GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 1

DATE:20-Sept-19 COURSE: ECE-2026

NAME: Solutions CanvasID: ____________
LAST, FIRST ex: gtJohnA

Circle your correct recitation section number - failing to do so will cost you 2 points

Recitation time	Mon	Tue	Wed	Thu
09:30:10:45		L12 Farahmand		L06 Causey
12:00-13:15		L07 Farahmand		L08 Barry
13:30-14:45		L09 Farahmand		L10 Barry
15:00-16:15	L01 Juang	L11 Farahmand	L02 Casinovi	
16:30-17:45			L04 Casinovi	

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK ON ALL PROBLEMS TO RECEIVE CREDIT. PROBLEMS WITH NO WORK AND JUST ANSWERS MAY RECEIVE 0 CREDIT, EVEN IF THE ANSWER IS CORRECT. YOU MUST SHOW SOME NUMERICAL WORK, REASONING, OR EXPLANATION FOR YOUR ANSWER. (I.E., DON'T JUST PUT AN ANSWER AND LEAVE THE WORK AREA BLANK)**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)**
- **ALL RADIAN ANSWERS MUST BE IN THE RANGE $(-\pi, \pi]$ FOR CREDIT.**

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
No/Wrong Recitation Circled	-2	
Total		

PROBLEM 1

Parts a and be can be solved independently of each other

(a) Consider the following complex signal defined as:

$$z(t) = Ae^{j(\omega_0 t+\varphi)}$$

with the following known information

1. $z(6) = Ae^{j\frac{\pi}{2}}$ (i.e., @ $t = 6$, the equivalent expression for $z(t)$ is $Ae^{j\frac{\pi}{2}}$)
2. The rotating phasor portion of $z(t)$ covers 9π radians every 30 seconds.
3. $z(t)z^*(t) = 64$

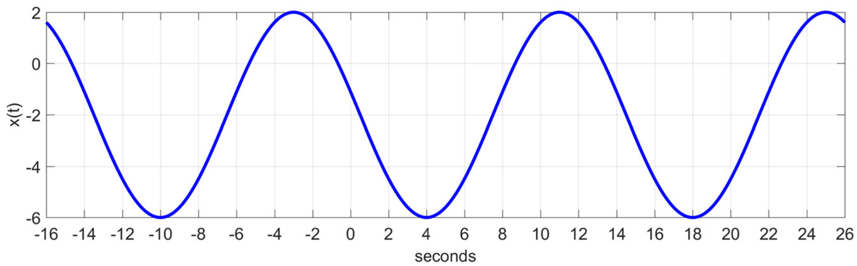
Find $A, \omega_0, \text{and } \varphi$. **(Recall all phase angles must be in the range $(-\pi, \pi]$ for credit)** (10 points)

Solution: From (2) - $\omega_0 = \frac{9\pi}{30} = \frac{3\pi}{10}$; From (3) - $zz^* = |z|^2 = A^2 = 64 \rightarrow A = 8$;

From (1): $z(6) = Ae^{j\left(\frac{3\pi}{10}(6)+\varphi\right)} = Ae^{j\left(\frac{18\pi}{10}+\varphi\right)} = Ae^{\frac{j\pi}{2}} \rightarrow \varphi = \frac{\pi}{2} - \frac{18\pi}{10} = -\frac{13\pi}{10} \rightarrow -\frac{13\pi}{10} + 2\pi = \frac{7\pi}{10}$

$A =$ ______8________ $\omega_0 =$ ______$3\pi/10$____________ $\varphi =$ _____$7\pi/10$_______

(b) From the plot below find $A, B, \omega_0, \varphi, \text{and } t_d$ such that the sinusoid may be expressed as $x(t) = B + A\cos(\omega_0 t + \phi) = B + A\cos(\omega_0(t - t_d))$ below. **(Recall all phase angles must be in the range $(-\pi, \pi]$ for credit)** (10 points)

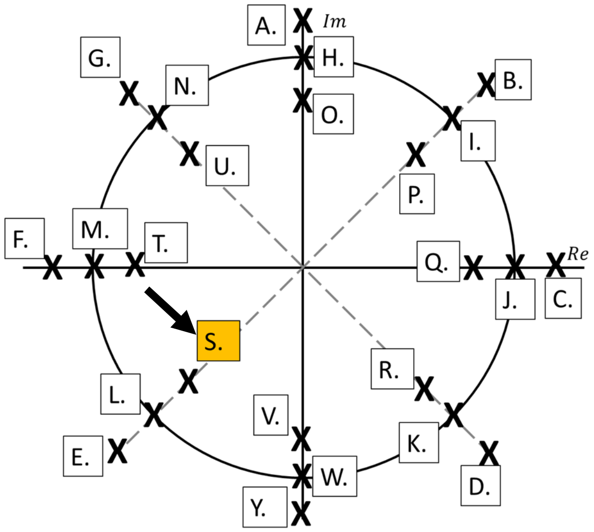


$A =$ ____4______ $B =$ ___-2_____ $\omega_0 =$ ____$\pi/7$_____ $\varphi =$ ___$3\pi/7$___ $t_d =$ ____-3___

