GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
QUIZ #1

DATE: 16-Sept-13 COURSE: ECE-2026

NAME: ______________________________ GT#: ______________
LAST, FIRST ex: gtaburDEll

Circle your correct **recitation section** number - failing to do so will cost you 3 points

L01: Mon - (Juang) L02: Wed - (Bloch) L03: Mon - (Casinovi)

L04: Wed - (Bloch) L05: Tues - (Bhatti) L06: Thurs - (Coyle)

L07: Tues - (Bhatti) L08: Thurs - (Coyle) L09: Tues - (Alregib)

L10: Thurs - (Ma) L11: Tues - (Causey) L12: Thurs - (Ma)

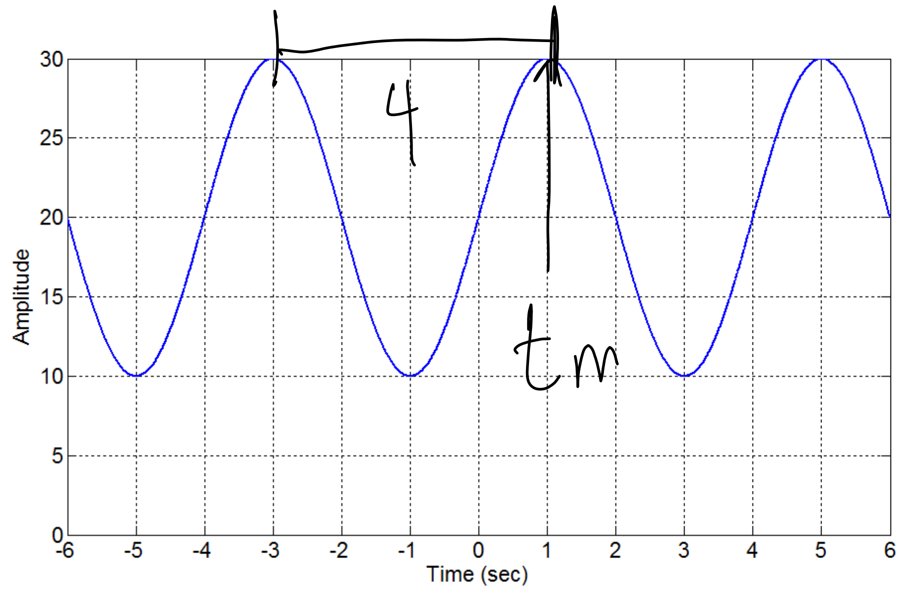
L13: Tues - (Causey) L14: Thurs - (Alregib)

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit. Explanations are also required to receive full credit for an answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. **Circle** your answers, or write them in the **boxes** provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM fa-13-Q.1.1:
The sinusoidal signal shown below is in the form:

$$x(t) = B + A\cos(\omega_0 t + \varphi)$$



(a) Determine *B*, *A*, ω_0, and φ (SHOW YOUR WORK)

B	20
A	10
ω_0	$\pi/2$
φ	$-\pi/2$

$$\omega_0 = 2\pi \cdot 1/4 = \pi/2$$

$$t_m = 1 = -\frac{\varphi}{\omega_0} \Rightarrow \varphi = -\pi/2$$

(b) Now suppose that $x(t) + K\cos(\omega_0 t) = C + \frac{20\sqrt{3}}{3}\cos(\omega_0 t + \theta)$. Determine C, K, and θ (C and K are REAL positive numbers and ω_0 is the same as in part (a)) (SHOW YOUR WORK)

C	20
K	$10\sqrt{3}/3$
θ	$-\pi/3$

$\rightarrow x(t) + K\cos(w_0 t) = C + \frac{20\sqrt{3}}{3}\cos(w_0 t + \theta)$

$\rightarrow 20 + 10\cos(\pi/2 t - \pi/2) + K\cos(\frac{\pi}{2}t) = C + \frac{20\sqrt{3}}{3}\cos(\frac{\pi}{2}t + \theta)$

→ The summation consists of a DC component and sinusoids @ $W_0 = \pi/2$.

→ C is a DC component and therefore must equal 20.

→ Complex addition can be used on the remaining sinusoids since they are at the same frequency.

