GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2023
Exam #1

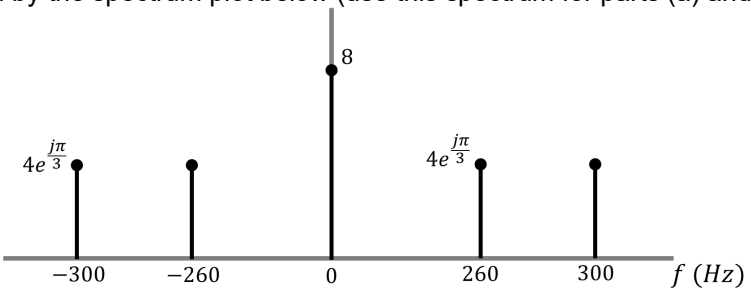
NAME: _____ GTemail: _____
FIRST LAST ex: gpburdell@gatech.edu

- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to received partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

Print Name (First Last) _____

PROBLEM 1:
Parts a and b (10 points each) can be solved independently of each other.
A signal $x(t)$ is described by the spectrum plot below (use this spectrum for parts (a) and (b))



(a) Express $x(t)$ in the form: $x(t) = A_0 + B\cos(\omega_\Delta t + \phi)\cos(\omega_c t)$ (i.e., DC term plus AM).

$x(t) =$ _____ + _____ cos(_____ t + _____)cos (_____ t)
A_0 B ω_Δ ϕ ω_c

(b) The Fourier series representation of $x(t)$ is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$.

Find f_0 (in Hz)

$f_0 =$

List the **non-zero** Fourier series coefficients (a_k) along with their k-index in the table below.

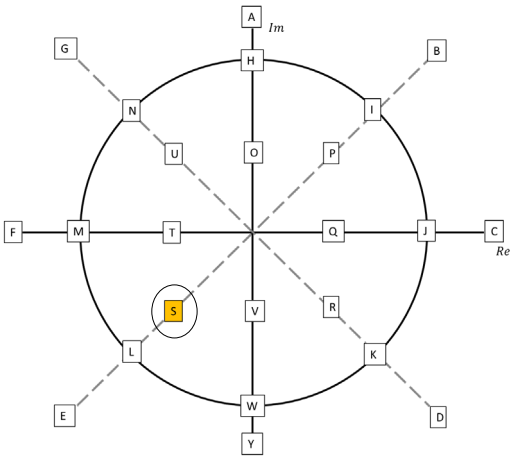
k							
a_k							

Print Name (First Last) _____

PROBLEM 2:

Parts a and b (10 points each) can be solved independently of each other.

(a) Consider the complex plane below with the unit circle shown and complex numbers labeled with a letter from A to Y. Assume that we start with a complex number $\boldsymbol{z = re^{j\theta}}$ at position **S** as indicated by the black circle and shaded box. We also define a new complex number, $\boldsymbol{z_1}$, that relates to $\boldsymbol{z}$ by the set of equations in the table below. Select the letter that best approximates the position of $\boldsymbol{z_1}$.



Equations ($z_1 =$)	Letter
z	**S**
$\frac{z^* z}{r^2}$	
rz^{-1}	
$\frac{1}{z^*}$	
$\frac{1}{4}\frac{(z^* + z)}{(r\cos(\theta))}$	

(b) Find an integer N in the range $21 < N < 40$, and also find $A > 0$ and $\phi \in (-\pi, \pi]$, such that the following equation is true for all time t:

$$x(t) = 8\cos(50\pi t - \pi/3) + 8\cos(50\pi t - 2\pi/3) + \sum_{k=-11}^{N} 6\cos(100\pi t + \pi k/8) = A\cos(50\pi t + \phi)$$

