GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 4-Feb-05 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM sp-05-Q.1.1:

For the following short answer questions, write your answers in the space provided or circle the correct answer. Provide a ***short justification*** for your answer.

(a) The periodic signal $x(t)$ has a spectrum containing frequency components (with nonzero magnitude) at $\omega = 0, \pm 1.8\pi$ and $\pm 2.4\pi$ rad/s. Determine the *fundamental period,* i.e., the shortest possible period. Make sure your answer has the correct units.

$T =$ ________

(b) $\boxed{\text{Circle}}$ the correct answer. The complex amplitude $5/j$ can be expressed in an equivalent form as:

(A) $5e^{-j\pi/2}$ **(B)** $5e^{-j\pi}$ **(C)** $5e^{j\pi/2}$ **(D)** $5e^{j2\pi}$ **(E)** 5

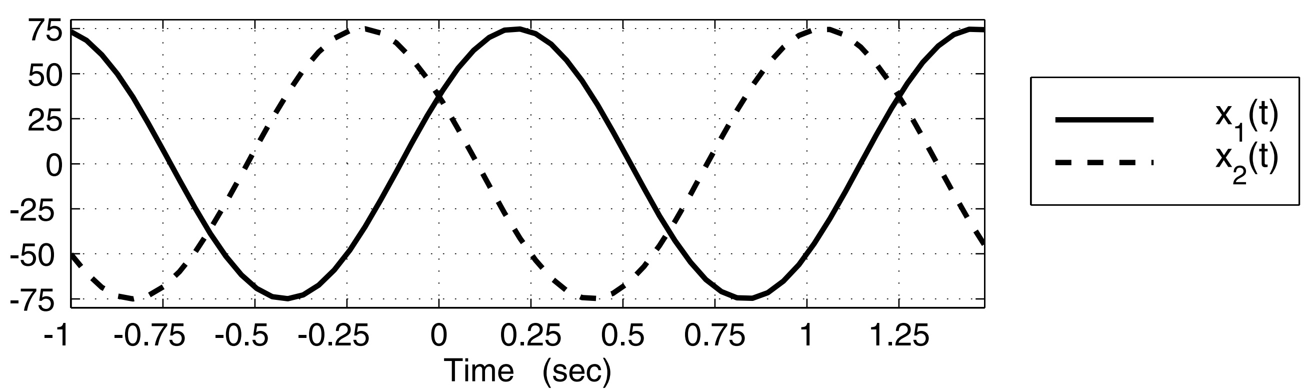
(c) $\boxed{\text{Circle}}$ the correct answer: For the sinusoid $x(t) = \cos(3\pi t)$, when you form the sum,

$$y(t) = 4x(t - 1/5) + 4x(t - 1/10),$$

the peak amplitude of $y(t)$ is:

(A) equal to 0, (B) equal to 8, (C) greater than 8, (D) less than 8, but not 0.

(d) In the figure below two sinusoidal signals are shown. Which one has a phase of $+\pi/3$?
$\boxed{\text{Circle}}$ the correct answer: $\mathbf{x_1(t)}$ or $\mathbf{x_2(t)}$.



(e) In the figure above both sinusoidal signals have the same frequency. What is the frequency (ω_0) in radians/sec ? $\boxed{\text{Circle}}$ the correct answer.

(A) 2.5π **(B)** 1.25π **(C)** 0.8 **(D)** $\pi/3$ **(E)** 1.6π

PROBLEM sp-05-Q.1.2:

For each of the following signals, pick one of the representations below that defines *exactly* the same signal. Write your answer ((a), (b), (c), (d), (e), or (f)) in the box next to each signal.

ANS = $\Re e\left\{\left|(-3+j3\sqrt{3})e^{j77\pi t}\right|\right\}$

ANS = $6\cos(77\pi t + 2\pi/3)$

ANS = $3e^{-j\pi/3}e^{-j77\pi t} + 3e^{j\pi/3}e^{j77\pi t}$

ANS = $3e^{j2\pi/3}e^{j77\pi t} + 3e^{-j2\pi/3}e^{-j77\pi t}$

ANS = $-6\cos(77\pi t + \pi/3)$

POSSIBLE ANSWERS:

Your answer will be one of the following choices.

