ECE 2026

FINAL EXAM

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

GEORGIA INSTITUTE OF TECHNOLOGY

AUGUST 3, 2026

Name: ______________________ ______________________ ______________________

FIRST LAST GTID

1. The final exam is closed book, closed notes,
   except for three double-sided handwritten sheets of notes.
2. No electronics (no watches/calculators/readers/phones/etc.)
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. A blank sheet is attached in case you need more room.
6. Only the *front* side of each page will be scanned and graded, use the back sides as scrap only.
7. Express all angles as a fraction of π. For example, write 0.1π instead of $18°$ or 0.3142 radians.

Problem	Points	Score
1	13	
2	13	
3	13	
4	13	
5	13	
6	13	
7	9	
8	13	
TOTAL:	100	

Since you do not have a calculator, this table may or may not be useful:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

PROB. Su26F.1. (The two parts of this problem are unrelated.)

(a) Let z^* denote the complex conjugate of $z = x + jy$. If $3 + 4j = 2jz + zz^*$ then the real and imaginary parts of z must be:

$x =$ $\boxed{}$,

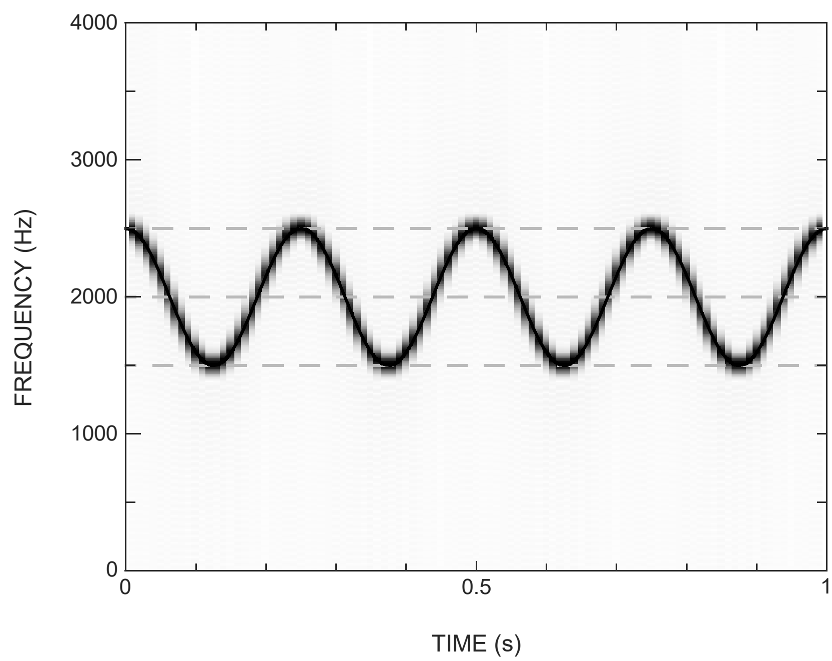
$y =$ $\boxed{}$.

(b) Specify *positive integer* values for the variables A, B, and C so that the following MATLAB code produces the plot below:

A = $\boxed{}$;

B = $\boxed{}$;

C = $\boxed{}$;



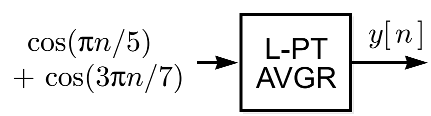

```
fs = 8000;

t = 0:1/fs:1;

x = A*cos(pi*A*B*C*t + B*sin(pi*C*t) + B);

spectrogram(x,160,[],1e4,fs,'yaxis');
```

PROB. Su26F.2.

(a) Find the *smallest* positive frequency f_0 (in Hz) for which the following equation is true for all time t:

$$\cos(2\pi f_0 t) + \cos\big(2\pi f_0 (t - \tfrac{1}{12})\big) = \sqrt{2}\cos(2\pi f_0 t - 0.25\pi) \quad \Rightarrow \quad \text{smallest } f_0 = \boxed{} \text{ Hz.}_{\,> 0}$$

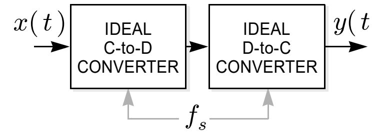
(b) Suppose the sequence $x[n] = \cos(\pi n/5) + \cos(3\pi n/7)$ is passed as an input to an L-point averager:



Find the *smallest* integer L so that both sinusoidal components are *nulled*, *i.e.*, so that the output is *zero*, $y[n] = 0$ for all n:

$$L = \boxed{}\,.$$

PROB. Su26F.3. Let $x(t) = \cos(24\pi t) + \cos(48\pi t) + \cos(84\pi t)$



be the input to an ideal sampling/reconstruction system with sampling rate f_s, as shown here, resulting in a reconstructed output $y(t)$:

(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s \geq$ $\boxed{}$ Hz.

