GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 8-Feb-08 COURSE: ECE-2025

NAME: ______________________________ GT username: ________________

LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Coyle)
L01:M-3pm (McClellan)	L09:Tues-3pm (Lanterman)	L02:W-3pm (Clements)	L10:Thur-3pm (Coyle)
	L11:Tues-4:30pm (Lanterman)	L04:W-4:30pm (Clements)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	40	
3	30	
No/Wrong Rec	−3	

PROBLEM sp-08-Q.1.1:
The sum of two sinusoids is another sinusoid:

$$A\cos(\omega t + \varphi) = 200\cos(6(t + 13)) + 300\cos(6t - 5\pi/6)$$

(a) Determine the complex amplitudes for both of the sinusoids above; call these X_1 and X_2.

$X_1 =$ ________

$X_2 =$ ________

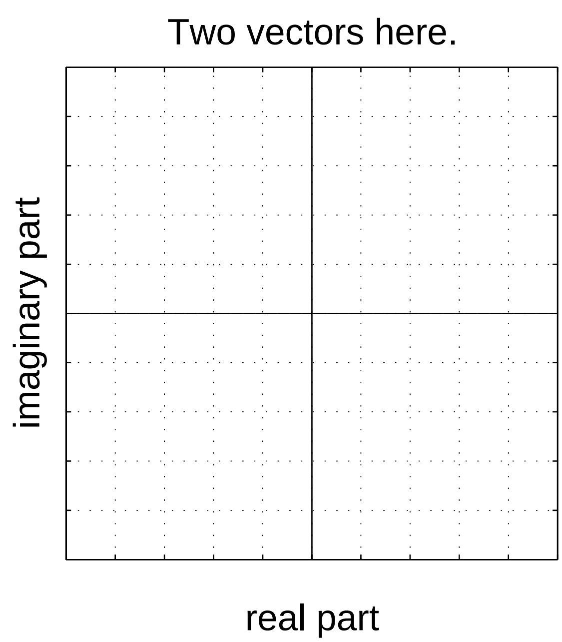
(b) Determine the the numerical values of A and φ, as well as ω (give the correct units).

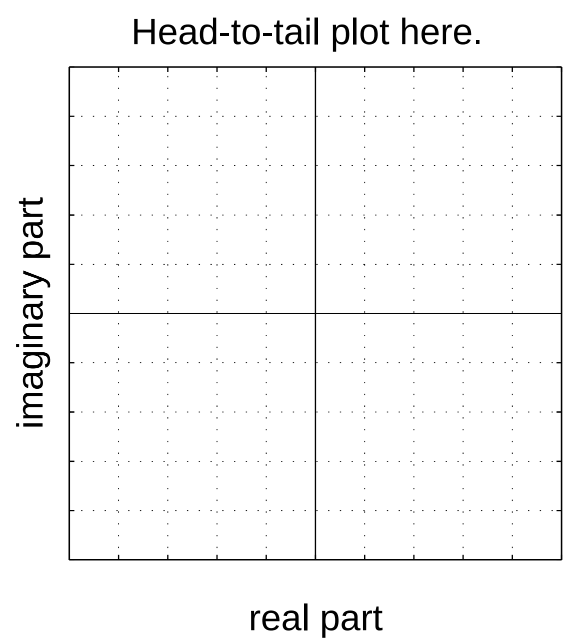
$A =$ ________

$\varphi =$ ________

$\omega =$ ________

(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were combined to solve part (a). On the first plot, show a vector plot of the two complex amplitudes whose values are given by the sinusoids on the **right** hand side of the equal sign; on the second plot, show a "head-to-tail" vector plot of those same two complex amplitudes plus the resultant vector that gives the solution. *Use an appropriate scale on the grids below.*





PROBLEM sp-08-Q.1.2:

(a) Suppose $x(t)$ is a *periodic* signal that is also real-valued, and we know partial information about its two-sided spectrum, given in the table below. Assume that $0 < \omega_a < \omega_b < \omega_c$.