Please note: some of the following signals could used more than one time to match the above signals.

(a) $x_a(t) = 6\cos(77\pi t + 7\pi/3)$

(b) $x_b(t) = 3\cos(77\pi t - \pi/3)$

(c) $x_c(t) = \Re e\left\{(-3+j3\sqrt{3})e^{j77\pi t}\right\}$

(d) $x_d(t) = 0$

(e) $x_e(t) = \Re e\left\{6e^{-j2\pi/3}e^{j77\pi t}\right\}$

(f) $x_f(t) = 6$

PROBLEM sp-05-Q.1.3:

The following MATLAB code defines two signals that are then summed:

```
tt = -10:0.001:10;
xx1 = 2 - 3*cos( 3*pi*(tt - 1/4 ) );
xx2 = -5 - 4*cos( 3*pi*tt + 11*pi/4 );
xx = xx1 + xx2;
```

(a) If the signal $x_1(t)$ corresponds to the MATLAB vector `xx1`, determine the complex amplitude X_1 that can be used to represent the sinusoidal part of $x_1(t)$ as $\Re e\{X_1 e^{j\omega t}\}$.

(b) If the signal $x(t)$ corresponds to the MATLAB vector `xx`, then it is possible to express $x(t)$ in the form

$$x(t) = \Re e\left\{X\, e^{j\alpha t} + Y\right\}$$

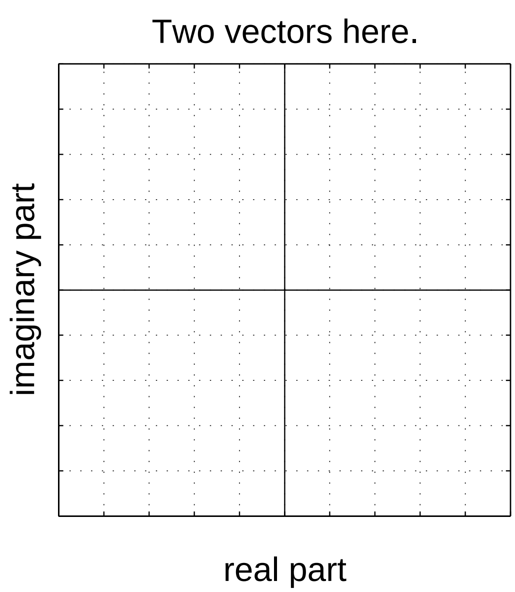
Determine the numerical values of X, Y, and α.

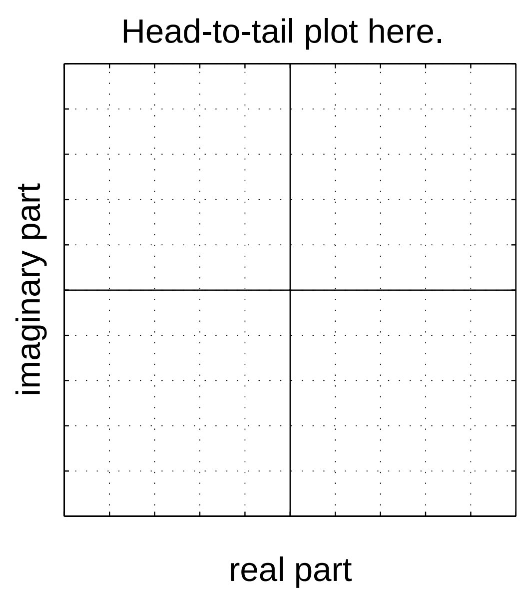
$X =$ ________

$Y =$ ________

$\alpha =$ ________

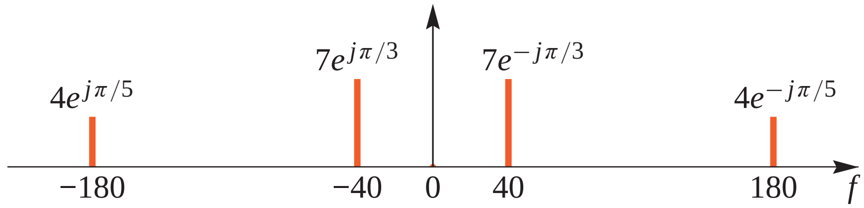
(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to find X in part (b). On the first plot, show only the two complex amplitudes (phasors) that were added to solve part (b); on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail). *Use an appropriate scale on the grid below.*