(b) For each sampling rate f_s listed below, the reconstructed output $y(t)$ will be *periodic*. Specify the corresponding *fundamental frequency* f_0 (in Hz) of the periodic output $y(t)$. (*Hint* — all answers are integers in the range $\{1, 2, 3, \ldots 18\}$):

(a) $f_s = 84$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

(b) $f_s = 83$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

(c) $f_s = 82$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

(d) $f_s = 81$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

(e) $f_s = 80$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

(f) $f_s = 79$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

(g) $f_s = 78$ Hz $\Rightarrow$ $f_0 =$ $\boxed{}$.

PROB. Su26F.4.

Consider a filter whose impulse response is $h[n] = 2\delta[n] + 2\delta[n-2] + 6\delta[n-3]$.

(a) Find values for the following outputs when the input is $x[n] = 2\delta[n] + 2\delta[n-2] + 6\delta[n-3]$:

$y[0] = \boxed{}$,

$y[1] = \boxed{}$,

$y[2] = \boxed{}$,

$y[3] = \boxed{}$,

$y[4] = \boxed{}$,

$y[5] = \boxed{}$,

$y[6] = \boxed{}$.

(b) The filter is [FIR][IIR], and its filter coefficients are:
(circle one)

$b_0 = \boxed{}$,

$b_1 = \boxed{}$,

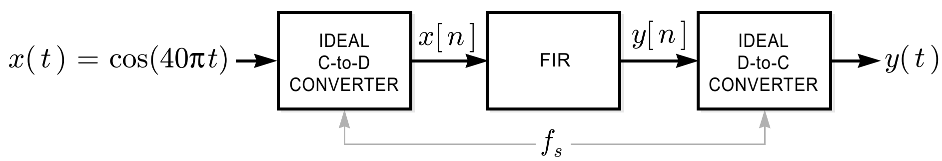
$b_2 = \boxed{}$,

$b_3 = \boxed{}$.

PROB. Su26F.5. Suppose that a sampled version $x[n]$ of the sinsusoid $x(t) = \cos(40\pi t)$ is passed through an FIR filter with difference equation:

$$y[n] = x[n] + \sqrt{3}\,x[n-1] + x[n-2],$$

and that the resulting filter output $y[n]$ is passed through an ideal D-to-C converter, resulting in a continuous-time output $y(t)$, as shown below:



Let f_s denote the same sampling rate parameter used by both the C-to-D and D-to-C converters.

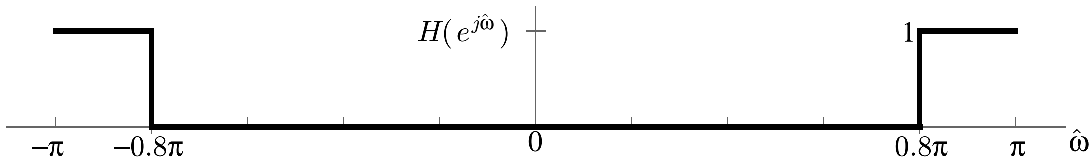
(a) Find the output $y(t)$ when the sampling rate parameter is $f_s = 20$ Hz, simplifying as much as possible:

$$y(t) = \boxed{\qquad\qquad} .$$

(b) Find the only integer-valued sampling rate parameter f_s that results in a *zero* output ($y(t) = 0$ for all t):

$$f_s = \boxed{\qquad\qquad} .$$

PROB. Su26F.6. Consider a filter whose frequency response is shown below:



(a) Circle the term that best describes this filter: [FIR][LPF][BPF][HPF][NOTCH][Dirichlet].

(b) The impulse response of this filter can be written in the following three different ways:

(i) $h[n] = \cos(\pi n)\dfrac{\sin(An)}{Bn}$, where $A =$ ☐ , $B =$ ☐ .

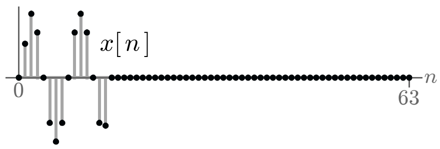
(ii) $h[n] = \delta[n] - \dfrac{\sin(Cn)}{Dn}$, where $C =$ ☐ , $D =$ ☐ .

(iii) $h[n] = \cos(0.9\pi n)\dfrac{\sin(Fn)}{Gn}$, where $F =$ ☐ , $G =$ ☐ .