Frequency (rad/sec)	Complex Amplitude
$-\omega_c$	$-3 - j3$
-60π	$5e^{j\phi_{-3}}$
$-\omega_a$	$\mu_{-2}\, e^{j\pi/5}$
ω_a	$7\, e^{j\phi_2}$
ω_b	$\mu_3\, e^{j\pi/3}$
ω_c	$a_5 = \mu_5\, e^{j\phi_5}$

In addition, its Fourier Series has only 6 nonzero coefficients, $\{a_k\}$, for $k = \pm 2, \pm 3, \pm 5$. Assume that the Fourier coefficients can be written *(in polar form)* as $a_k = \mu_k\, e^{j\phi_k}$. Determine the numerical values of the following Fourier coefficients *(in polar form)* and frequencies:

$a_2 =$ ________

$a_3 =$ ________

$a_5 =$ ________

$\omega_a =$ ________ rad/s

$\omega_b =$ ________ rad/s

$\omega_c =$ ________ rad/s

(b) The signal $x(t)$ has the spectrum in part (a); determine the fundamental period (T_0) of $x(t)$.

$T_0 =$ ________ secs.

(c) The AM signal $s(t) = 8\cos(400t + 2)\cos(6t - 1)$ will have a nonzero component in its spectrum at $\omega = 394$ rad/s. Determine the complex amplitude (X_{394}) of this one spectral component.

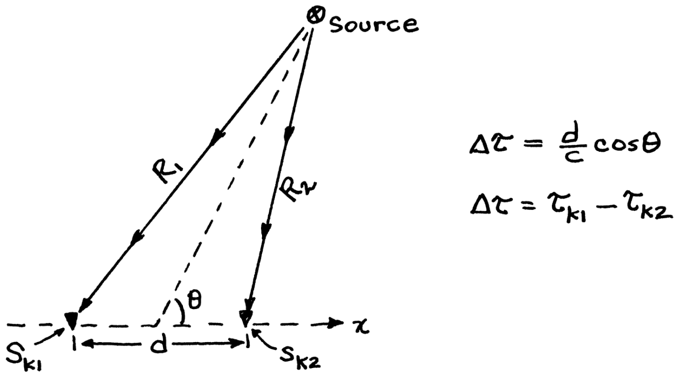
$X_{394} =$ ________

(d) The instantaneous frequency of the linear-FM chirp signal $v(t) = 7\cos(1000\pi t^2 + 400\pi t)$ will change by F Hz per second. Determine F (in Hz/sec).

$F =$ ________ Hz/s

PROBLEM sp-08-Q.1.3:

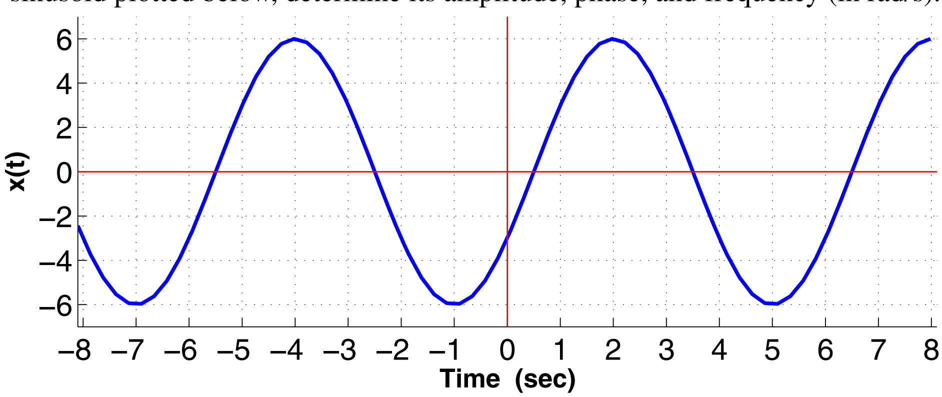
(a) Recall Lab #2 where the angle of arrival (θ) is determined from two receivers as shown below. If the signals are $s_1(t) = 3\cos(200\pi t + \pi/4)$ and $s_2(t) = 3\cos(200\pi t + \pi/3)$, determine θ when the receivers are located at $(-0.2, 0)$ m and $(0.2, 0)$ m, and the velocity of sound is 300 m/s.