PROBLEM sp-05-Q.1.4:

The two-sided spectrum representation of a real-valued signal $x_1(t)$ is shown below:

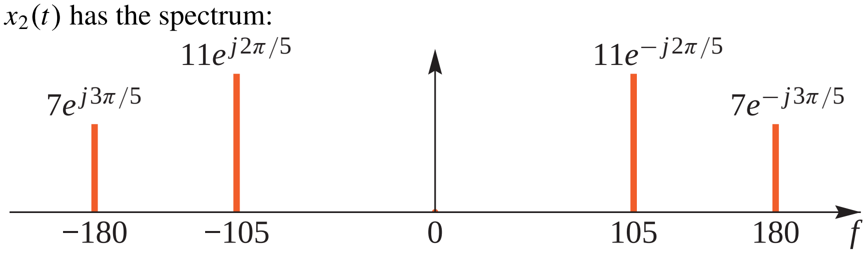


(a) Determine the *fundamental frequency* for $x_1(t)$. *Note:* the frequency f is given in Hz.

(b) Write the formula for $x_1(t)$ as a sum of real-valued sinusoids.

(c) Make a (well-labeled) sketch of the spectrum of the delayed signal $x_1(t - 1/60)$. Simplify the numerical values for the complex amplitudes.

(d) A second signal $x_2(t)$ has the spectrum:



If we define a third signal as

$$y(t) = x_2(t) - \beta x_1(t - \lambda)$$

Determine values of β and λ such that the spectrum for $y(t)$ is zero at $f = \pm 180$ Hz. Make β positive.

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 4-Feb-05 COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT #: Ver - 1

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM sp-05-Q.1.1:

For the following short answer questions, write your answers in the space provided or circle the correct answer. Provide a ***short justification*** for your answer.

(a) The periodic signal $x(t)$ has a spectrum containing frequency components (with nonzero magnitude) at $\omega = 0, \pm 1.8\pi$ and $\pm 2.4\pi$ rad/s. Determine the *fundamental period,* i.e., the shortest possible period. Make sure your answer has the correct units.

$T =$ 10/3 secs $\qquad \gcd(\pm 1.8\pi, \pm 2.4\pi) = 0.6\pi$ rad/s

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{0.6\pi} = \frac{2}{3/5} = \frac{10}{3} \text{ secs.}$$

(b) $\boxed{\text{Circle}}$ the correct answer. The complex amplitude $5/j$ can be expressed in an equivalent form as:

(A) $5e^{-j\pi/2}$ (B) $5e^{-j\pi}$ (C) $5e^{j\pi/2}$ (D) $5e^{j2\pi}$ (E) 5

$$\frac{5}{j} = \frac{5}{e^{j\pi/2}} = 5e^{-j\pi/2}$$

(c) $\boxed{\text{Circle}}$ the correct answer: For the sinusoid $x(t) = \cos(3\pi t)$, when you form the sum,

$$y(t) = 4x(t - 1/5) + 4x(t - 1/10),$$

the peak amplitude of $y(t)$ is:

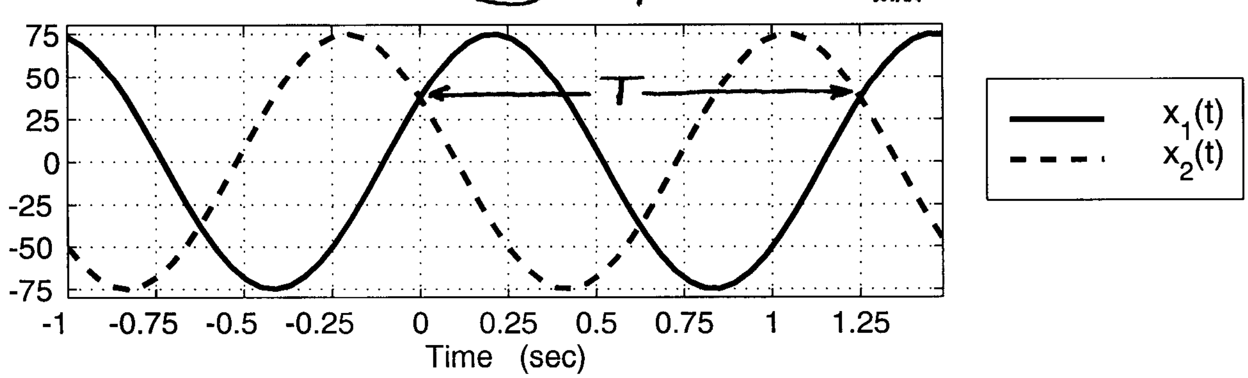
(A) equal to 0, (B) equal to 8, (C) greater than 8, (D) less than 8, but not 0.