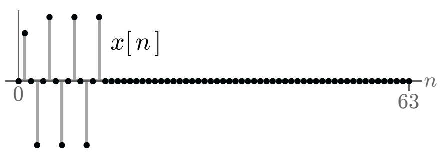
PROB. Su26F.7. Shown on the left are the stem plots of nine length-64 signal segments $\{x[0], \ldots\ x[63]\}$. Shown on the right are the stem plots of the 64-point DFT magnitudes $\{|X[0]|, \ldots\ |X[63]|\}$, labeled `A` ... `I`, but in a scrambled order. Match each signal to its corresponding DFT plot. Indicate answers by writing a letter from {`A`, `B`, ... `I`} into each answer box.

(i)

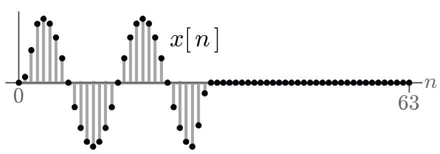
(ii)



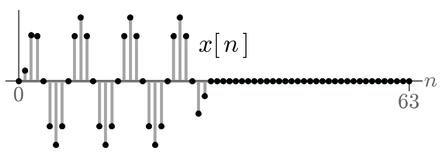
(iii)



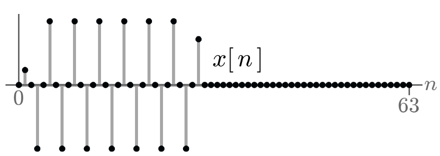
(iv)



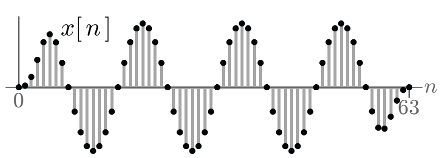
(v)



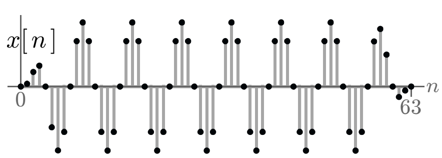
(vi)



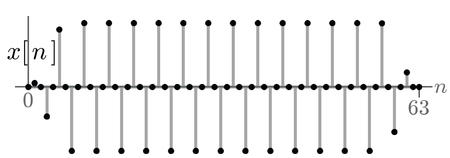
(vii)

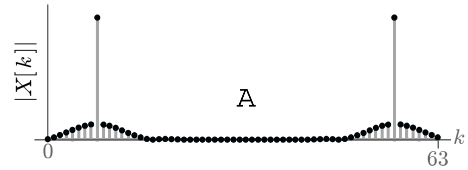


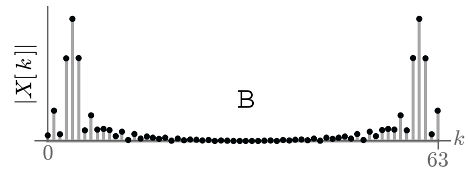
(viii)

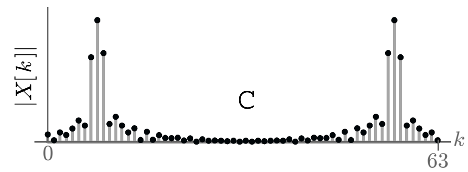


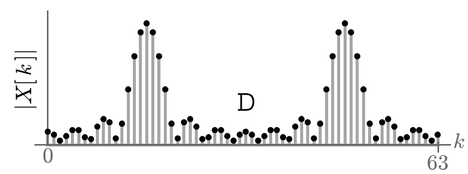
(ix)

