GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 1

DATE:18-Sept-17 COURSE: ECE-2026

NAME: ______________________________ TSquareID: ______________
LAST, FIRST ex: gtJohnA

Circle your correct recitation section number - failing to do so will cost you 2 points

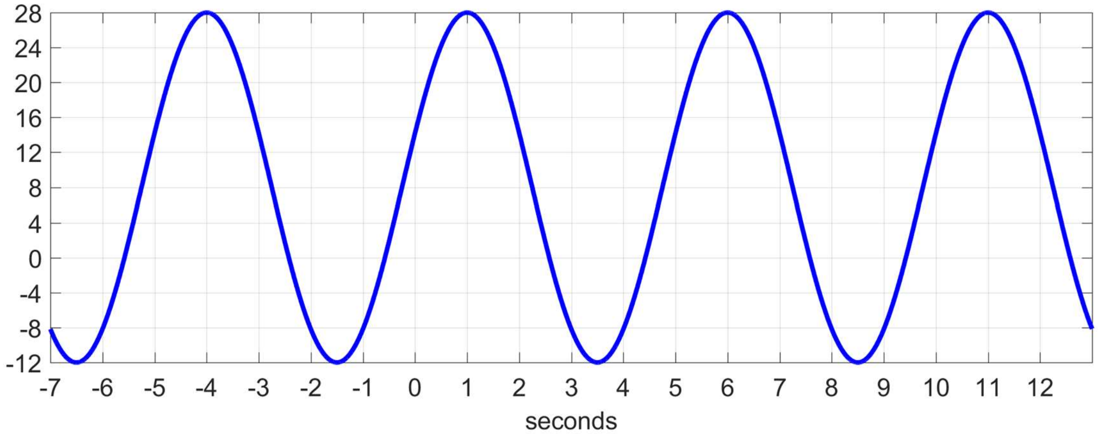
Recitation time	Mon	Tue	Wed	Thu
09:30:10:45				L06 Harper
12:00-13:15		L07 Causey		L08 Harper
13:30-14:45		L09 Yang		L10 Stuber
15:00-16:15	L01 Juang	L11 Yang	L02 Causey	L12 Stuber
16:30-17:45	L03 Marenco		L04 Causey	

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK ON ALL PROBLEMS TO RECEIVE CREDIT**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π instead of 1.257)**
- **ALL RADIAN ANSWERS SHOULD BE IN THE RANGE (-π, π].**

Problem	*Value*	*Score*
1	20	
2	20	
3	10	
No/Wrong Recitation Circled	-2	
Total		

PROBLEM 1:

The sinusoidal signal shown below is in the form: $x(t) = B + A\cos(\omega_0 t + \varphi)$



(a) Find B, A, ω_0, and φ (SHOW YOUR WORK) (12 points)

To find B: (28+-12)/2 = 8

To find A: (28+12)/2 = 20

To find ω_0: $T = (1+4) = 5\,\text{sec} \rightarrow \omega_0 = \frac{1}{5} * 2\pi = \frac{2\pi}{5} = 0.4\pi$

To find φ: $t_d = 1\,\text{sec} \rightarrow \varphi = -\omega_0 t_d = -0.4\pi$

$A =$ ____20_________ $B =$ ___8___________ $\omega_0 =$ ____0.4π__________ $\varphi =$ ___ -0.4π_______

(b) A sinusoid is defined as $x(t) = \Re e\left\{3e^{j\theta}e^{-j\frac{2\pi}{5}}e^{j80\pi t}\right\}$. Find θ such that the closest peak to zero for $x(t)$ is located at **-0.01 seconds**. (SHOW YOUR WORK) (8 points) (Make sure that $\theta \in (-\pi, \pi]$)

$$x(t) = \Re e\left\{3e^{j\theta}e^{-j\frac{2\pi}{5}}e^{j80\pi t}\right\} = 3\cos(80\pi t + \theta - 2\pi/5) \rightarrow t_d = -\frac{\left(\theta - \frac{2\pi}{5}\right)}{80\pi} = -0.01$$