Determine the angle θ (in radians).

$\theta =$ ________ rad

(b) For the sinusoid plotted below, determine its amplitude, phase, and frequency (in rad/s).



$A =$ ________

$\varphi =$ ________

$\omega =$ ________ rad/s

(c) The MATLAB expression `xt=real((-5-12j)*exp(j*200*pi*tt))` defines a signal vector that can be expressed as a sinusoid, $A\cos(\omega t + \varphi)$. Determine the values of A, φ, and ω (in rad/s).

$A =$ ________ $\varphi =$ ________ rad $\omega =$ ________ rad/s

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PROBLEM sp-08-Q.1.1:

The sum of two sinusoids is another sinusoid:

$$A\cos(\omega t + \varphi) = \underbrace{200\cos(6(t+13))}_{x_1(t)} + \underbrace{300\cos(6t - 5\pi/6)}_{x_2(t)}$$

(a) Determine the complex amplitudes for both of the sinusoids above; call these X_1 and X_2.

$X_1 = 200e^{j2.602}$ $\quad X_1 = 200\,e^{j6(13)} = 200\,e^{j78}$

$X_2 = 300e^{-j5\pi/6}$ $\quad 78 \text{ rads} = 2.602, \text{ or } 0.828\pi, \text{ or } 149.07°$

(b) Determine the the numerical values of A and φ, as well as ω (give the correct units).

$A = 433.9$ $\quad X_1 + X_2$ on a calculator

$\varphi = -3.033$ rad

$\omega = 6$ rad/s $\quad 200e^{j2.602} + 300e^{-j2.618}$

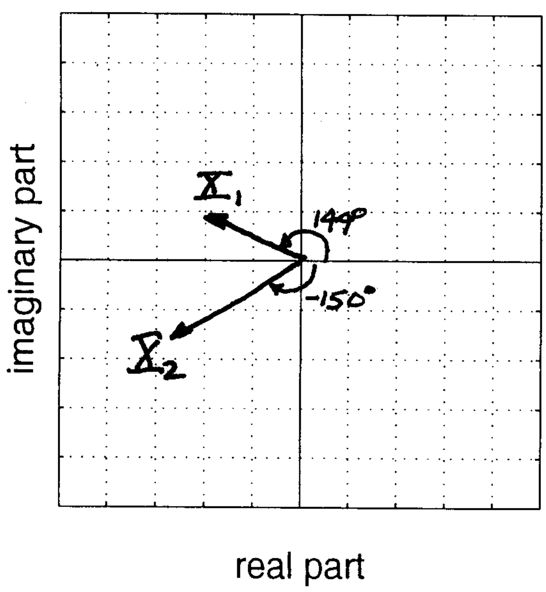
$= 433.9\,e^{-j3.033}$

$-3.033 \text{ rads} = -173.76°$

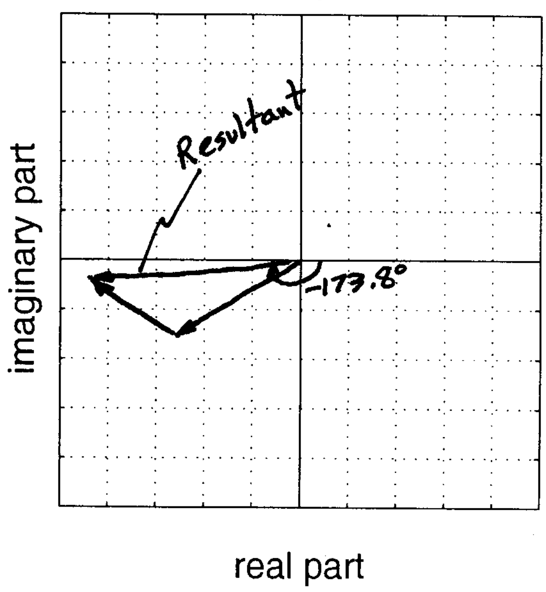
$= -0.965\pi$ rad

(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were combined to solve part (a). On the first plot, show a vector plot of the two complex amplitudes whose values are given by the sinusoids on the **right** hand side of the equal sign; on the second plot, show a "head-to-tail" vector plot of those same two complex amplitudes plus the resultant vector that gives the solution. *Use an appropriate scale on the grids below.*

Two vectors here.