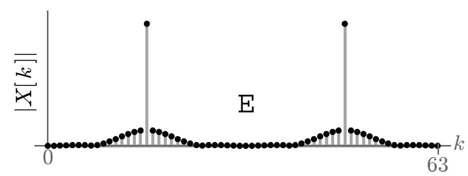


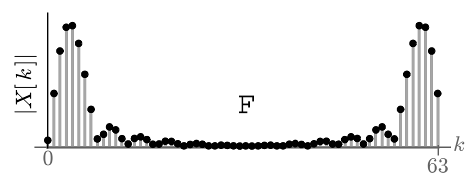


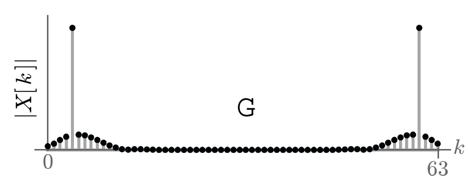


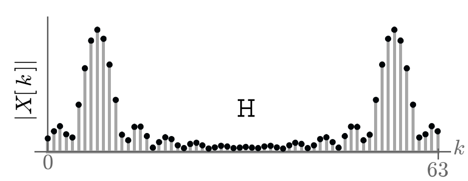


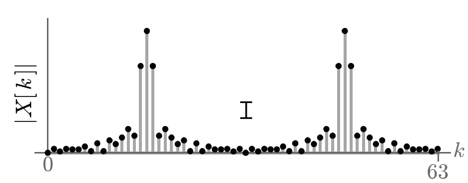




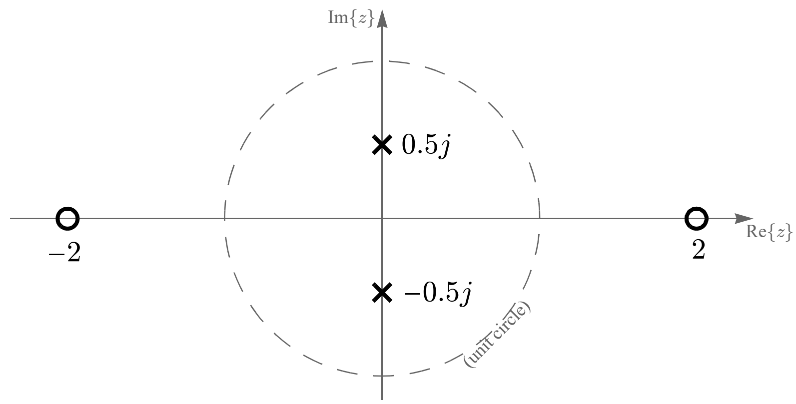








PROB. Su26F.8. Consider an LTI filter whose system function $H(z)$ has the following pole-zero plot, with zeros at ± 2 and poles at $\pm 0.5j$:



(a) The filter is [FIR][IIR].
(circle one)

(b) If the difference equation for this filter has the form $y[n] = 5x[n] + Bx[n-2] + Ay[n-2]$,

then the unspecified filter coefficients must be:

$B = \boxed{}$,

$A = \boxed{}$.

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SOLUTIONS

Name: ______________________ ______________________ ______________________
FIRST LAST GTID

1. The final exam is closed book, closed notes,
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7. Express all angles as a fraction of π. For example, write 0.1π instead of $18°$ or 0.3142 radians.

Problem	Points	Score
1	13	
2	13	
3	13	
4	13	
5	13	
6	13	
7	9	
8	13	
TOTAL:	100	

Since you do not have a calculator, this table may or may not be useful:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

PROB. Su26F.1. (The two parts of this problem are unrelated.)

(a) Let z^* denote the complex conjugate of $z = x + jy$. If $3 + 4j = 2jz + zz^*$ then the real and imaginary parts of z must be:

$x = \boxed{2}$,

$y = \boxed{1}$.

$$3 + 4j = 2j(x + jy) + x^2 + y^2$$
$$= x^2 + y^2 - 2y + 2jx$$

1. Equate imaginary parts $\Rightarrow x = 2$

2. Equate real parts $\Rightarrow 4 + y^2 - 2y = 3$

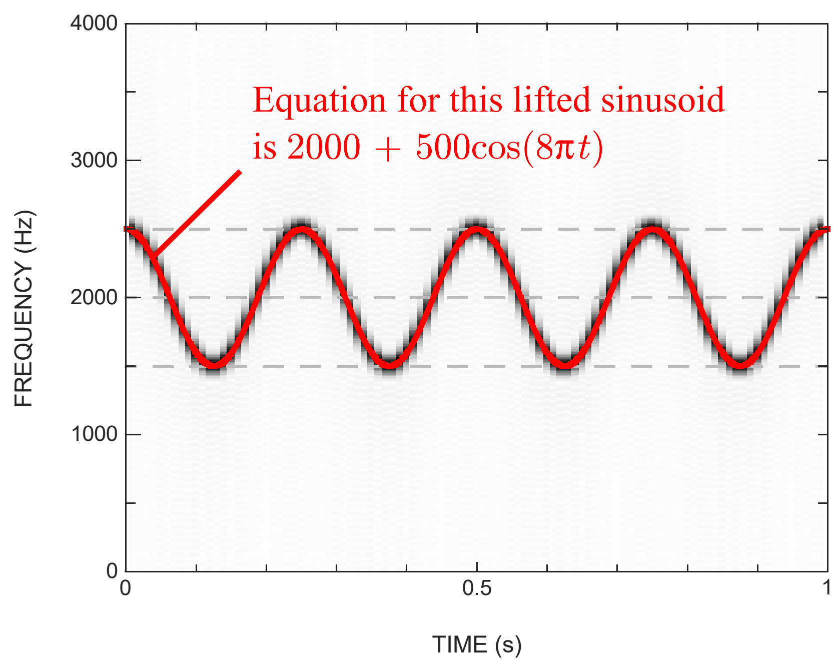
$$\Rightarrow \quad (y - 1)^2 = 0 \quad \Rightarrow \quad y = 1$$

