

PROBLEM sp-04-Q.1.1:

For the following short answer questions, write your answers in the space provided or circle the correct answer. Provide a *short justification* for your answer.

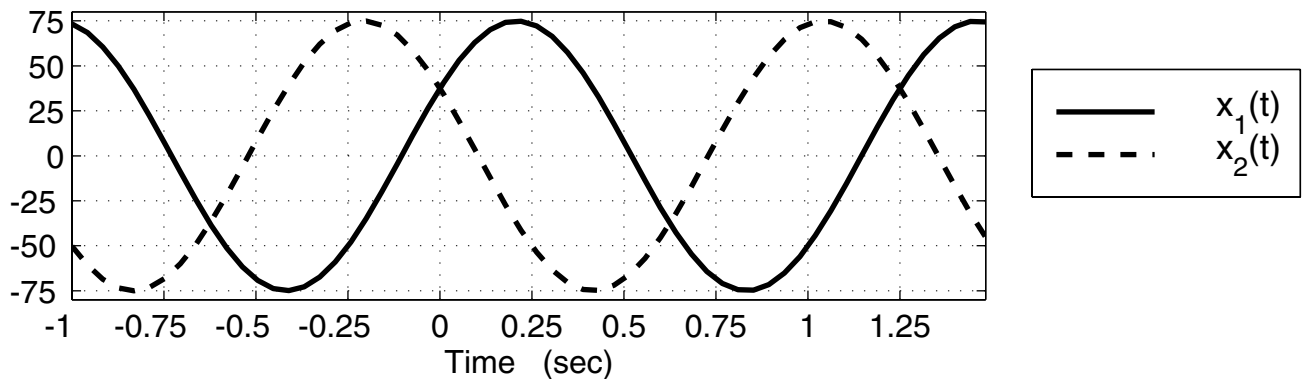
- (a) The periodic signal $x(t)$ has a spectrum containing frequency components (with nonzero magnitude) at $f = 0, \pm 1.8$ and ± 2.4 Hz. Determine the *fundamental period*, i.e., the shortest possible period. Make sure your answer has the correct units.

$T =$ _____

- (b) **TRUE** or **FALSE**: “If the spectrum of a signal $x(t)$ has frequency components (with nonzero magnitudes) only at $f = \pm 2$ Hz, then the spectrum of a new signal defined by $y(t) = x(t) \cos(2\pi(100)t)$ has frequency components (with nonzero magnitudes) at $f = \pm 102$ Hz and $f = \pm 98$ Hz.”

- (c) the correct answer: When you add $4 \cos(2t + 2\pi/3) + 4 \cos(2t - 5\pi/3)$ the maximum value of the resulting signal is:
(A) equal to 0, (B) equal to 8, (C) greater than 8, (D) less than 8, but not 0.

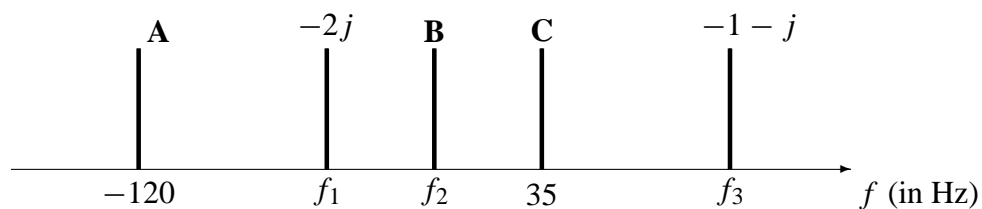
- (d) In the figure below two sinusoidal signals are shown. Which one has a phase of $+\pi/3$?
 the correct answer: $x_1(t)$ or $x_2(t)$.



- (e) In the figure above both sinusoidal signals have the same frequency. What is the frequency (ω_0) in radians/sec? the correct answer.
(A) 2.5π (B) 1.25π (C) 1.25 (D) $\pi/3$ (E) 1.6π

PROBLEM sp-04-Q.1.2:

The two-sided spectrum representation of a real-valued signal $x(t)$ is shown below, but it is missing some information:



Assume that the time signal $x(t)$ for this spectrum is real-valued, and that the DC value of $x(t)$ is $\frac{1}{2}$.