$\rightarrow 10\cos(\pi/2 t - \pi/2) + K\cos(\pi/2 t) = \frac{20\sqrt{3}}{3}\cos(\pi/2 t + \theta)$

$10e^{-j\pi/2} + K = \frac{20\sqrt{3}}{3}e^{j\theta} = \frac{20\sqrt{3}}{3}\cos(\theta) + j\frac{20\sqrt{3}}{3}\sin(\theta)$

$K - j10 = \frac{20\sqrt{3}}{3}\cos(\theta) + j\frac{20\sqrt{3}}{3}\sin(\theta)$

$\therefore -10 = \frac{20\sqrt{3}}{3}\sin(\theta) \rightarrow \theta = -\pi/3$

$K = \frac{20\sqrt{3}}{3}\cos(\theta) \rightarrow K = 10\sqrt{3}/3$

(c) Now suppose that $x(t) + Z\cos(\omega_0 t + \beta) = D$. Determine D, Z, and θ (Z and D are REAL positive numbers and ω_0 is the same as in part (a)) (SHOW YOUR WORK)

D	20
Z	10
β	$\pi/2$

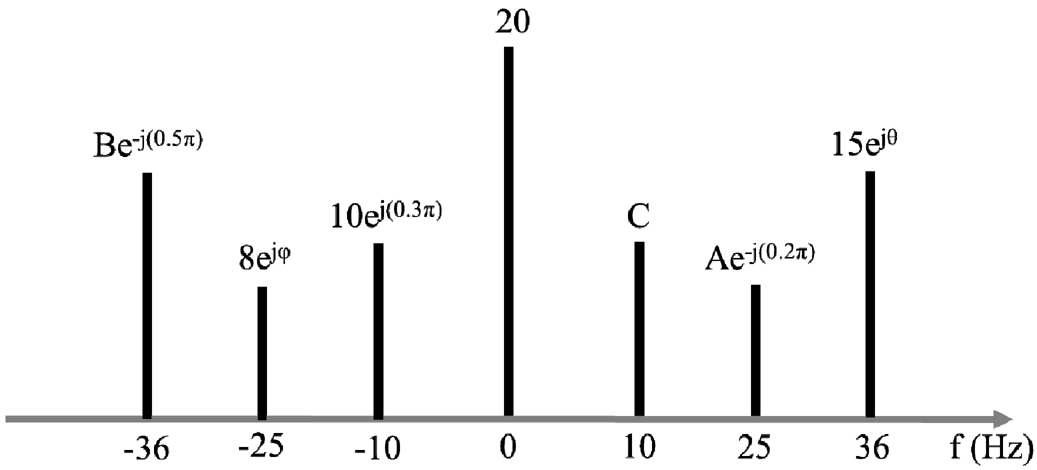
$10\cos(\frac{\pi}{2}t - \pi/2) = -Z\cos(w_0 t + \beta)$

$= Z\cos(w_0 t + \beta + \pi)$

$Z = 10$

$\beta = \pi/2$

PROBLEM fa-13-Q.1.2:
A real signal $x(t)$ has the following two-sided spectrum.



(a) Determine A, B, C, θ, and φ (Explain your reasoning)

$$x(t) = 20 + 20\cos(20\pi t - 0.3\pi) + 16\cos(50\pi t - 0.2\pi) + 30\cos(72\pi t + \pi/2)$$

A	B	C	θ	φ
8	15	$10e^{-j0.3\pi}$	0.5π	0.2π

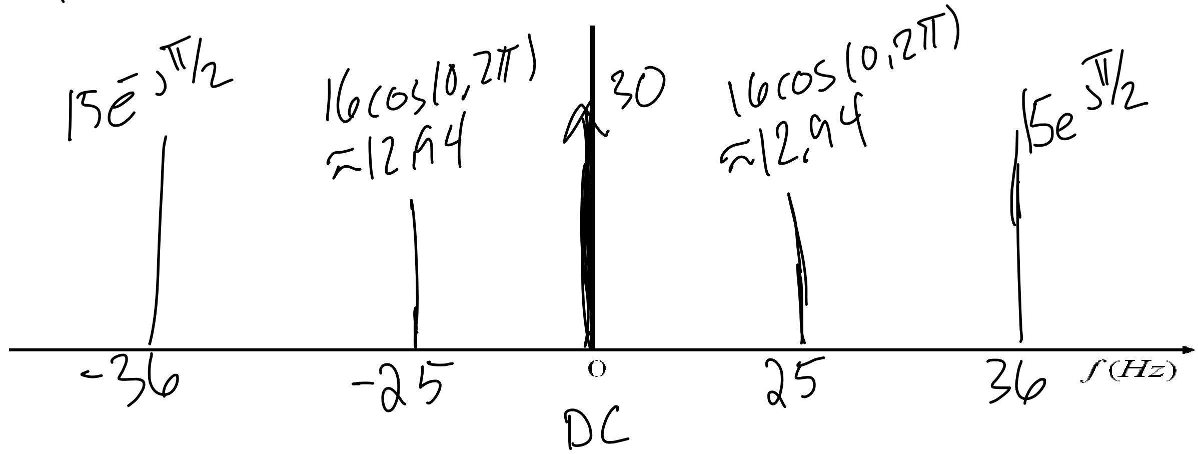
(b) Plot the spectrum for the signal $x(t) + 10 + 20\cos(20\pi t + 0.7\pi) + 16\cos(50\pi t + 0.2\pi)$ (Show your work) add components with same frequency.