(b) Specify *positive integer* values for the variables `A`, `B`, and `C` so that the following MATLAB code produces the plot below:

`A =` $\boxed{4}$;

`B =` $\boxed{125}$;

`C =` $\boxed{8}$;




```
fs = 8000;

t = 0:1/fs:1;

x = A*cos(pi*A*B*C*t + B*sin(pi*C*t) + B);
```

$$\Rightarrow \psi(t) = \pi ABCt + B\sin(\pi Ct) + B$$

```
spectrogram(x,160,[],1e4,fs,'yaxis');
```

Equate $2\pi(2000 + 500\cos(8\pi t)) = \frac{d}{dt}\psi(t) = \pi ABC + \pi BC\cos(\pi Ct)$

$\Rightarrow C = 8$

$\Rightarrow B = 125$

$\Rightarrow A = 4$

PROB. Su26F.2.

(a) Find the *smallest* positive frequency f_0 (in Hz) for which the following equation is true for all time t:

$$\cos(2\pi f_0 t) + \cos\big(2\pi f_0 (t - \tfrac{1}{12})\big) = \sqrt{2}\cos(2\pi f_0 t - 0.25\pi) \;\Rightarrow\; \text{smallest } f_0 = \boxed{3}\ \text{Hz.}_{> 0}$$

Corresponding phasor equation: $1 + e^{-j2\pi f_0/12} = \sqrt{2}\,e^{-j0.25\pi} \overset{\text{rectangular}}{=} 1 - j$

$$\Rightarrow e^{-j2\pi f_0/12} = -j \overset{\text{polar}}{=} e^{-j0.5\pi}$$

$$\Rightarrow 2\pi f_0/12 = 0.5\pi + \ell 2\pi$$

$$\Rightarrow f_0 = 3 + 12\ell, \quad \text{smallest when } \ell = 0$$

(b) Suppose the sequence $x[n] = \cos(\pi n/5) + \cos(3\pi n/7)$ is passed as an input to an L-point averager:

$$\cos(\pi n/5) + \cos(3\pi n/7) \longrightarrow \boxed{\text{L-PT AVGR}} \xrightarrow{y[n]}$$

Find the *smallest* integer L so that both sinusoidal components are *nulled*, *i.e.*, so that the output is *zero*, $y[n] = 0$ for all n:

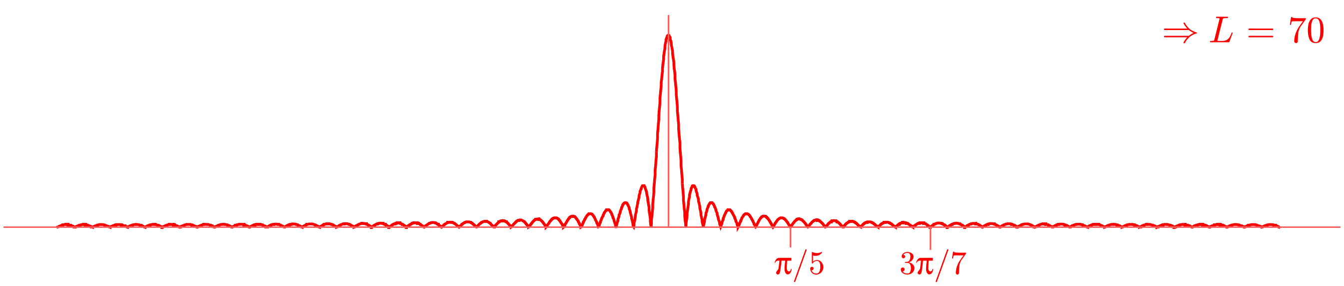
$$L = \boxed{70}.$$

To null $\pi/5$ we must have $\pi/5 = m(\frac{2\pi}{L}) \;\Rightarrow\; L = 10m \in \{10, 20, 30, 40, \ldots$

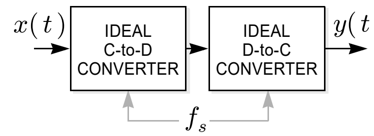
To null $3\pi/7$ we must have $3\pi/7 = n(\frac{2\pi}{10m}) \;\Rightarrow\; \frac{15}{7} = \frac{n}{m}$

Smallest m for which this is true is $m = 7$

$\Rightarrow L = 70$



PROB. Su26F.3. Let $x(t) = \cos(24\pi t) + \cos(48\pi t) + \cos(84\pi t)$ be the input to an ideal sampling/reconstruction system with sampling rate f_s, as shown here, resulting in a reconstructed output $y(t)$:



(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s \geq \boxed{2f_{\max} = 84}$ Hz.

