GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2024
Exam #1

NAME: Solution ______________________ GTemail: ______________________
FIRST LAST ex: gAburdell@gatech.edu

- Write your name at the top of EACH PAGE.
- DO not unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to received partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles/phase must expressed in the range $(-\pi, \pi]$ for full credit.
- You must show your work in the space provided on the exam paper itself. Only these answers with shown work can received credit. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

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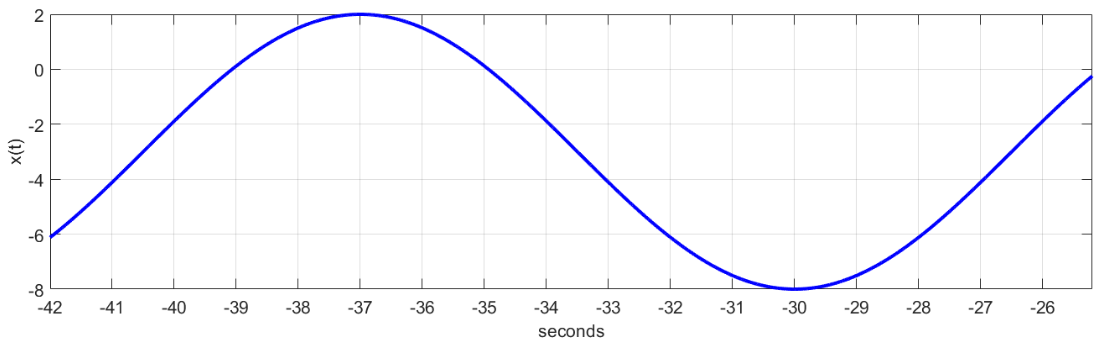
PROBLEM 1:

Parts a and b can be solved independently of each other.

(a) (15 points) A portion of a sinusoidal signal $x(t)$ is shown the plot below and may be expressed as:

$$x(t) = B + A\cos(\omega_0 t + \varphi) = B + A\cos\big(\omega_0(t - t_d)\big)$$

Find $B, A > 0, \omega_0, -\pi < \varphi \leq \pi,$ and t_d (where t_d represents the delay value **closest to** $t = 0$ that matches the plot)



$$B = \frac{2 + (-8)}{2} = -3$$

$$A = \frac{2 - (-8)}{2} = 5$$

$$T = 14 \to \omega_0 = \frac{2\pi}{14} = \frac{\pi}{7}$$

$$t_d = -37 + (3 * 14) = 5$$

$$\varphi = -\omega_0 t_d = -\frac{5\pi}{7}$$

$\underline{\qquad\qquad}$ B $\quad$ $\underline{\qquad\qquad}$ $A > 0$ $\quad$ $\underline{\qquad\qquad}$ ω_0 $\quad$ $\underline{\qquad\qquad}$ φ $\quad$ $\underline{\qquad\qquad}$ t_d

(b) (5 points) A sinusoid is defined as

$$x(t) = \Re e\left\{8e^{j\varphi}e^{-j\frac{\pi}{8}}e^{j40\pi t}\right\}$$

Find $-\pi < \varphi \leq \pi$ such that $x(-0.005) = 8$.

$$x(t) = 8\cos\left(40\pi t - \frac{\pi}{8} + \varphi\right)$$

$$40\pi(-0.005) - \frac{\pi}{8} + \varphi = 0 \to \varphi = 0.2\pi + 0.125\pi = 0.325\pi$$

$\varphi =$ $\underline{\qquad\qquad}$

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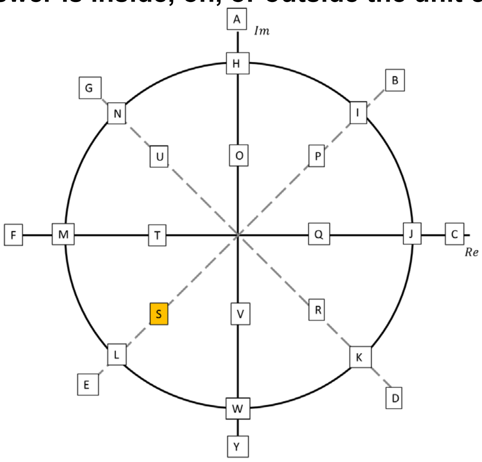
PROBLEM 2:

Parts a and b (10 points each) can be solved independently of each other.