(a) Determine the values for the missing frequencies (in Hz):

$$f_1 = \underline{\hspace{2cm}}$$

$$f_2 = \underline{\hspace{2cm}}$$

$$f_3 = \underline{\hspace{2cm}}$$

(b) Determine the values for the missing complex amplitudes:

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

$$C = \underline{\hspace{2cm}}$$

(c) Write an equation for $x(t)$ using real-valued quantities only.

PROBLEM sp-04-Q.1.3:

The following MATLAB code defines several signals that are then multiplied and summed:

```
tt = -10:0.001:10;  
xxe = exp(-tt.*tt/5);  
xx1 = 3*cos( 3*pi*(tt - 1/4 ) );  
xx2 = 4*cos( 3*pi*tt + 11*pi/4 );  
xx = xxe.*xx1 + xxe.*xx2;
```

(a) If the signal $x_1(t)$ corresponds to the MATLAB vector `xx1`, determine the complex amplitude of $x_1(t)$.

(b) If the signal $x(t)$ corresponds to the MATLAB vector `xx`, then it is possible to express $x(t)$ in the form

$$x(t) = \Re \left\{ X e^{\alpha t^2 + \beta t} \right\}$$

Determine the numerical values of X , α , and β . *Hint: Use phasor addition.*

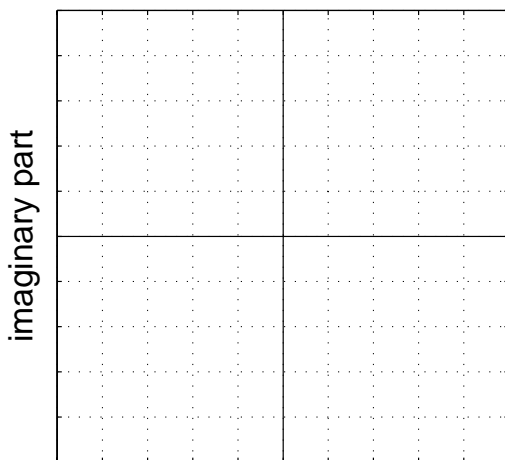
$X =$ _____

$\alpha =$ _____

$\beta =$ _____

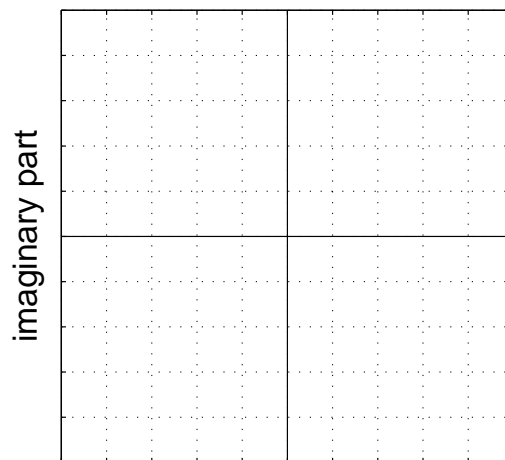
(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (b). On the first plot, show only the two complex amplitudes (phasors) that were added to solve part (b); on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail). *Use an appropriate scale on the grid below.*

Two vectors here.



real part

Head-to-tail plot here.



real part

PROBLEM sp-04-Q.1.4:

For each of the following signals, pick one of the representations below that defines *exactly* the same signal. Write your answer ((a), (b), (c), (d), (e), or (f)) in the box next to each signal.

$$\text{ANS} = \boxed{} \quad \Re \left\{ \left| (-1 + j\sqrt{3})e^{j80\pi t} \right| \right\}$$

$$\text{ANS} = \boxed{} \quad 2 \cos(80\pi t + 2\pi/3)$$

$$\text{ANS} = \boxed{} \quad e^{j\pi/3}e^{j80\pi t} + e^{-j\pi/3}e^{-j80\pi t}$$

$$\text{ANS} = \boxed{} \quad e^{j2\pi/3}e^{j80\pi t} + e^{-j2\pi/3}e^{-j80\pi t}$$

$$\text{ANS} = \boxed{} \quad 2 \cos(80\pi t + 4\pi/3)$$

POSSIBLE ANSWERS:

Your answer will be one of the following choices.