$$y(t) = 4\cos(3\pi(t - 1/5)) + 4\cos(3\pi(t - 1/10))$$
$$= 4\cos(3\pi t - 3\pi/5) + 4\cos(3\pi t - 3\pi/10)$$

phase difference $= -3\pi/5 - (-3\pi/10) = -3\pi/10$

Since the phase diff is neither 0, nor π, $y(t) \neq 0$ & peak amp $\neq 8$

(d) In the figure below two sinusoidal signals are shown. Which one has a phase of $+\pi/3$?
$\boxed{\text{Circle}}$ the correct answer: $\mathbf{x_1(t)}$ or $\mathbf{x_2(t)}$. $\qquad \varphi = +\pi/3 \Rightarrow t_{MAX} < 0$



(e) In the figure above both sinusoidal signals have the same frequency. What is the frequency (ω_0) in radians/sec ? $\boxed{\text{Circle}}$ the correct answer.

(A) 2.5π (B) 1.25π (C) 0.8 (D) $\pi/3$ (E) 1.6π

$$T = 1.25\text{s} \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1.25} = 1.6\pi \text{ rad/s}$$

PROBLEM sp-05-Q.1.2:

For each of the following signals, pick one of the representations below that defines *exactly* the same signal. Write your answer ((a), (b), (c), (d), (e), or (f)) in the box next to each signal.

ANS = f $\quad \Re e\left\{\left|(-3+j3\sqrt{3})e^{j77\pi t}\right|\right\}$ $\quad$ Mag is real $\left.\begin{array}{l}\\ |e^{j77\pi t}|=1\end{array}\right\} \Rightarrow$ ans $= |-3+j3\sqrt{3}| = \sqrt{3^2+3^2\cdot 3} = 6$

ANS = c $\quad 6\cos(77\pi t+2\pi/3) = \Re e\{6e^{j2\pi/3}e^{j77\pi t}\}$

ANS = a $\quad 3e^{-j\pi/3}e^{-j77\pi t}+3e^{j\pi/3}e^{j77\pi t} = 6\cos(77\pi t+\pi/3)$

ANS = c $\quad 3e^{j2\pi/3}e^{j77\pi t}+3e^{-j2\pi/3}e^{-j77\pi t} = 6\cos(77\pi t+2\pi/3)$

ANS = e $\quad -6\cos(77\pi t+\pi/3) = 6\cos(77\pi t+\pi/3-\pi) = 6\cos(77\pi t-2\pi/3)$

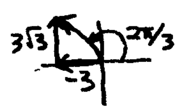
POSSIBLE ANSWERS:

Your answer will be one of the following choices.

Please note: some of the following signals could used more than one time to match the above signals.

(a) $x_a(t) = 6\cos(77\pi t+7\pi/3) = 6\cos(77\pi t+7\pi/3-2\pi) = 6\cos(77\pi t+\pi/3)$

(b) $x_b(t) = 3\cos(77\pi t-\pi/3)$

(c) $x_c(t) = \Re e\left\{(-3+j3\sqrt{3})e^{j77\pi t}\right\}$ 

$= \Re e\{6e^{j2\pi/3}e^{j77\pi t}\}$

$= 6\cos(77\pi t+2\pi/3)$

(d) $x_d(t) = 0$

(e) $x_e(t) = \Re e\left\{6e^{-j2\pi/3}e^{j77\pi t}\right\} = 6\cos(77\pi t-2\pi/3)$

