\alpha) = \frac{e^{-\alpha}+e^{\alpha}}{2}$. For $\alpha = -\pi/7$, $x = 1.1024$.

(d) Find a complex numerical value for:

$$z = \sum_{k=1}^{11} e^{j(2\pi k/12+\pi^{-1})}$$

$\sum_{k=0}^{11} e^{j(2\pi k/12+\pi^{-1})} = 0$. Therefore $\sum_{k=1}^{11}(*) = 0 - e^{j\pi^{-1}}$

Problem Q1.2:

All parts are mutually unrelated.

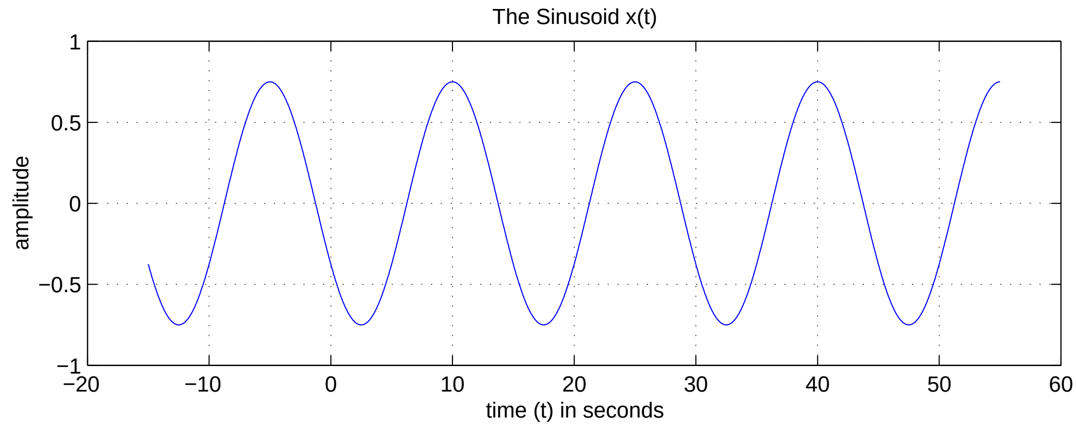
(a) (9 pts) Express $3\cos(3\pi t + \pi/3) + 2\cos(3\pi t - \pi/3)$ as $A\cos(\omega_0 t + \phi)$.

$A =$ <u>2.6458</u> $\phi =$ <u>0.3335</u>(in radians) $\omega_0 = \underline{3\pi = 9.425}$ (in radians/second)

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

$Ae^{j\phi} = 3e^{j\pi/3} + 2e^{-j\pi/3} = 2.6458e^{j0.3335}$

(b) (9 pts) Consider the sinusoidal signal plotted below.



This signal may be written in the form $A\cos(\omega t + \phi)$. Find A, ω, and ϕ.

$A =$ <u>0.75</u> $\omega =$ <u>0.4189</u> $\phi =$ <u>2.0944</u>

Be sure that A is a positive, real number. Also express ϕ as a decimal number, $-\pi < \phi \leq \pi$.

A looks like 0.75. Two periods are 30 seconds, so $T = 15, \omega = 2\pi/15$. The sinusoid is advanced by 1/3 of a period, so $\phi = 2\pi/3$.

(c) (7 pts) For $P = 3e^{-j\pi/5}$, express $\Re e\{Pe^{-j10\pi t}\}$ in standard *cosine* form, i.e., as $A\cos(\omega t + \phi)$.

$= \Re e\{3e^{-j\pi/5}e^{-j10\pi t}\} = 3\Re e\{e^{-j(10\pi t + \pi/5)}\} = 3\cos(10\pi t + \pi/5)$

(d) (7 pts) Simplify the following expression:

$$x(t) = \sum_{k=0}^{40} \cos\left(206\pi t + \frac{k}{10}2\pi\right)$$

You should be able to do this without doing any explicit calculations!

Since $\sum_{k=0}^{39}(*) = 0$, $\sum_{k=0}^{40}(*) = \cos\left(206\pi t + \frac{40}{10}2\pi\right) = \cos(206\pi t)$

Problem Q1.3:

All parts are mutually unrelated.

(a) (9 points) Consider the signal:

$$x(t) = \cos^2(60\pi t)$$

which can be written as:

$$x(t) = A + B\cos(\alpha t)$$

Using the Euler and/or inverse Euler relations, find A, B, and α.

$\cos^2(\theta) = \left(\frac{e^{j\theta}+e^{-j\theta}}{2}\right)^2 = \frac{e^{j2\theta}}{4} + \frac{e^{-j2\theta}}{4} + \frac{1}{2} = \frac{1}{2} + \frac{\cos(120\pi t))}{2}$

$A =$ <u>0.5</u> $B =$<u>0.5</u> $\alpha = \underline{120\pi}$

(b) (8 pts) Find a complex-valued signal $z_1(t) = (Ae^{j\varphi})e^{j\omega t}$ such that

$$\Re e\{\tfrac{d}{dt}z_1(t)\} = 7\cos(50\pi(t-0.01))$$

(Express your answer in decimal form where $A \geq 0$ and $-\pi < \varphi \leq \pi$ in radians)

$A = \underline{\frac{7}{50\pi}}$ $\varphi = \underline{\pi}.$ $\omega = \underline{50\pi}$

Fist of all, $\omega = 50\pi$. Also $7\cos(50\pi(t-0.01)) = 7\cos(50\pi t - \pi/2)) = 7\sin(50\pi t)$. This is the derivative of $\frac{-7}{50\pi}\cos(50\pi t)$. Therefore $z_1(t)$ corresponds to a cosine of complex amplitude $Ae^{j\varphi} = \frac{7}{50\pi}e^{j\pi}$ and frequency $\omega = 50\pi$.

(c) (8 pts) Consider the complex integral:

$$z = \int_{0.01}^{t_0} e^{j(100\pi t + \pi/11)} dt$$

Find the smallest value of $t_0 > 0.01$ such that $z = 0$. (You should not have to perform an integral.)

The complex exponential in question repeats every 0.02 seconds. Its integral over an interval of width 0.02 will therefore be zero. Hence $t_0 = 0.03$.

(d) (8 pts) Consider the following simultaneous equations by using complex amplitudes, i.e., phasors,

$$\begin{aligned} \sin(\omega_0 t + \pi/3) &= A_1 \cos(\omega_0 t + \varphi_1) + A_2 \cos(\omega_0 t + \varphi_2 - \pi/3) \\ 2\cos(\omega_0 t - \pi/7) &= A_1 \cos(\omega_0 t + \varphi_1) + A_2 \cos(\omega_0 t + \varphi_2 - \pi/3 - \pi) \end{aligned}$$

where the unknown amplitudes (A_k) are positive, and the unknown phases (φ_k) lie between $\pm\pi$. Solve for A_1 and φ_1. (Once you set the problem up correctly, the solution is straightforward.) Express your answers in decimal form.

$A_1 = \underline{1.4991}$. $\quad\quad \varphi_1 = \underline{-0.4737}$.

$$\begin{aligned} e^{j(\pi/3-\pi/2)} &= A_1 e^{j\varphi_1} + A_2 e^{j(\varphi_2 - \pi/3)} \\ 2e^{-j(\pi/7)} &= A_1 e^{j\varphi_1} - A_2 e^{j(\varphi_2 - \pi/3)} \end{aligned}$$

Adding the two equations results in:
$2A_1 e^{j\varphi_1} = e^{j(\pi/3-\pi/2)} + 2e^{-j(\pi/7)}$, $A_1 = 1.4991$, $\varphi = -0.4737$
