GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 1-Feb-10 COURSE: ECE-2025

NAME: _____ GT username: _____

LAST, FIRST (ex: gpburdell3)

3 points | 3 points | 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Michaels)	L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Michaels)	L08:Thur-1:30pm (Bhatti)
L01:M-3pm (Lee)	L09:Tues-3pm (Fekri)	
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	50	
2	30	
3	20	
No/Wrong Rec	−3	

PROBLEM sp-10-Q.1.1:

Evaluate the expressions below, where angles are given in radians and frequencies in rad/s. In the answers, the magnitudes, r, or amplitudes, A, ***must be nonnegative***; and the angles, θ or φ, ***must be in radians***, and lie between $-\pi$ and $+\pi$. Use a calculator; only the answers will be graded—no explanations necessary.

(a) Determine r and θ, such that $re^{j\theta} = -4$.

$r =$	$\theta =$

(b) Determine r and θ, such that $re^{j\theta} = 512j$.

$r =$	$\theta =$

(c) Determine r and θ, such that $re^{j\theta} = 2e^{j1.5} + 3e^{-j2}$.

$r =$	$\theta =$

(d) Determine r and θ, such that $re^{j\theta} = (-50 - j70)e^{-j2}$.

$r =$	$\theta =$

(e) Express this signal, $\Re\{-5je^{j8\pi t}\}$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$

(f) Express this signal, $\Re\{(-21 - j8)e^{j0.7t}\}$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$

(g) Express this signal, $\Re\{789e^{-j0.3}e^{j8\pi t}\}$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$

(h) Express this signal, $3\cos(5t + 1.2) + 7\cos(5t + 2.5)$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$	$\omega_0 =$

PROBLEM sp-10-Q.1.2:

(a) Evaluate this definite integral, and express the answer in polar form: $\int_{5}^{10} e^{j\,0.1\pi t}\,dt = r\,e^{j\theta}$

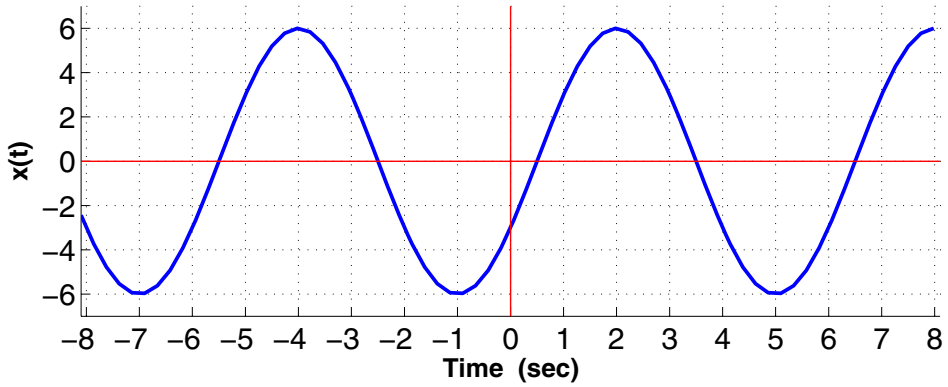
$r =$	$\theta =$

(b) Find the *real* numbers p and q such that the following equation is true: $\dfrac{jp}{5+jq} = 8e^{j\,3\pi/4}$

$p =$	$q =$

(c) Values of the sinusoid shown below can be generated via the following MATLAB statements:

```
tt = -8:0.01:8;  XX = ??;  ww = ??;  xt = real( XX * exp(j*ww*tt) );
```



Write the appropriate MATLAB statements needed to define `XX` and `ww`.

`XX=` _____

`ww=` _____

PROBLEM sp-10-Q.1.3:

(a) Recall that the following sum: $\sum_{k=1}^{L} e^{j2\pi k/N}$ is equal to 0 when $L = N$.