(a) Consider the complex plane below showing the unit circle (i.e., the black circle indicating a radius 1) and complex numbers in the complex space labeled with letters from A to Y. Assume that we start with a complex number $\boldsymbol{z} = \boldsymbol{r}e^{j\theta}$ at position **S** as indicated by the shaded box. We also define a new complex number, $\boldsymbol{z_1}$, that relates to $\boldsymbol{z}$ by the set of equations in the table below. Select the letter that best approximates the position of $\boldsymbol{z_1}$. **While the unit circle is represented in the graph, the box locations are not exactly to scale so you should choose letters that are the best approximation based on approximate angle and whether the answer is inside, on, or outside the unit circle**.



Equations ($z_1 =$)	**Letter**
z	**S**
$-(z^*)^{-1}$	B
$\frac{1}{2}\frac{j(z^*+z)}{(r\cos(\theta))}$	H
$\frac{z^*z}{r^2}e^{j\pi}$	M
$(r^{-1})z^*$	N

(b) Consider the following expression for $x(t)$

$$x(t) = 8\cos(30\pi t - \pi/4) + 4\cos(30\pi t + 3\pi/8) + \sum_{k=-3}^{N} 6\cos(10\pi t + \pi k/17) = A\cos(30\pi t + \varphi)$$

For this equation to be valid, it is necessary that

$$\sum_{k=-3}^{N} 6\cos(10\pi t + \pi k/17) = 0$$

Find the **three smallest** possible values for $N > 0$ such that above equation is true

$N =$ ________ or ________ or ________

$$\sum_{k=-3}^{N} 6\cos(10\pi t + \pi k/17) \rightarrow (phasor\ addition)$$

$$\sum_{k=-3}^{N} 6e^{j\frac{2\pi k}{34}} = 0 \rightarrow N = -3 + 33 = \mathbf{30} \text{ OR } \sum_{k=-3}^{N} 6e^{j\frac{2\pi k}{34}} = 0 \rightarrow N = -3 + 33 + 34 = \mathbf{64}$$

$$\sum_{k=-3}^{N} 6e^{\frac{j2\pi}{34}} = 0 \rightarrow N = -3 + 33 + 34 + 34 = \mathbf{98}$$

For one of the values of N above, find $A > 0$ and ϕ.

$A =$

$\varphi =$ ____________ π

$$8\cos(30\pi t - \pi/4) + 4\cos(30\pi t + 3\pi/8) \rightarrow (phasor\ addition)$$

$$Ae^{j\varphi} = 8e^{-\frac{j\pi}{4}} + 4e^{\frac{j3\pi}{8}} = 7.45e^{-j0.0848\pi}$$

$$A = 7.45$$

$$\varphi = -0.0848\pi$$

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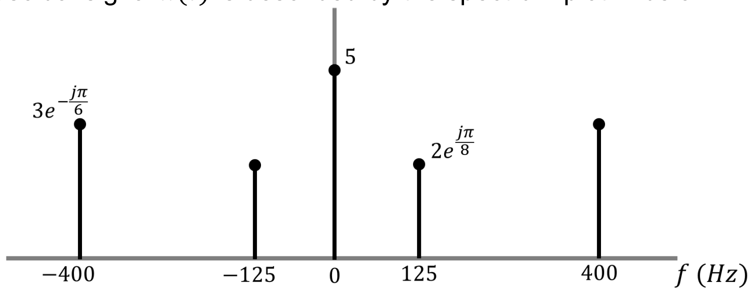
PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) (10 points) A real sinusoidal signal $x(t)$ is described by the spectrum plot in below



(a) Let $y(t) = 2x(t - 0.002)$. Use the spectrum plot above to express $y(t)$ as a sum of sinusoids in the standard form shown below

$$y(t) = A_0 + A_1 \cos(\omega_1 t + \varphi_1) + A_2 \cos(\omega_2 t + \varphi_2) + \cdots + A_N \cos(\omega_N t + \varphi_N)$$

where N represents total number of sinusoids present.

$$x(t) = 5 + 4\cos\left(250\pi t + \frac{\pi}{8}\right) + 6\cos\left(800\pi t + \frac{\pi}{6}\right)$$

$$y(t) = 2x(t - 0.002) = 10 + 8\cos\left(250\pi(t - 0.002) + \frac{\pi}{8}\right) + 12\cos\left(800\pi(t - 0.002) + \frac{\pi}{6}\right)$$

$$y(t) = 10 + 8\cos(250\pi t - 0.375\pi) + 12\cos(800\pi t - 1.4333\pi)$$

$$y(t) = 10 + 8\cos(250\pi t - 0.375\pi) + 12\cos(800\pi t + 0.5667\pi)$$

$y(t) =$

(b) (10 points) Plot the two-side spectrum for the sinusoid defined below (make sure to label all axes and values).

$$x(t) = \Re e\left\{4 + 12e^{j(50\pi(t-0.01))} + 6e^{j\frac{\pi}{5}}e^{j100\pi t}\right\}$$