DC: $20 + 10 = 30$

$W = 20\pi$: $20e^{-j0.3\pi} + 20e^{j0.7\pi} = 20(e^{-j0.3\pi} + e^{j\pi}e^{-j0.3\pi}) = 20(e^{j0.3\pi} - e^{j0.3\pi}) = 0$

$W = 50\pi$: $16e^{-j0.2\pi} + 16e^{j0.2\pi} = 16\cdot 2\cos(0.2\pi) = 32\cos(0.2\pi)$

$\Rightarrow 30 + (32\cos(0.2\pi))\cos(50\pi t) + 30\cos(72\pi t + \pi/2)$

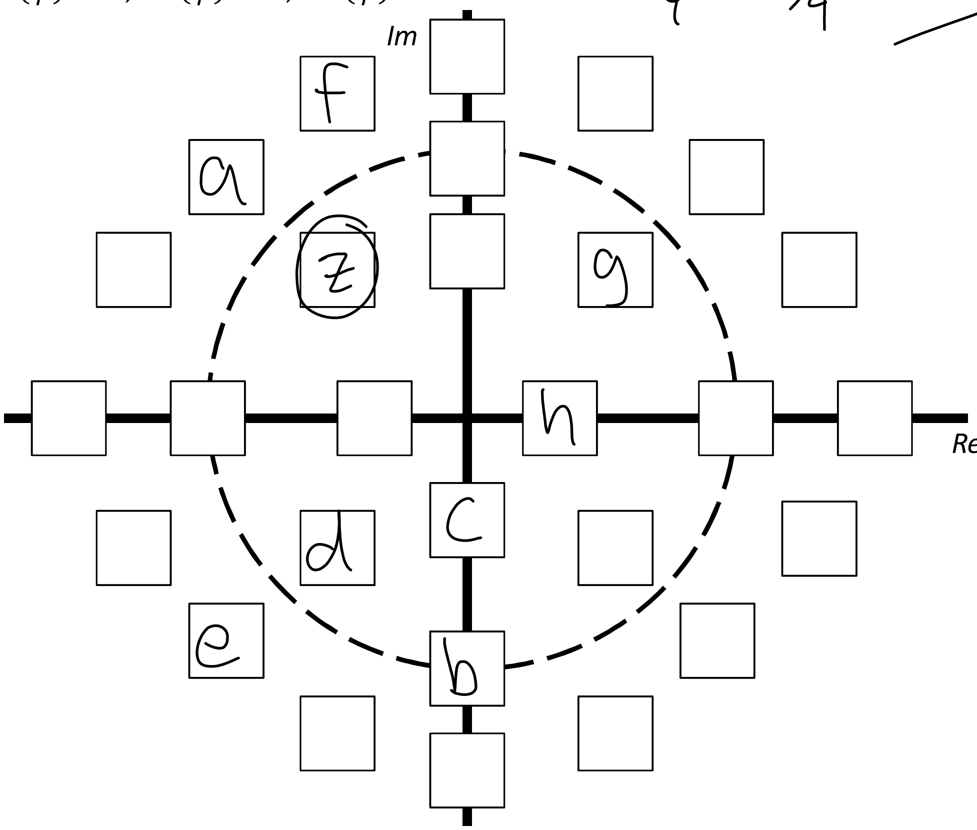


PROBLEM fa-13-Q.1.3:

Consider a complex number defined as $z = re^{j\varphi}$. It has the following properties

- $\sum_{k=0}^{\infty} r^k < \infty$
- $\cos(\varphi) < 0, \sin(\varphi) > 0, \tan(\varphi) = -1$

$r < 1$ $\quad \varphi = 3\pi/4 \quad z = re^{j3\pi/4}$



A complex plane is represented above with the dashed circle representing a radius of r=1. Match each letter below to the *best* location of each of the following operations on $z = re^{j\varphi}$. (NOTE: There are **more spaces than letters** so many spaces will be **blank**.)