Please note: some of the following signals could be used more than one time to match the above signals.

(a) $x_a(t) = \cos(80\pi t - \pi/3)$

(b) $x_b(t) = 2 \cos(80\pi t + 7\pi/3)$

(c) $x_c(t) = \Re \left\{ (-1 + j\sqrt{3})e^{j80\pi t} \right\}$

(d) $x_d(t) = \Re \left\{ 2e^{-j2\pi/3}e^{j80\pi t} \right\}$

(e) $x_e(t) = 0$

(f) $x_f(t) = 2$

PROBLEM sp-04-Q.1.1:

For the following short answer questions, write your answers in the space provided or circle the correct answer:

- (a) The periodic signal $x(t)$ has a spectrum containing frequency components (with nonzero magnitude) at $f = 0, \pm 1.8$ and ± 2.4 Hz. Determine the *fundamental period*, i.e., the shortest possible period. Make sure your answer has the correct units.

$$T = 5/3 \text{ sec}$$

$$f_0 = \text{gcd}(1.8, 2.4) = 0.6 \text{ Hz}$$

$$T = 1/f_0 = 1/0.6 = 10/6 = 5/3 \text{ sec}$$

- (b) **TRUE** or **FALSE**: "If the spectrum of a signal $x(t)$ has frequency components (with nonzero magnitudes) only at $f = \pm 2$ Hz, then the spectrum of a new signal defined by $y(t) = x(t) \cos(2\pi(100)t)$ has frequency components (with nonzero magnitudes) at $f = \pm 102$ Hz and $f = \pm 98$ Hz."

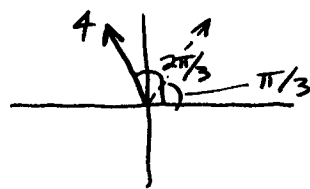
$$y(t) = (Ae^{j4\pi t} + Be^{-j4\pi t}) \left(\frac{1}{2} e^{j200\pi t} + \frac{1}{2} e^{-j200\pi t} \right)$$

$$= \frac{A}{2} e^{j204\pi t} + \frac{B}{2} e^{j196\pi t} + \frac{A}{2} e^{-j196\pi t} + \frac{B}{2} e^{-j204\pi t}$$

$204\pi \text{ rad/s} = 102 \text{ Hz}$
 $196\pi \text{ rad/s} = 98 \text{ Hz}$

- (c) Circle the correct answer: When you add $4 \cos(2t + 2\pi/3) + 4 \cos(2t - 5\pi/3)$ the maximum value of the resulting signal is:

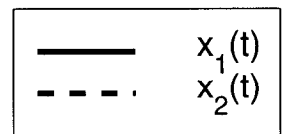
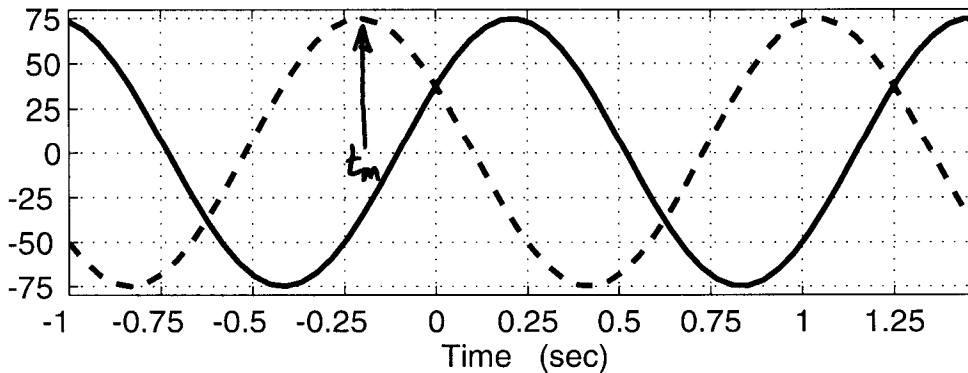
(A) equal to 0, (B) equal to 8, (C) greater than 8, **(D) less than 8, but not 0.**



$$4e^{-j5\pi/3} = 4e^{j\pi/3}$$

- (d) In the figure below two sinusoidal signals are shown. Which one has a phase of $+\pi/3$?

