



**PROBLEM Fall-25-Q.1.1.1:**

Many signals can be expressed as sum of sinusoids as

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(\omega_k t + \phi_k)$$

where  $A_k \geq 0$ ,  $-\pi < \phi_k \leq \pi$  for  $k = 1, 2, \dots, N$ , and all  $\omega_k$ s are distinct. This is often called a canonical or standard form. Reduce the following signals to their canonical form.

(a)  $x_1(t) = 4 \cos(20\pi(t - \frac{1}{6})) - 4 \sin(20\pi t - \frac{15\pi}{6})$

$$x_1(t) = 4 \cos(20\pi t - \frac{10\pi}{3}) - 4 \sin(20\pi t - \frac{\pi}{2})$$

$$x_1(t) = 4 \cos(20\pi t + \frac{2\pi}{3}) + 4 \cos(20\pi t)$$

$$4e^{j2\pi/3} + 4 = 4e^{j\pi/3}$$

$$x_1(t) = 4 \cos(20\pi t + \pi/3)$$

(b)  $x_2(t) = \sum_{k=0}^2 2(k+1) \cos(50k\pi t + \frac{k\pi}{4}) + 4 \cos(50\pi t - \frac{\pi}{4})$ .

$$x_2(t) = \sum_{k=0}^2 2(k+1) \cos(50k\pi t + \frac{k\pi}{4}) + 4 \cos(50\pi t - \frac{\pi}{4})$$

$$= 2 + 4 \cos(50\pi t + \frac{\pi}{4}) + 6 \cos(100\pi t + \frac{\pi}{2}) + 4 \cos(50\pi t - \frac{\pi}{4})$$

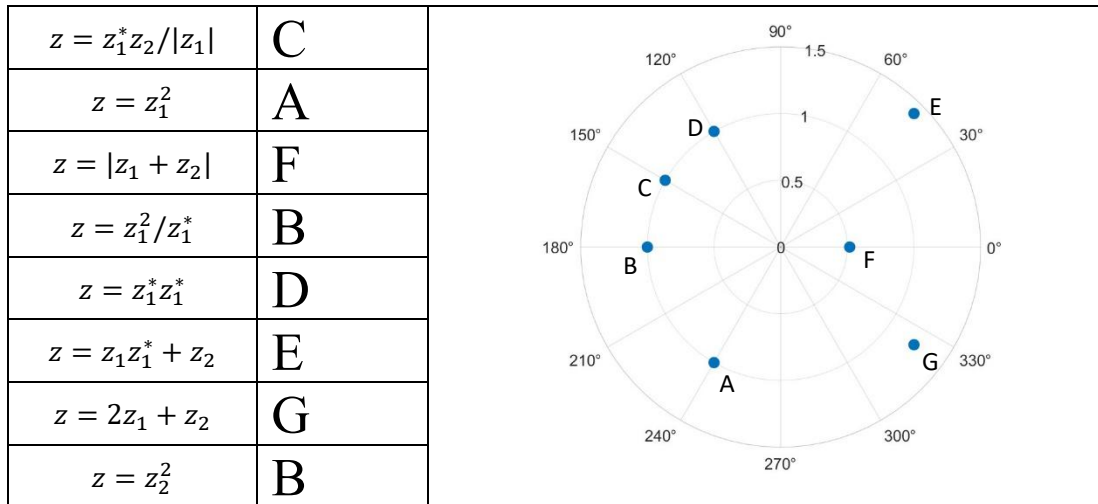
$$4e^{j\pi/4} + 4e^{-j\pi/4} = 8 \cos(\pi/4) = 4\sqrt{2}$$

$$x_2(t) = 2 + 4\sqrt{2} \cos(50\pi t) + 6 \cos(100\pi t + \frac{\pi}{2})$$

$$x_2(t) = 2 + 4\sqrt{2} \cos(50\pi t) + 6 \cos(100\pi t + \frac{\pi}{2})$$

**PROBLEM Fall-25-Q.1.1.2:**

Let  $z_1 = e^{-j\pi/3}$ , but  $z_2$  is to be found from the provided clue. Find the corresponding solid (filled) point in the plot, designated in capital letter, for each of the expressions in the table below. (Note a point may correspond to multiple expressions.)



From the given answer of C, it is found that  $z_2 = j$

**PROBLEM Fall-25-Q.1.1.3:**

The following MATLAB code generates a sinusoidal signal once **zx**, a single complex number, is defined.

```
tt = -0.1 : 1/10240 : 0.1;
Fo = 16;
zx = ??;
zz = zx*exp(j*(2*pi*Fo*tt + pi/4));
xx = real( zz );
```

When the entire signal **xx** is plotted, it is observed that the signal attains a positive peak with value 3 at **tt(345)**.

Now give answers to the following:

- What is the length of **xx**?
- Find **zx**.
- Write a mathematical expression for the real signal  $x(t)$  that corresponds to the signal array **xx**.

Answers:

- With  $-0.1 \leq t \leq 0.1$ , a time span of 0.2s, and a time increment of 1/10240 represented in the **tt** array, we have  $0.2 \times 10240 = 2048$  increments altogether. Counting the two endpoints, we conclude that the length of **tt** is 2049, with **tt(1) = -0.1** and with **tt(2049) = 0.1**. The data array **xx** has the same length as 2049.
- Given **Fo = 16**, we have period  $T = \frac{1}{f_0} = \frac{1}{16}$ . A peak occurs at **tt(345)**, which corresponds to  $t = \frac{344}{10240} - 0.1 = -\frac{68}{1024}$ . To find the phase, we recognize the fact that  $\cos \theta$  attains its peak value when  $\theta = 0$ . Note that the code for **zz** already has **pi/4**, therefore to find the phase, we invoke  $\omega t_m + \frac{\pi}{4} + \varphi = 0$  and obtain  $\varphi = -2\pi \times 16 \times \left(-\frac{68}{1024}\right) - \frac{\pi}{4} = \frac{15\pi}{8}$  or  $-\frac{\pi}{8}$ . Peak value is 3,  $|z_x| = 3$ . Therefore, **zx = 3\*exp(-j\*pi/8)**
- Finally the signal's phase is  $\frac{\pi}{8} + \frac{\pi}{4} = \frac{3\pi}{8}$ ,  $x(t) = 3 \cos(32\pi t + \frac{3\pi}{8})$

Answers:

- Length is 2049;
- zx = 3\*exp(-j\*pi/8)**
- $x(t) = 3 \cos(32\pi t + \frac{3\pi}{8})$