GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 6-Feb-09 COURSE: ECE-2025

NAME: ______________________________ GT username: ______________
LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L05:Tues-Noon (Bhatti) L06:Thur-Noon (Barry)

L07:Tues-1:30pm (Bhatti) L08:Thur-1:30pm (Barry)

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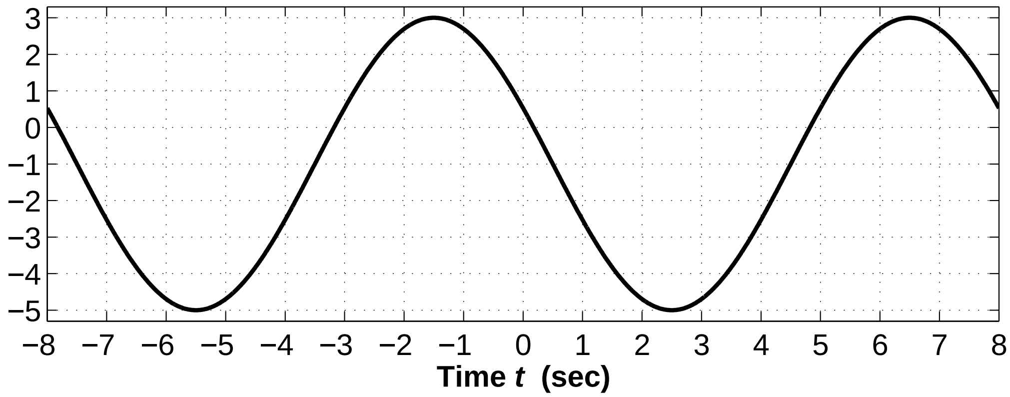
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	40	
2	30	
3	30	
No/Wrong Rec	−3	

PROBLEM sp-09-Q.1.1:

(a) The signal plotted below is the sum of two sinusoids; determine the amplitudes and phases, as well as the frequencies in rad/s.



$A_1 =$ __________

$\phi_1 =$ __________

$\omega_1 =$ __________ rad/s

$A_2 =$ __________

$\phi_2 =$ __________

$\omega_2 =$ __________ rad/s

(b) Express this signal as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$$s_a(t) = 5e^{-j\pi/3}e^{j30\pi t} + 5e^{j\pi/3}e^{-j30\pi t}$$

(c) Express this signal as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$$s_b(t) = \Re e\left\{(-10 - j30)e^{j631t}\right\}$$

PROBLEM sp-09-Q.1.2:

The following MATLAB code defines a periodic signal as the sum of sinusoids:

```
tt = -10:0.00001:10;  %- in seconds
xxt = -5*ones(size(tt));
kk=12; xxt = xxt + (30/kk)*cos( 1.2*pi*kk*(tt-1/6) );
kk=10; xxt = xxt + real( (-5-12/j)*exp(j*1.6*pi*kk*tt) );
```

In the following questions, the signal $x(t)$ corresponds to the MATLAB vector xxt.

(a) Determine the *fundamental frequency* of $x(t)$. $\omega_0 =$ __________ rad/s

(b) The signal $x(t)$ is periodic, so it has a Fourier Series

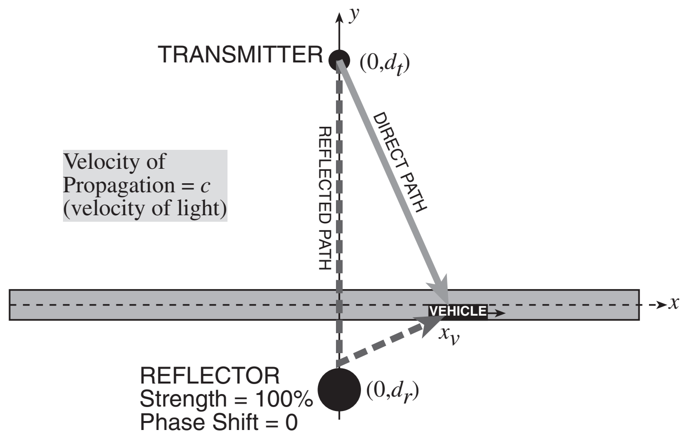
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}$$

Determine the values of all the nonzero coefficients $\{a_k\}$, and list them in the table below. Write the Fourier coefficients *in polar form* as $a_k = \mu_k e^{j\varphi_k}$, where $\mu_k \geq 0$ and $-\pi < \varphi_k \leq \pi$.

Index k	k^{th} Fourier coefficient a_k in polar form

PROBLEM sp-09-Q.1.3:

(a) Recall Lab #2 where one sinusoid is broadcast from a transmitter, but a vehicle receives two signals, one directly from the transmitter and the other via a reflected path. Using the fact that the resulting signal in the vehicle is the sum of these two, the magnitude of the received signal can be calculated. Suppose that the time delay along the direct path from the transmitter to the vehicle is 3.1 μs, and along the reflected path the time delay is 3.8 μs. If the transmitter's frequency is 6 MHz, determine the magnitude of the signal at the receiver on the vehicle. The **transmitted sinusoid has an amplitude of one**, and the reflector is assumed perfect with no phase shift.