(f) $x_f(t) = 6$

PROBLEM sp-05-Q.1.3:

The following MATLAB code defines two signals that are then summed:

```
tt = -10:0.001:10;
xx1 = 2 - 3*cos( 3*pi*(tt - 1/4 ) );
xx2 = -5 - 4*cos( 3*pi*tt + 11*pi/4 );
xx = xx1 + xx2;
```

(a) If the signal $x_1(t)$ corresponds to the MATLAB vector xx1, determine the complex amplitude of sinusoidal part of $x_1(t)$. $x_1(t) = 2 - 3\cos(3\pi t - 3\pi/4)$

$X_1 = -3e^{-j\pi 3/4} = 3e^{j\pi}e^{-j3\pi/4} = 3e^{j\pi/4}$ when used in $\Re e\{X_1 e^{j3\pi t}\}$

(b) If the signal $x(t)$ corresponds to the MATLAB vector xx, then it is possible to express $x(t)$ in the form

$$x(t) = \Re e\left\{X\, e^{j\alpha t} + Y\right\}$$

Determine the numerical values of X, Y, and α. *Hint:* Use phasor addition.

$X = 5e^{-j0.142}$

$Y = 2-5=-3$

$\alpha = 3\pi$

$X_2 = -4e^{j11\pi/4} = 4e^{-j\pi}e^{j11\pi/4} = 4e^{j7\pi/4}$

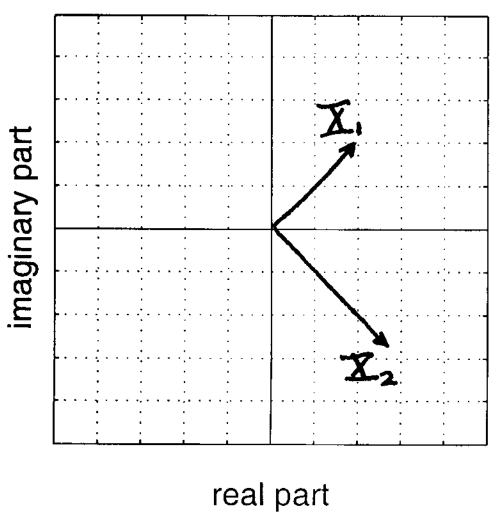
$X = X_1 + X_2 = 3e^{j\pi/4} + 4e^{-j\pi/4}$

$= 5e^{-j0.142} = 5e^{-j0.045\pi}$

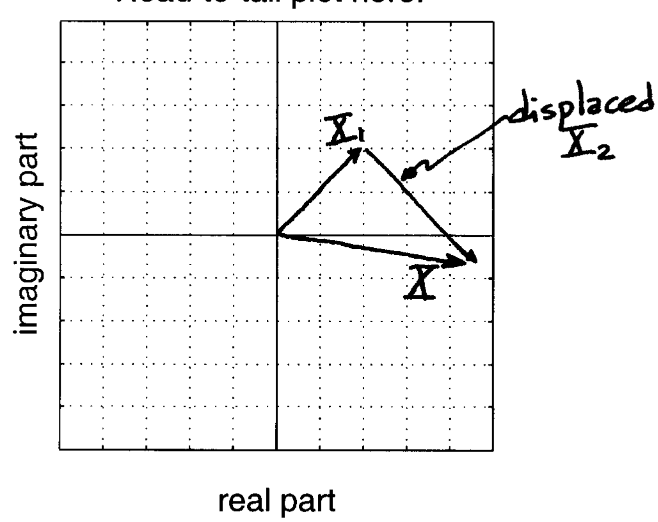
$-8.13°$

(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to find X in part (b). On the first plot, show only the two complex amplitudes (phasors) that were added to solve part (b); on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail). *Use an appropriate scale on the grid below.*

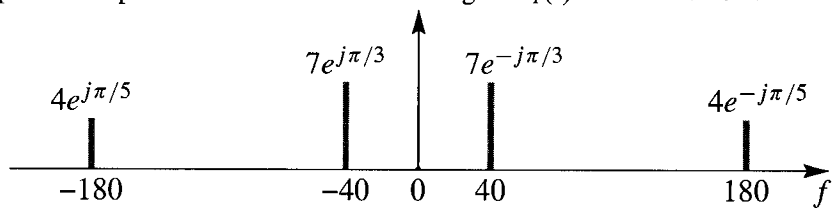
Two vectors here.