(b) For each sampling rate f_s listed below, the reconstructed output $y(t)$ will be *periodic*. Specify the corresponding *fundamental frequency* f_0 (in Hz) of the periodic output $y(t)$. (*Hint* — all answers are integers in the range $\{1, 2, 3, \ldots 18\}$):

$\gcd\{12, 24, 42\} = 6$ (a) $f_s = 84$ Hz $\Rightarrow$ $f_0 = \boxed{6}$.

$\gcd\{12, 24, 41\} = 1$ (b) $f_s = 83$ Hz $\Rightarrow$ $f_0 = \boxed{1}$.

$\gcd\{12, 24, 40\} = 4$ (c) $f_s = 82$ Hz $\Rightarrow$ $f_0 = \boxed{4}$.

$\gcd\{12, 24, 39\} = 3$ (d) $f_s = 81$ Hz $\Rightarrow$ $f_0 = \boxed{3}$.

$\gcd\{12, 24, 38\} = 2$ (e) $f_s = 80$ Hz $\Rightarrow$ $f_0 = \boxed{2}$.

$\gcd\{12, 24, 37\} = 1$ (f) $f_s = 79$ Hz $\Rightarrow$ $f_0 = \boxed{1}$.

$\gcd\{12, 24, 36\} = 12$ (g) $f_s = 78$ Hz $\Rightarrow$ $f_0 = \boxed{12}$.

The 42-Hz component aliases to *these* frequencies

PROB. Su26F.4.

Consider a filter whose impulse response is $h[n] = 2\delta[n] + 2\delta[n-2] + 6\delta[n-3]$.

(a) Find values for the following outputs when the input is $x[n] = 2\delta[n] + 2\delta[n-2] + 6\delta[n-3]$:

Convolve $\underline{2}026$ with $\underline{2}026$:

$\underline{4}$	0	4	12			
$\underline{0}$	0	4	0	4	12	
$\underline{0}$	0	0	12	0	12	36
$\underline{4}$	0	8	24	4	24	36

$y[0] = \boxed{4}$,

$y[1] = \boxed{0}$,

$y[2] = \boxed{8}$,

$y[3] = \boxed{24}$,

$y[4] = \boxed{4}$,

$y[5] = \boxed{24}$,

$y[6] = \boxed{36}$.

(b) The filter is [FIR] [IIR], and its filter coefficients are:
(circle one)

$b_0 = \boxed{2}$,

$b_1 = \boxed{0}$,

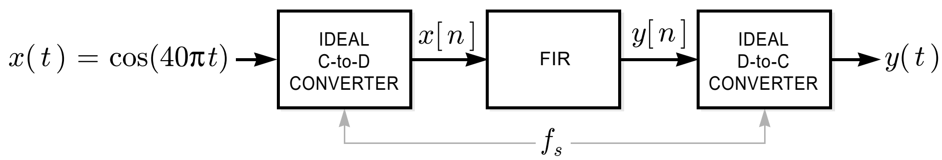
$b_2 = \boxed{2}$,

$b_3 = \boxed{6}$.

PROB. Su26F.5. Suppose that a sampled version $x[n]$ of the sinsusoid $x(t) = \cos(40\pi t)$ is passed through an FIR filter with difference equation:

$$y[n] = x[n] + \sqrt{3}\,x[n-1] + x[n-2],$$

and that the resulting filter output $y[n]$ is passed through an ideal D-to-C converter, resulting in a continuous-time output $y(t)$, as shown below:



Let f_s denote the same sampling rate parameter used by both the C-to-D and D-to-C converters.

(a) Find the output $y(t)$ when the s ampling rate parameter is $f_s = 20$ Hz, simplifying as much as possible:

$$y(t) = \boxed{\sqrt{3} + 2}.$$

$f_s = 20 \Rightarrow x[n] = \cos(40\pi \frac{n}{20}) = \cos(2\pi n) = 1$ ... constant!