Head-to-tail plot here.



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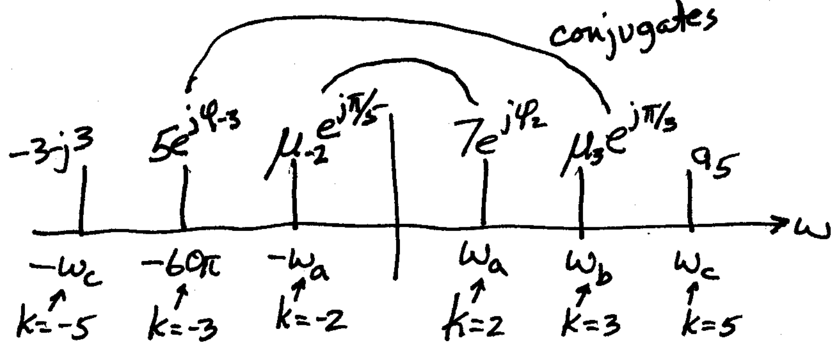


In addition, its Fourier Series has only 6 nonzero coefficients, $\{a_k\}$, for $k = \pm 2, \pm 3, \pm 5$. Assume that the Fourier coefficients can be written *(in polar form)* as $a_k = \mu_k\, e^{j\phi_k}$. Determine the numerical values of the following Fourier coefficients *(in polar form)* and frequencies:

$a_2 = 7e^{-j\pi/5}$

$a_3 = 5e^{j\pi/3}$

$a_5 = 3\sqrt{2}\,e^{j3\pi/4}$

$\omega_a = 40\pi$ rad/s

$\omega_b = 60\pi$ rad/s

$\omega_c = 100\pi$ rad/s

$a_{-5} = -3 - j3 = 3\sqrt{2}\,e^{-j3\pi/4}$ $\quad a_5 = a_{-5}^*$

$-60\pi = -3\omega_0 \Rightarrow \omega_0 = 20\pi$ rad/s

$\omega_k = k\omega_0$

(b) The signal $x(t)$ has the spectrum in part (a); determine the fundamental period (T_0) of $x(t)$.

$T_0 = 1/10$ secs. $\quad T_0 = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{20\pi} = \dfrac{1}{10}$ s

(c) The AM signal $s(t) = 8\cos(400t + 2)\cos(6t - 1)$ will have a nonzero component in its spectrum at $\omega = 394$ rad/s. Determine the complex amplitude (X_{394}) of this one spectral component.

$X_{394} = 2e^{j3}$ $\quad$ Need the e^{j394t} term

$$s(t) = 8\left(\tfrac{1}{2}\right)\left(e^{j400t}e^{j2} + e^{-j400t}e^{-j2}\right)\left(\tfrac{1}{2}\right)\left(e^{j6t}e^{-j} + e^{-j6t}e^{j}\right)$$

$$8\left(\tfrac{1}{2}\right)e^{j400t}e^{j2}\left(\tfrac{1}{2}\right)e^{-j6t}e^{j} = 2e^{j394t}e^{j3}$$

(d) The instantaneous frequency of the linear-FM chirp signal $v(t) = 7\cos(1000\pi t^2 + 400\pi t)$ will change by F Hz per second. Determine F (in Hz/sec).