Head-to-tail plot here.



PROBLEM sp-05-Q.1.4:
The two-sided spectrum representation of a real-valued signal $x_1(t)$ is shown below:



(a) Determine the *fundamental frequency* for $x_1(t)$. *Note:* the frequency f is given in Hz.

$\gcd(40, 180) = 20\,\text{Hz}$

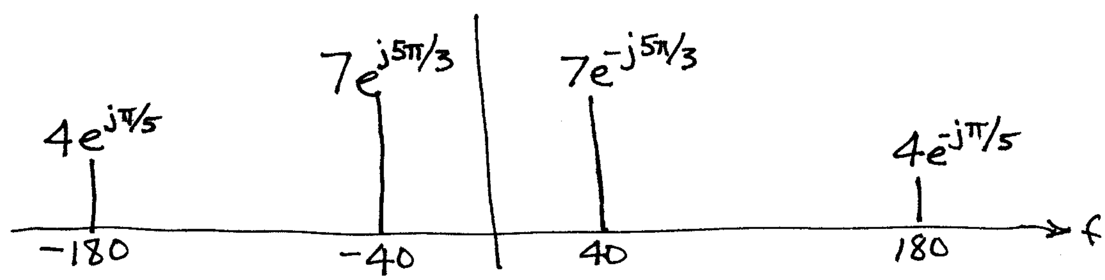
(b) Write the formula for $x_1(t)$ as a sum of real-valued sinusoids.

$$x_1(t) = 14\cos(2\pi(40)t - \pi/3) + 8\cos(2\pi(180)t - \pi/5)$$

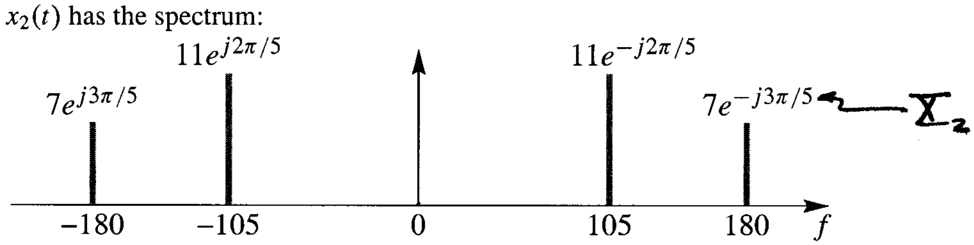
(c) Make a (well-labeled) sketch of the spectrum of the delayed signal $x_1(t - 1/60)$. Simplify the numerical values for the complex amplitudes.

$$x_1(t - 1/60) = 14\cos(80\pi(t - 1/60) - \pi/3) + 8\cos(360\pi(t - 1/60) - \pi/5)$$
$$= 14\cos(80\pi t \underbrace{- 4\pi/3 - \pi/3}_{-5\pi/3}) + 8\cos(360\pi t - 6\pi - \pi/5)$$

MULTIPLE of 2π



(d) A second signal $x_2(t)$ has the spectrum:



If we define a third signal as

$$y(t) = x_2(t) - \beta x_1(t - \lambda)$$

Determine values of β and λ such that the spectrum for $y(t)$ is zero at $f = \pm 180$ Hz. Make β positive.

The spectrum lines at $f = 180$Hz must add to zero

$$Y = X_2 - \beta e^{-j\omega\lambda} X_1 = X_2 - \beta e^{-j360\pi\lambda} X_1 = 0$$

$$\Rightarrow \beta e^{-j360\pi\lambda} = \frac{X_2}{X_1} = \frac{7e^{-j3\pi/5}}{4e^{-j\pi/5}} = \frac{7}{4} e^{-j2\pi/5} \Rightarrow \boxed{\beta = \frac{7}{4}}$$

$360\pi\lambda = 2\pi/5$

$$\Rightarrow \lambda = \frac{2\pi}{5(360\pi)} = \frac{1}{900} \text{ secs.}$$