Circle the correct answer: $x_1(t)$ or **$x_2(t)$** . $\varphi = -\omega t_m \Rightarrow t_m < 0$



- (e) In the figure above both sinusoidal signals have the same frequency. What is the frequency (ω_0) in radians/sec? Circle the correct answer.

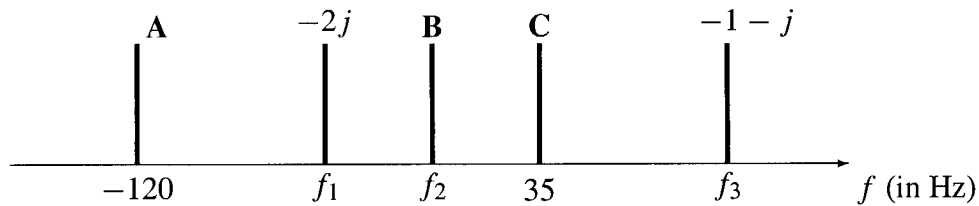
(A) 2.5π (B) 1.25π (C) 1.25 (D) $\pi/3$ **(E) 1.6π**

$$T = 1 - (-0.25) = 1.25 \text{ sec}$$

$$\omega_0 = 2\pi/T = 2\pi/5/4 = 8\pi/5 = 16\pi/10 = 1.6\pi \text{ rad/s}$$

PROBLEM sp-04-Q.1.2:

The two-sided spectrum representation of a real-valued signal $x(t)$ is shown below, but it is missing some information:



Assume that the time signal $x(t)$ for this spectrum is real-valued, and that the DC value of $x(t)$ is $\frac{1}{2}$.

- (a) Determine the values for the missing frequencies (in Hz):

$$\begin{aligned} \underline{f_1} &= -35 \text{ Hz} && \text{Use complex conjugate symmetry} \\ \underline{f_2} &= 0 \text{ Hz} \\ \underline{f_3} &= 120 \text{ Hz} \end{aligned}$$

- (b) Determine the values for the missing complex amplitudes:

$$\begin{aligned} \underline{A} &= -1 + j = \sqrt{2} e^{j3\pi/4} \\ \underline{B} &= 1/2 \\ \underline{C} &= 2j = 2 e^{j\pi/2} \end{aligned}$$

- (c) Write an equation for $x(t)$ using real-valued quantities only.

$$\begin{aligned} x(t) &= \frac{1}{2} + 4 \cos(2\pi(35)t + \pi/2) \\ &\quad + 2\sqrt{2} \cos(2\pi(120)t - 3\pi/4) \end{aligned}$$

PROBLEM sp-04-Q.1.3:

The following MATLAB code defines several signals that are then multiplied and summed:

```
tt = -10:0.001:10;
xxe = exp(-tt.*tt/5);
xx1 = 3*cos( 3*pi*(tt - 1/4 ) );
xx2 = 4*cos( 3*pi*tt + 11*pi/4 );
xx = xxe.*xx1 + xxe.*xx2;
```

- (a) If the signal $x_1(t)$ corresponds to the MATLAB vector `xx1`, determine the complex amplitude of $x_1(t)$.

$$x_1(t) = 3 \cos(3\pi t - 3\pi/4) \quad \underline{X_1} = 3e^{-j3\pi/4}$$

- (b) If the signal $x(t)$ corresponds to the MATLAB vector `xx`, then it is possible to express $x(t)$ in the form