PROBLEM 2:

Parts a and b can be solved independently of each other.

(a) Consider the complex plane below with complex numbers marked by an **X** and labeled with a letter from A to Y. Assume that we start with a complex number $\boldsymbol{z = re^{j\theta}}$ at position **S** as indicated by the black arrow and colored box. We also define a new complex number, $\boldsymbol{z_1}$, that relates to $\boldsymbol{z}$ by the set of equations in the table below. Based on these equations, select the letter that best approximates the position of $\boldsymbol{z_1}$. (**You may show work on the picture**) (10 points)



Equations ($\boldsymbol{z_1}$ =)	**Letter**
z	**S**
$(z^*)(z)(r^{-2})$	J
$\frac{rj}{z^*}$	K
$-\frac{1}{z}$	D
$0.5(r\sin(\theta))^{-1}(z^* - z)$	W

(b) Consider the following expression for $x(t)$. Simplify the summation terms and rewrite $x(t)$ using as few sinusoids as possible (all radian should be in the range $(-\pi, \pi]$). (10 points)

$$x(t) = \sum_{k=39}^{55} 2\cos(300\pi t + \pi k/8) + \sum_{k=53}^{77} 2\cos(300\pi t + \pi k/12)$$

Solution:

$$x(t) = \sum_{k=39}^{55} 2\cos\left(300\pi t + \frac{\pi k}{8}\right) + \sum_{k=53}^{77} 2\cos\left(300\pi t + \frac{\pi k}{12}\right)$$

$$= 2\cos\left(300\pi t + \frac{55\pi}{8}\right) + 2\cos\left(300\pi t + \frac{77\pi}{12}\right)$$

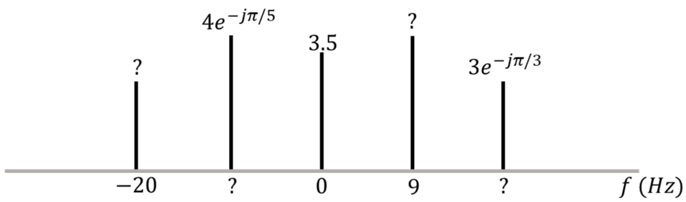
$$= 2\cos\left(300\pi t + \frac{7\pi}{8}\right) + 2\cos\left(300\pi t + \frac{5\pi}{12}\right) \ (use\ phasor\ addition)$$

$$\approx 3\cos(300\pi t + 0.6458\pi)$$

$x(t) =$	$3\cos(300\pi t + 0.6458\pi)$

PROBLEM 3:
Parts a and b can be solved independently of each other

(a) The spectrum of a real signal $x(t)$ is shown below. Write the signal $x(t)$ as a sum of sinusoids in the standard form $x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(\omega_k t + \varphi_k)$ (10 points)



$$x(t) = \quad 3.5 + 8\cos\left(18\pi t + \frac{\pi}{5}\right) + 6\cos\left(40\pi t - \frac{\pi}{3}\right)$$

(b) Consider the signal $x(t) = Re\left\{3je^{j\left(30\pi t + \frac{\pi}{3}\right)}\right\} + Re\left\{3e^{j\left(30\pi t - \frac{\pi}{6}\right)}\right\} + Re\left\{-4je^{j\left(40\pi t + \frac{\pi}{6}\right)}\right\}$. Write the signal $x(t)$ as a sum of sinusoids in the standard form $x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(\omega_k t + \varphi_k)$ (10 points)

Solution:

$$x(t) = 3\cos\left(30\pi t + \frac{5\pi}{6}\right) + 3\cos\left(30\pi t - \frac{\pi}{6}\right) + 4\cos\left(40\pi t - \frac{\pi}{3}\right)$$

$$x(t) = 4\cos\left(40\pi t - \frac{\pi}{3}\right)$$

$$x(t) = \qquad 4\cos\left(40\pi t - \frac{\pi}{3}\right)$$