$N =$	$A =$	$\phi =$_____ π

Print Name (First Last) _____

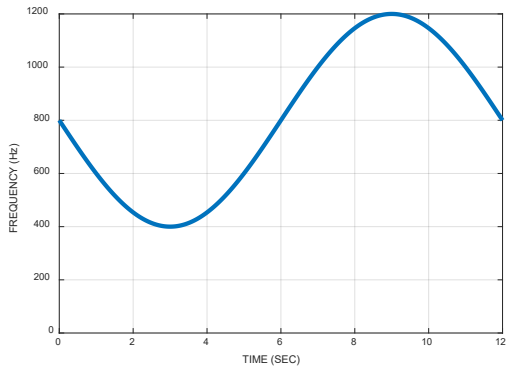
PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) Find numerical values for the constants A, B, and C so that the spectrogram of the signal

$$x(t) = 2023\cos(\pi At + B\cos(\pi Ct))$$

looks like this:



$A =$	$B =$	$C =$

(b) A sinusoid is defined as $x(t) = \Re e\left\{6e^{j\theta}e^{-j\frac{\pi}{4}}e^{j10\pi t}\right\} = A\cos(\omega_0(t+0.03)) = A\cos(\omega_0 t + \varphi)$. Find A, ω_0, and θ.

$A\ (>0) =$	$\omega_0 (\geq 0) =$	$\theta \in (-\pi, \pi] =$ _____ π

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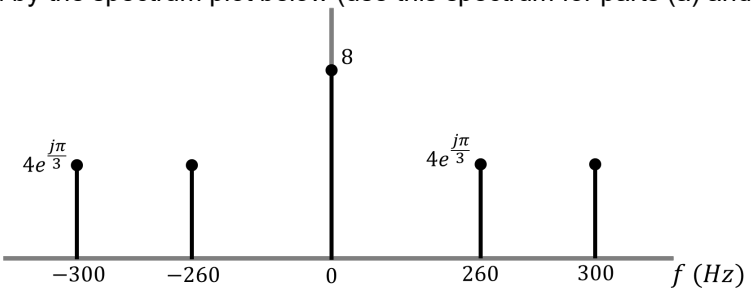

(a) Express $x(t)$ in the form: $x(t) = A_0 + B\cos(\omega_\Delta t + \phi)\cos(\omega_c t)$ (i.e., DC term plus AM).

$x(t) =$ **8** (A_0) + **16** (B) cos(**40π** (ω_Δ) t + **$-\frac{\pi}{3}$** (ϕ))cos (**560π** (ω_c) t)

(b) The Fourier series representation of $x(t)$ is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$.

Find f_0 (in Hz)

gcd(260,300)=20

$f_0 =$ 20 Hz

List the **non-zero** Fourier series coefficients (a_k) along with their k-index in the table below.

k	**0**	**-13**	**13**	**-15**	**15**		
a_k	**8**	**$4e^{-\frac{j\pi}{3}}$**	**$4e^{\frac{j\pi}{3}}$**	**$4e^{\frac{j\pi}{3}}$**	**$4e^{-\frac{j\pi}{3}}$**		

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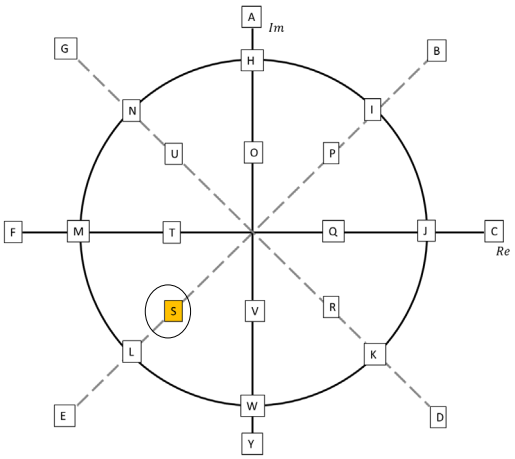
PROBLEM 2:

Parts a and b (10 points each) can be solved independently of each other.

(a) Consider the complex plane below with the unit circle shown and complex numbers labeled with a letter from A to Y. Assume that we start with a complex number $z = re^{j\theta}$ at position **S** as indicated by the black circle and shaded box. We also define a new complex number, z_1, that relates to z by the set of equations in the table below. Select the letter that best approximates the position of z_1.