$$x(t) = \Re \left\{ X e^{\alpha t^2 + \beta t} \right\}$$

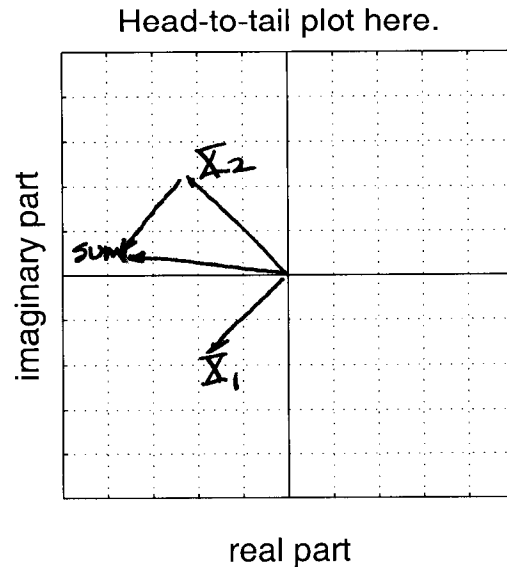
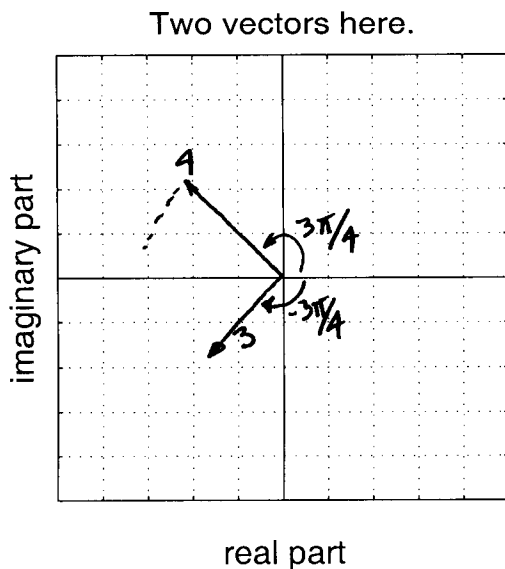
Determine the numerical values of X , α , and β . *Hint: Use phasor addition.*

$$\begin{aligned} \underline{X} &= 5e^{j3} \\ \alpha &= -1/5 \\ \underline{\beta} &= j3\pi \end{aligned} \quad \begin{aligned} x(t) &= e^{-t^2/5} \left(3 \cos(3\pi t - 3\pi/4) + 4 \cos(3\pi t + 11\pi/4) \right) \\ &= 3e^{-j3\pi/4} + 4e^{j3\pi/4} \\ &= 5e^{j3} = 5e^{j0.955\pi} \end{aligned}$$

↑ Same as $3\pi/4$

↑ 171.9°

- (c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (b). On the first plot, show only the two complex amplitudes (phasors) that were added to solve part (b); on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail). Use an appropriate scale on the grid below.



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For each of the following signals, pick one of the representations below that defines *exactly* the same signal.

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$$\text{ANS} = \mathbf{f} \quad \Re \left\{ \left| (-1 + j\sqrt{3})e^{j80\pi t} \right| \right\} = 2 \quad -1 + j\sqrt{3} = 2e^{j\pi/3}$$

$$\text{ANS} = \mathbf{c} \quad 2 \cos(80\pi t + 2\pi/3)$$

$$\text{ANS} = \mathbf{b} \quad e^{j\pi/3}e^{j80\pi t} + e^{-j\pi/3}e^{-j80\pi t} = 2 \cos(80\pi t + \pi/3)$$

$$\text{ANS} = \mathbf{c} \quad e^{j2\pi/3}e^{j80\pi t} + e^{-j2\pi/3}e^{-j80\pi t} = 2 \cos(80\pi t + 2\pi/3)$$

$$\text{ANS} = \mathbf{d} \quad 2 \cos(80\pi t + 4\pi/3) = 2 \cos(80\pi t - 2\pi/3)$$

POSSIBLE ANSWERS:

Your answer will be one of the following choices.

Please note: some of the following signals could be used more than one time to match the above signals.

(a) $x_a(t) = \cos(80\pi t - \pi/3)$

(b) $x_b(t) = 2 \cos(80\pi t + 7\pi/3) = 2 \cos(80\pi t + \pi/3)$

(c) $x_c(t) = \Re \left\{ (-1 + j\sqrt{3})e^{j80\pi t} \right\} = \Re \left\{ 2e^{j\pi/3}e^{j80\pi t} \right\} = 2 \cos(80\pi t + 2\pi/3)$

(d) $x_d(t) = \Re \left\{ 2e^{-j2\pi/3}e^{j80\pi t} \right\} = 2 \cos(80\pi t - 2\pi/3) = 2 \cos(80\pi t + 4\pi/3)$

(e) $x_e(t) = 0$

(f) $x_f(t) = 2$