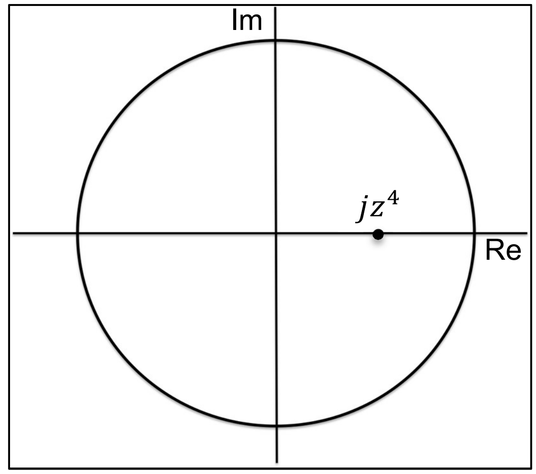
$$t_d = -\frac{\left(\theta - \frac{2\pi}{5}\right)}{80} = -0.01 \rightarrow \theta = 0.8\pi + 0.4\pi = 1.2\pi \rightarrow -0.8\pi$$

$\theta =$ ___ -0.8π______

PROBLEM 2:

Parts a and b can be solved independently of each other.

(a) Consider the plot below where $z = re^{j\theta}$. Based on the information provided in the plot find all possible values of $\theta \in (-\pi, \pi]$. (NOTE: Assume $jz^4 = pe^{j\varphi}$) (10 points)



$jz^4 = e^{\frac{j\pi}{2}}\left(r^4 e^{j4\theta}\right) = pe^{j0}$

$r^4 e^{j4\theta} = pe^{-j\frac{\pi}{2}+j2\pi k}$

$r = \pm\sqrt[4]{p} \rightarrow z = \pm\sqrt[4]{p}\, e^{-\frac{j\pi}{8}+\frac{j2\pi k}{4}}$

$r = \sqrt[4]{p} \rightarrow z = \sqrt[4]{p}\, e^{-\frac{j\pi}{8}+\frac{j2\pi k}{4}}$

$\rightarrow k = 0, 1, 2, 3: \theta = -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}$

(NOTE: $\theta = \frac{11\pi}{8} \rightarrow -\frac{5\pi}{8}$)

$r = -\sqrt[4]{p} \rightarrow z = -\sqrt[4]{p} e^{-\frac{j\pi}{8}+j\pi+\frac{j2\pi k}{4}} = \sqrt[4]{p} e^{\frac{j7\pi}{8}+\frac{j2\pi k}{4}}$

$\rightarrow k = 0, 1, 2, 3: \theta = \frac{7\pi}{8}, \frac{11\pi}{8}, -\frac{\pi}{8}, \frac{19\pi}{8}$

(NOTE: $\theta = \frac{19\pi}{8} \rightarrow \frac{3\pi}{8}$)

Therefore:

$\theta = -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, -\frac{5\pi}{8}$

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(b) Solve the following equation for K (express you answer in the polar form with any potential radian angles bound by $(-\pi, \pi]$). (10 points)

$$\sum_{k=28}^{39} 10e^{j\left(\frac{2\pi}{27}k\right)} + \sum_{k=41}^{54} 10e^{j\left(\frac{2\pi}{27}k\right)} = K$$

The only vector missing from this summation to make it equal 0 is $k = 40$. Therefore, the answer is:

$$K = -10e^{\frac{j2\pi}{27}(40)} = -10e^{\frac{j2\pi}{27}(13)} = 10e^{\frac{j2\pi(13)}{27}-j\pi} = 10e^{\frac{j2\pi(13)}{27}-\frac{j27\pi}{27}} = 10e^{-\frac{j\pi}{27}}$$

$K =$ ____$10e^{\frac{-j\pi}{27}}$______________

PROBLEM 3:

The following MATLAB code is used to generate a plot of a sinusoidal signal ($\boldsymbol{y(t)}$) represented by the MATLAB vector `yy`. Write a mathematical formula for the signal in standard sinusoidal form (i.e., $\boldsymbol{y(t) = A\cos(\omega_0 t + \varphi)}$ **with** $\boldsymbol{\varphi} \in (-\boldsymbol{\pi}, \boldsymbol{\pi}]$) inside the box provided and determine how many periods, $\boldsymbol{N}$**,** will be plotted by the MATLAB code (note this may not be an integer). (10 points)

```
dt = 1/1000;
tt = -0.04:dt:0.12;
Fo = 30;
yy = real(3 + 1.5*exp(j*(2*pi*Fo*(tt +0.02))));
plot( tt, yy );
axis tight;
```

$y(t) = \Re e\{3 + 1.5e^{j(2\pi 30(t+0.02))}\} = 3 + 1.5\cos(60\pi t + 1.2\pi) = 3 + 1.5\cos(60\pi t - 0.8\pi)$

$N = \frac{0.16}{\frac{1}{30}} = 4.8$ periods

$y(t) =$ ___$3 + 1.5\cos(60\pi t - 0.8\pi)$___________________ $N =$ ___4.8______________