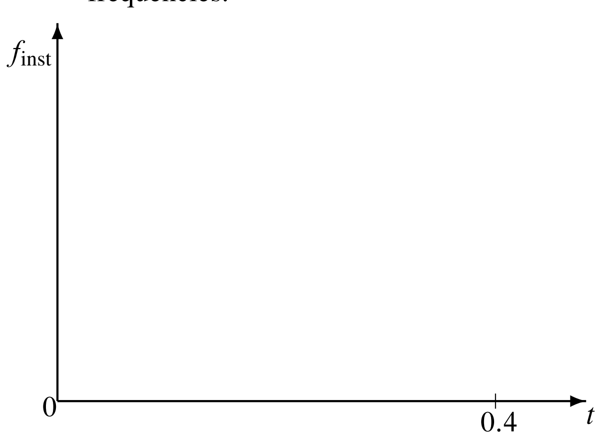
Determine the received signal's magnitude: _________

Explain how you obtain the answer.

(b) A frequency-modulated (FM) signal can be used to make a sound whose frequency changes with time. For the FM signal

$$y(t) = \cos(25\sin(10\pi t) + 1000\pi t)$$

plot the *instantaneous frequency* in hertz over the time interval $0 \le t \le 0.4$ s. Plot only the positive frequencies.



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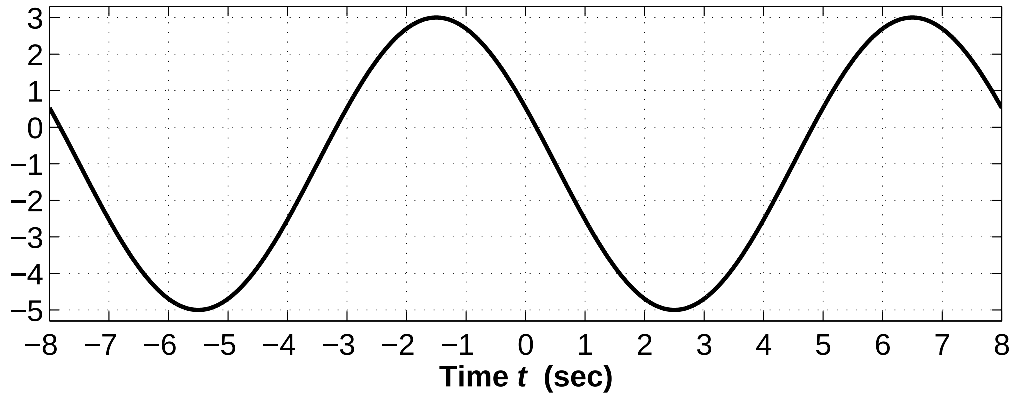
PROBLEM sp-09-Q.1.1:

(a) The signal plotted below is the sum of two sinusoids; determine the amplitudes and phases, as well as the frequencies in rad/s.



$$-1 + 4\cos((2\pi/8)t + 3\pi/8)$$

(b) Express this signal as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$$s_a(t) = 5e^{-j\pi/3}e^{j30\pi t} + 5e^{j\pi/3}e^{-j30\pi t}$$

Pick off the values from the positive frequency component to get

$$s_a(t) = 10\cos(30\pi t - \pi/3)$$

(c) Express this signal as a sinusoid in standard form, i.e., $A\cos(\omega_0 t + \varphi)$.

$$s_b(t) = \Re e\left\{(-10 - j30)e^{j631t}\right\}$$

The complex amplitude in polar form is $(-10 - j30) = 31.62e^{-j1.893}$, so

$$s_b(t) = 31.62\cos(631t - 1.893)$$

PROBLEM sp-09-Q.1.2:

The following MATLAB code defines a periodic signal as the sum of sinusoids:

```
tt = -10:0.00001:10;  %- in seconds
xxt = -5*ones(size(tt));
kk=12; xxt = xxt + (30/kk)*cos( 1.2*pi*kk*(tt-1/6) );
kk=10; xxt = xxt + real( (-5-12/j)*exp(j*1.6*pi*kk*tt) );
```

In the following questions, the signal $x(t)$ corresponds to the MATLAB vector `xxt`.