$z = re^{j3\pi/4}$

(a)	$1/z^*$
(b)	z/z^*
(c)	z^2
(d)	jz
(e)	$1/z$
(f)	$z + j$
(g)	$z + 1$
(h)	zz^*

$1/re^{-j3\pi/4} \rightarrow \frac{1}{r}e^{j3\pi/4}$

$re^{j3\pi/4} \cdot \frac{1}{r}e^{j3\pi/4} = e^{j3\pi/2} = -j$

$(re^{j3\pi/4})^2 = r^2e^{j3\pi/2} = -jr^2$

$jz = e^{j\pi/2}re^{j3\pi/4} = re^{j5\pi/4} = re^{-j3\pi/4}$

$1/z = 1/r\, e^{-j3\pi/4}$

increase imag part by 1

increase Real part by 1

$r^2 < 1$

(r is a real number less than 1)

PROBLEM fa-13-Q.1.4:

Consider the following lines of MATLAB code:

```
tt =-5:0.001:5;
xxe=exp(-tt/3);
xt=0;
for k=1:99;
    xt=xt+real(xxe.*exp(j*(6*pi*tt+k*pi/25)));
end
```

(a) If the signal $x(t)$ corresponds to the MATLAB vector `xt` then it is possible to express $x(t)$ in the form

$$x(t) = \sum_{k=1}^{99} \Re e\{A_k e^{j(\omega t+\varphi_k)+\alpha t}\} = \Re e\{A e^{j(\omega t+\varphi)+\alpha t}\}$$

Find the parameters specified below. (Show your work)

$\rightarrow x(t) = \sum_{k=1}^{99} \Re e\{A_k e^{j(\omega t+\varphi_k) - \frac{1}{3}t}\} \Rightarrow e^{-\frac{1}{3}t} \sum_{k=1}^{99} \underbrace{\Re e\{e^{j\frac{\pi k}{25}} e^{j6\pi t}\}}_{\text{phasor addition}}$

$\sum_{k=1}^{99} e^{j\frac{\pi k}{25}} \rightarrow$ This represents phasors equally spaced by $\pi/25$ around a circle of radius 2π. Essentially $\varphi_k = \frac{\pi}{25}k = \frac{2\pi}{50}k$

$\rightarrow \sum_{k=1}^{50} e^{j\frac{\pi k}{25}} + \sum_{k=51}^{99} e^{j\frac{\pi k}{25}} = 2\underbrace{\sum_{k=1}^{50} e^{j\frac{\pi k}{25}}}_{=0} - e^{j\left(\frac{\pi}{25}\right)50} = -e^{-j2\pi} = e^{j\pi}$

Recall:

$\sum_{k=0}^{N-1} \alpha^k = \frac{1-\alpha^N}{1-\alpha}$

$\underbrace{\sum_{k=1}^{50} e^{j\frac{\pi k}{25}} = \sum_{k=0}^{49} e^{j\frac{\pi k}{25}}}_{\text{Both represent 50 phasors spaced by } \pi/25} = \frac{1-e^{j\frac{\pi}{25}50}}{1-e^{j\pi/25}} = \frac{1-1}{1-e^{j\pi/25}} = 0$

A	ω	φ	α
1	6π	π	$-1/3$

(b) The MATLAB variable `k` consists of the set of values {1,2,3,…,99}. If we define a signal $x_k(t)$ as

$$x_k(t) = \Re e\{A_k e^{j(\omega t+\varphi_k)+\alpha t}\}$$

find all values of `k` within the set of values {1,2,3,…,99} that result in an equivalent expression to $x_5(t)$.

$x_5(t) = e^{\alpha t}\cos\left(\omega t + \frac{5\pi}{25}\right)$ repeats every $2\pi p$ change in phase (p integer)

$= e^{\alpha t}\cos\left(\omega t + \frac{5\pi}{25} + 2\pi p\right)$

$= e^{\alpha t}\cos\left(\omega t + \frac{\pi(5+50p)}{25}\right)$

k =	55

only $p=0,1$ fit $k \le 99$ so