The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$.

```
tt = 0:1:1000;
xx = 0*tt;
for kk=3:11
   xx = xx + 5*cos(0.006*pi*tt + 0.25*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

The plot made from the vector `xx` is a single sinusoid, which can be written as $A\cos(\omega_0 t + \varphi)$. Determine the parameters for the sinusoid in the vector `xx`. Also, identify the value of N.

$N =$	$A =$	$\varphi =$	$\omega_0 =$

(b) The simultaneous sinusoidal equations below can be converted into simultaneous linear equations involving complex variables and complex numbers.

$$\begin{aligned} 8\cos(\omega_0 t - \pi/5) &= 4A_1\cos(\omega_0 t + \varphi_1) + 3A_2\cos(\omega_0 t + \varphi_2) \\ 3\cos(\omega_0 t + 3\pi/5) &= A_1\cos(\omega_0 t + \varphi_1) + 7A_2\cos(\omega_0 t - 3\pi/7 + \varphi_2) \end{aligned}$$

Then MATLAB can obtain the solution for $\{A_1, \varphi_1, A_2, \varphi_2\}$ via its backslash operator, or `inv`:

```
AA = [ ?, ?; ?, ? ];
bb = [ ?; ? ];
zz = AA \ bb;
amps = abs(zz), angles = angle(zz)
```

Determine the correct MATLAB statements for the matrix `AA`, and the vector `bb`.
Note: it is not necessary to solve the equations; just set up the MATLAB code.

`AA =` _____

`bb =` _____

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PROBLEM sp-10-Q.1.1:

Evaluate the expressions below, where angles are given in radians and frequencies in rad/s. In the answers, the magnitudes, r, or amplitudes, A, ***must be nonnegative***; and the angles, θ or φ, ***must be in radians***, and lie between $-\pi$ and $+\pi$. Use a calculator; only the answers will be graded—no explanations necessary.

(a) Determine r and θ, such that $re^{j\theta} = -4$.

$r = 4$ $\theta = \pm\pi$ rads

(b) Determine r and θ, such that $re^{j\theta} = 512j$.

$r = 512$ $\theta = \pi/2$ rads

(c) Determine r and θ, such that $re^{j\theta} = 2e^{j1.5} + 3e^{-j2}$.

$r = 1.328$ $\theta = -2.557$ rads

(d) Determine r and θ, such that $re^{j\theta} = (-50 - j70)e^{-j2}$.

$r = 86.02$ $\theta = 2.092$ rads

(e) Express this signal, $\Re\{-5je^{j8\pi t}\}$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A = 5$ $\varphi = -\pi/2$ rads

(f) Express this signal, $\Re\{(-21 - j8)e^{j0.7t}\}$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A = 22.47$ $\varphi = -2.778$ rads

(g) Express this signal, $\Re\{789e^{-j0.3}e^{j8\pi t}\}$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A = 789$ $\varphi = -0.3$ rads

(h) Express this signal, $3\cos(5t + 1.2) + 7\cos(5t + 2.5)$, as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$A = 8.321$ $\varphi = 2.145$ rads $\omega_0 = 5$ rad/s

PROBLEM sp-10-Q.1.2:

(a) Evaluate this definite integral, and express the answer in polar form: $\int_5^{10} e^{j\,0.1\pi t}\,dt = re^{j\theta}$

$$re^{j\theta} = \frac{\sqrt{2}}{0.1\pi}\,e^{j\,3\pi/4} = 4.5016\,e^{j\,2.3562}$$

Approach: The integral of an exponential is an exponential, but you end up with a j in the denominator because the exponent contains a j. After evaluating at the limits of the definite integral, the numerator has a complex number. Finally, convert the complex numerator-denominator into polar form with a calculator.

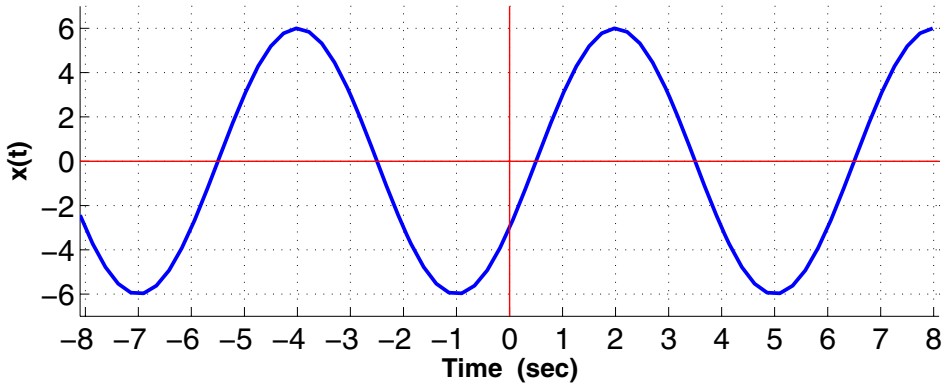
(b) Find the *real* numbers p and q such that the following equation is true: $\dfrac{jp}{5+jq} = 8e^{j\,3\pi/4}$

$\boxed{p = 40\sqrt{2} = 56.5685}$ $\boxed{q = -5}$

Approach: Express the right-hand side (RHS) in Cartesian form using Euler's formula. Then cross multiply by the denominator of the LHS to get a single complex-valued equation. Equate the real and imaginary parts to get two (linear) equations in the two unknowns, p and q. One of these equations has only p, or only q, so solve it and then substitute into the other equation.

(c) Values of the sinusoid shown below can be generated via the following MATLAB statements:

```
tt = -8:0.01:8;  XX = ??;  ww = ??;  xt = real( XX * exp(j*ww*tt) );
```



Write the appropriate MATLAB statements needed to define `XX` and `ww`.

```
XX = 6*exp(-j*2*pi/3)
```

```
ww = 2*pi/6
```

Approach: Measure the period to obtain $T = 6$ s, and measure the location of a positive peak, $t_m = 2$ s. Measure the amplitude, A, from the height of a positive peak.
Calculate the frequency (in rad/s) via $\omega = 2\pi/T = 2\pi/6$, and then the phase (in rads) via $\varphi = -\omega t_m = -2\pi(2)/6$.
Finally, use A and φ to define `XX` from the complex amplitude $Ae^{j\varphi}$.

PROBLEM sp-10-Q.1.3:

(a) Recall that the following sum: $\sum_{k=1}^{L} e^{j2\pi k/N}$ is equal to 0 when $L = N$.

The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$.

```
tt = 0:1:1000;
xx = 0*tt;
for kk=3:11
   xx = xx + 5*cos(0.006*pi*tt + 0.25*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

The plot made from the vector `xx` is a single sinusoid, which can be written as $A\cos(\omega_0 t + \varphi)$. Determine the parameters for the sinusoid in the vector `xx`. Also, identify the value of N.

$N = 8$	$A = 5$	$\varphi = 0.75\pi$ rads	$\omega_0 = 0.006\pi$ rad/s

Approach: The `for` loop adds 9 sinusoids, which can be done as the phasor addition of 9 complex amplitudes. The phases of the sinusoids are $2\pi k/8$, i.e., the angular difference between successive complex amplitudes is $2\pi/8$. The identity tells us that adding 8 successive complex exponentials will give zero, so we have one left over. That "left over" one is the complex amplitude of the answer; you can choose the first one, or the last one. Since the range of k is `3:11`, the first one is at $k = 3$, so it is $5e^{j2\pi(3)/8}$ which will be the complex amplitude $Ae^{j\varphi}$, giving A and φ for the surviving sinusoid.

(b) The simultaneous sinusoidal equations below can be converted into simultaneous linear equations involving complex variables and complex numbers.

$$\begin{aligned} 8\cos(\omega_0 t - \pi/5) &= 4A_1\cos(\omega_0 t + \varphi_1) + 3A_2\cos(\omega_0 t + \varphi_2) \\ 3\cos(\omega_0 t + 3\pi/5) &= A_1\cos(\omega_0 t + \varphi_1) + 7A_2\cos(\omega_0 t - 3\pi/7 + \varphi_2) \end{aligned}$$

Then MATLAB can obtain the solution for $\{A_1, \varphi_1, A_2, \varphi_2\}$ via its backslash operator, or `inv`:

```
AA = [ ?, ?; ?, ? ];
bb = [ ?; ? ];
zz = AA \ bb;
amps = abs(zz), angles = angle(zz)
```

Determine the correct MATLAB statements for the matrix `AA`, and the vector `bb`.
Note: it is not necessary to solve the equations; just set up the MATLAB code.

```
AA = [ 4, 3; 1, 7*exp(-j*3*pi/7) ]
```

```
bb = [ 8*exp(-j*pi/5); 3*exp(3i*pi/5)]
```

Approach: First, you can convert the two sinusoidal equations into two phasor equations. If you define a variable z_i for the complex amplitude $z_i = A_i e^{j\varphi_i}$ then the phasor equations can be written as *linear* equations in the unknowns z_i, i.e., without exponents and without explicit use of the A_i and φ_i.

$$\begin{bmatrix} 8e^{j(-\pi/5)} \\ 3e^{j(+3\pi/5)} \end{bmatrix} = \begin{bmatrix} 4e^{j0} & 3e^{j0} \\ e^{j0} & 7e^{j(-3\pi/7)} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$