(a) Determine the *fundamental frequency* of $x(t)$. $\omega_0 = 1.6\pi$ rad/s

The signal $x(t)$ is the sum of three sinusoids with frequencies: 0, $12(1.2\pi) = 14.4\pi$, and $10(1.6\pi) = 16.0\pi$. Using the greatest common divisor (GCD), we obtain the fundamental frequency as

$$\omega_0 = 0.1\pi \,\text{gcd}(0, 144, 160) = 1.6\pi \text{ rad/s}$$

(b) The signal $x(t)$ is periodic, so it has a Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}$$

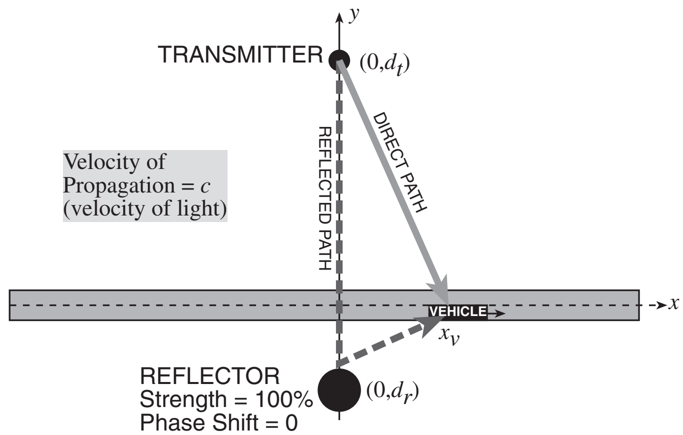
Determine the values of all the nonzero coefficients $\{a_k\}$, and list them in the table below. Write the Fourier coefficients *in polar form* as $a_k = \mu_k e^{j\varphi_k}$, where $\mu_k \geq 0$ and $-\pi < \varphi_k \leq \pi$.

$x(t) = -5 + 2.5\cos(9(1.6\pi)t - 12(1.2)\pi/6) + \Re e\{13e^{j1.966}e^{j10(1.6\pi)t}\}$

We have DC ($k = 0$), and the ninth and tenth harmonics. The phase of the second term is -2.4π, but must be changed to -0.4π to be in the correct range.

Index k	k^{th} Fourier coefficient a_k in polar form
0	$5e^{j\pi}$
9	$1.25e^{-j0.4\pi}$
-9	$1.25e^{j0.4\pi}$
10	$6.5e^{j1.966}$
-10	$6.5e^{-j1.966}$

PROBLEM sp-09-Q.1.3:



(a) Recall Lab #2 where one sinusoid is broadcast from a transmitter, but a vehicle receives two signals, one directly from the transmitter and the other via a reflected path. Using the fact that the resulting signal in the vehicle is the sum of these two, the magnitude of the received signal can be calculated. Suppose that the time delay along the direct path from the transmitter to the vehicle is 3.1 μs, and along the reflected path the time delay is 3.8 μs. If the transmitter's frequency is 6 MHz, determine the magnitude of the signal at the receiver on the vehicle. The **transmitted sinusoid has an amplitude of one**, and the reflector is assumed perfect with no phase shift.

Determine the received signal's magnitude: ________

Explain how you obtain the answer.

The time delays are equivalent to a phase change in the sinusoids according to the equation: $\varphi = -2\pi f \tau$. For the direct path, the phase is $\varphi_d = -2\pi(6\times10^6)(3.1\times10^{-6})$, for the reflected path $\varphi_r = -2\pi(6\times10^6)(3.8\times10^{-6})$ Then we do the phasor addition:

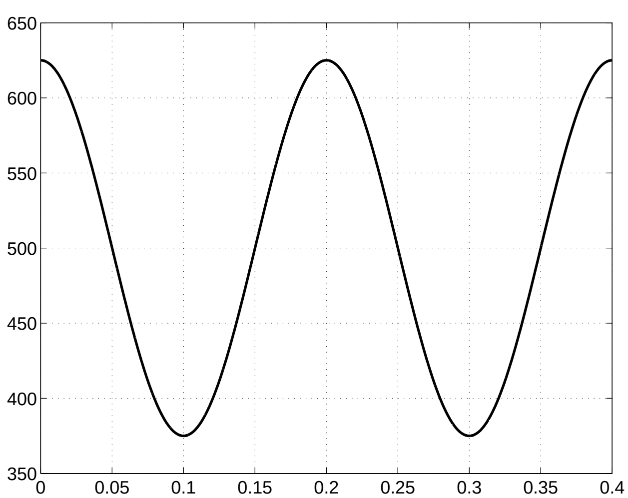
$$1e^{j\varphi_d} + 1e^{j\varphi_r} = e^{-j37.2\pi} + e^{-j45.6\pi} = 1.618e^{j1.885}$$

The magnitude of the complex sum is the amplitude of the sinusoid, i.e., 1.618.

(b) A frequency-modulated (FM) signal can be used to make a sound whose frequency changes with time. For the FM signal

$$y(t) = \cos(25\sin(10\pi t) + 1000\pi t)$$

plot the *instantaneous frequency* in hertz over the time interval $0 \le t \le 0.4$ s. Plot only the positive frequencies.



The angle function is

$$\psi(t) = 25\sin(10\pi t) + 1000\pi t$$

and we take the derivative to get the instantaneous frequency:

$$\frac{d}{dt}\psi(t) = 250\pi\cos(10\pi t) + 1000\pi$$

To convert to hertz, we divide by 2π, so

$$\frac{1}{2\pi}\frac{d}{dt}\psi(t) = 125\cos(10\pi t) + 500$$