$F = 1000$ Hz/s

$$\psi(t) = 1000\pi t^2 + 400\pi$$

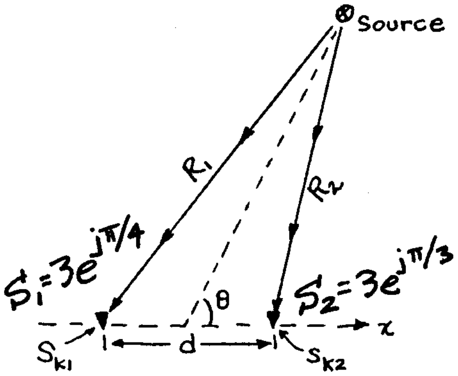
$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t) = \frac{1}{2\pi}\left(2000\pi t + 400\pi\right)$$

$$= 1000t + 200 \text{ Hz}$$

slope is Hz/s

PROBLEM sp-08-Q.1.3:

(a) Recall Lab #2 where the angle of arrival (θ) is determined from two receivers as shown below. If the signals are $s_1(t) = 3\cos(200\pi t + \pi/4)$ and $s_2(t) = 3\cos(200\pi t + \pi/3)$, determine θ when the receivers are located at $(-0.2, 0)$ m and $(0.2, 0)$ m, and the velocity of sound is 300 m/s.



$\Delta\tau = \frac{d}{c}\cos\theta$

$\Delta\tau = \tau_{k1} - \tau_{k2}$

$\Delta\tau = t_{m1} - t_{m2}$

$= \frac{\varphi_1}{-\omega} - \frac{\varphi_2}{-\omega} = \frac{\varphi_1 - \varphi_2}{-\omega}$

$= \frac{\pi/4 - \pi/3}{-200\pi} = \frac{1/3 - 1/4}{200} = \frac{1/12}{200}$ s

$\cos\theta = \frac{c}{d}\Delta\tau = \left(\frac{300}{0.4}\right)\left(\frac{1/12}{200}\right) = \frac{15}{48} = \frac{5}{16}$

$= 0.3125$

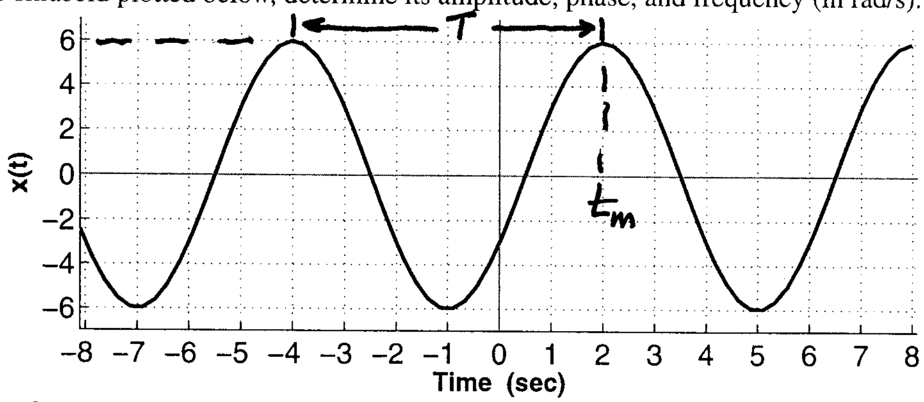
Determine the angle θ (in radians).

$\theta =$ 1.253 rad

$\theta = \cos^{-1}\left(\frac{5}{16}\right) = 1.253$ rad

$= 71.8°$

(b) For the sinusoid plotted below, determine its amplitude, phase, and frequency (in rad/s).



$A =$ 6

$\varphi =$ $-2\pi/3$ rad

$\omega =$ $\pi/3$ rad/s

$T = 2 - (-4) = 6s \Rightarrow \omega = \frac{2\pi}{6} = \frac{\pi}{3}$ rad/s

$t_m = 2s$

$\varphi = -\omega t_m = -(\pi/3)2 = -2\pi/3$ rad

(c) The MATLAB expression `xt=real((-5-j12)*exp(j*200*pi*tt))` defines a signal vector that can be expressed as a sinusoid, $A\cos(\omega t + \varphi)$. Determine the values of A, φ, and ω (in rad/s).

$A =$ 13 $\quad \varphi =$ -1.97 rad $\quad \omega =$ 200π rad/s

$-5 - j12 = 13e^{-j1.966}$

$= -0.626\pi$ rad $= -112.62°$