Equations ($z_1 =$)	Letter
z	**S**
$\frac{z^* z}{r^2}$	**J**
rz^{-1}	**N**
$\frac{1}{z^*}$	**E**
$\frac{1}{4}\frac{(z^* + z)}{(r\cos(\theta))}$	**Q**

(b) Find an integer N in the range $21 < N < 40$, and also find $A > 0$ and $\phi \in (-\pi, \pi]$, such that the following equation is true for all time t:

$$x(t) = 8\cos(50\pi t - \pi/3) + 8\cos(50\pi t - 2\pi/3) + \sum_{k=-11}^{N} 6\cos(100\pi t + \pi k/8) = A\cos(50\pi t + \phi)$$

$$\sum_{k=-11}^{N} 6\cos(100\pi t + \pi k/8) = 0 \rightarrow 6\sum_{k=-11}^{N} e^{\frac{j2\pi k}{16}} = 0$$

This function will equal zero for values of $N = 4,20,36,52,$ ...etc. But in the given range the answer is $N = 36$

$$8\cos(50\pi t - \pi/3) + 6\cos(50\pi t - 2\pi/3) = A\cos(50\pi t + \phi)$$

Use Phasor addition:

$$8e^{-\frac{j\pi}{3}} + 8e^{-\frac{j2\pi}{3}} = 8\sqrt{3}e^{-j0.5\pi}$$

$N = 36$	$A = 8\sqrt{3}$	$\phi =$ **-0.5** π

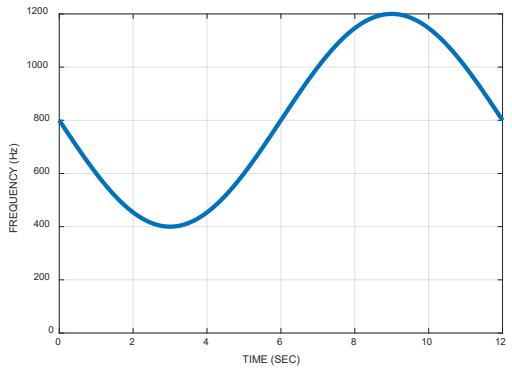
Print Name (First Last) _____

PROBLEM 3:
Parts a and b (10 points each) can be solved independently of each other.

(a) Find numerical values for the constants A, B, and C so that the spectrogram of the signal

$$x(t) = 2023\cos(\pi A t + B\cos(\pi C t))$$

looks like this:



$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(\pi A t + B\cos(\pi C t))$$
$$= \frac{A}{2} - \frac{BC}{2}\sin(\pi C t)$$

From the plot:

$$\pi C = \frac{2\pi}{12} \rightarrow C = \frac{1}{6}$$
$$\frac{A}{2} = 800 \rightarrow A = 1600$$
$$\frac{BC}{2} = 400 \rightarrow \frac{B}{2}\left(\frac{1}{6}\right) = 400 \rightarrow B = 4800$$

A	B	C
$= 1600$	$= 4800$	$= \frac{1}{6}$

(b) A sinusoid is defined as $x(t) = \Re e\left\{6e^{j\theta}e^{-j\frac{\pi}{4}}e^{j10\pi t}\right\} = A\cos(\omega_0(t+0.03)) = A\cos(\omega_0 t + \varphi)$. Find A, ω_0, and θ.

$$\Re e\left\{6e^{j\theta}e^{-j\frac{\pi}{4}}e^{j10\pi t}\right\} = 6\cos\left(10\pi t - \frac{\pi}{4} + \theta\right) = A\cos(\omega_0 t + \varphi)$$

$$\omega_0 = 10\pi$$
$$A = 6$$
$$\varphi = \omega_0(0.03) = -\frac{\pi}{4} + \theta \rightarrow \theta = (10\pi)(0.03) + 0.25\pi = 0.3\pi + 0.25\pi = 0.55\pi$$
$$\theta = 0.55\pi$$

$A\ (>0)$	$\omega_0 (\geq 0)$	$\theta \in (-\pi, \pi]$
$= \mathbf{6}$	$= \mathbf{10\pi}$	$=$ **0.55** π