$\Rightarrow y[n] = \sqrt{3} + 2$ (dc gain)

$\Rightarrow y(t) =$ same

(b) Find the only integer-valued sampling rate parameter f_s that results in a *zero* output ($y(t) = 0$ for all t):

$$f_s = \boxed{48}.$$

FIR nulling filter $[1, b_1, 1]$ nulls $\hat{\omega}_0$ when $b_1 = -2\cos(\hat{\omega}_0)$

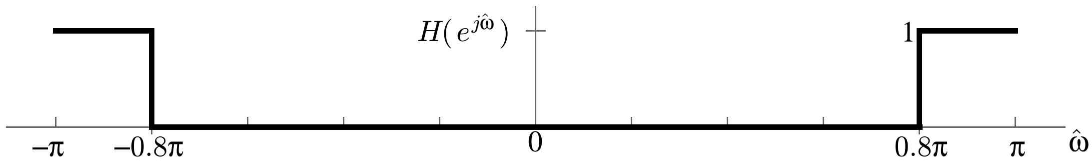
$\Rightarrow \cos(\hat{\omega}_0) = -\sqrt{3}/2$

$\Rightarrow \hat{\omega}_0 = 5\pi/6 + \ell 2\pi$ or $\hat{\omega}_0 = 7\pi/6 + \ell 2\pi$

But $\hat{\omega}_0 = \frac{40\pi}{f_s} \Rightarrow f_s = \frac{40\pi}{5\pi/6 + \ell 2\pi} = \frac{240}{5 + \ell 12} = 48$ (integer) only when $\ell = 0$

(The other option doesn't work: $f_s = \frac{40\pi}{7\pi/6 + \ell 2\pi} = \frac{240}{7 + \ell 12}$ is never an integer, for any ℓ)

PROB. Su26F.6. Consider a filter whose frequency response is shown below:



(a) Circle the term that best describes this filter: [FIR][LPF][BPF][HPF][NOTCH][Dirichlet].

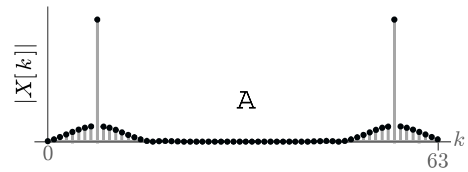
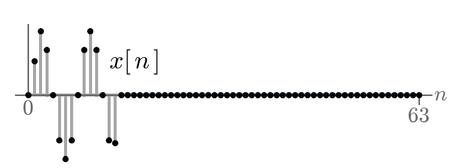
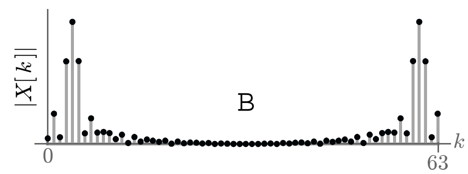
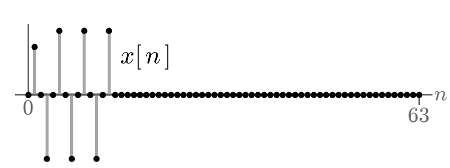
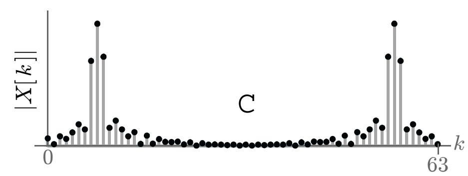
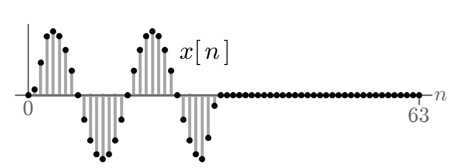
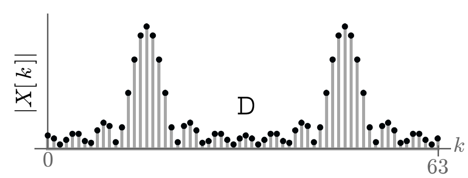
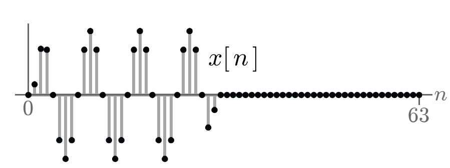
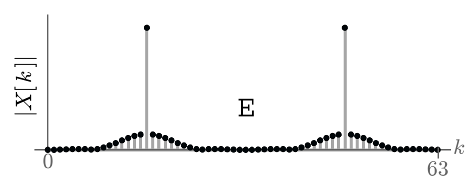
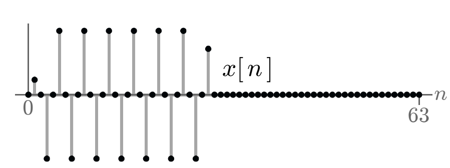
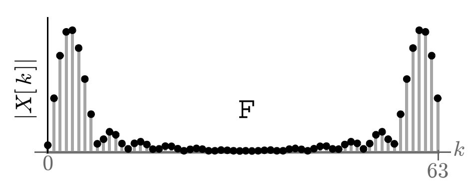
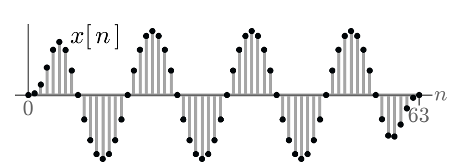
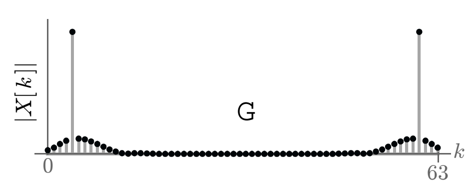
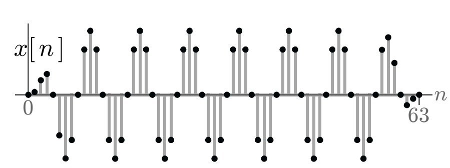
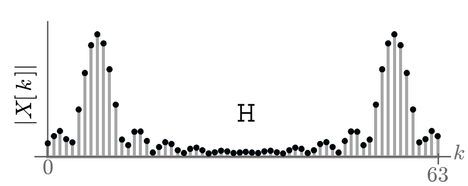
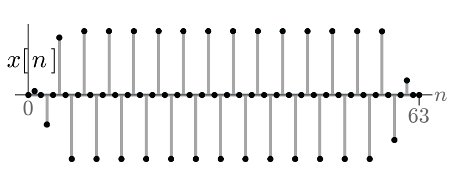
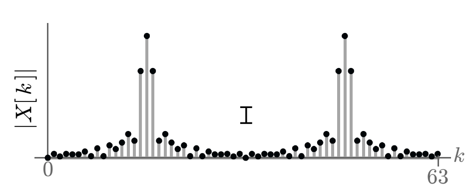
(b) The impulse response of this filter can be written in the following three different ways:

(i) $h[n] = \cos(\pi n)\dfrac{\sin(An)}{Bn}$, where $A =$ $\boxed{0.2\pi}$, $B =$ $\boxed{\pi}$.

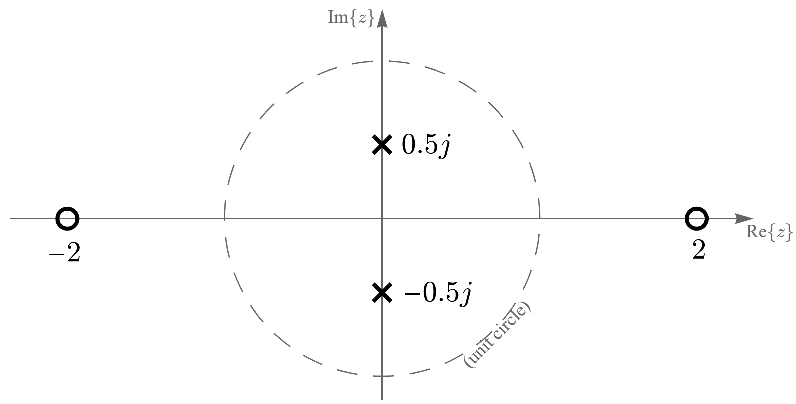
(ii) $h[n] = \delta[n] - \dfrac{\sin(Cn)}{Dn}$, where $C =$ $\boxed{0.8\pi}$, $D =$ $\boxed{\pi}$.

(iii) $h[n] = \cos(0.9\pi n)\dfrac{\sin(Fn)}{Gn}$, where $F =$ $\boxed{0.1\pi}$, $G =$ $\boxed{0.5\pi}$.

PROB. Su26F.7. Shown on the left are the stem plots of nine length-64 signal segments $\{x[0], \ \dots \ x[63]\}$. Shown on the right are the stem plots of the 64-point DFT magnitudes $\{|X[0]|, \dots |X[63]|\}$, labeled `A` ... `I`, but in a scrambled order. Match each signal to its corresponding DFT plot. Indicate answers by writing a letter from $\{$`A`, `B`, ... `I`$\}$ into each answer box.

F	(i)		
H	(ii)		
D	(iii)		
B	(iv)		
C	(v)		
I	(vi)		
G	(vii)		
A	(viii)		
E	(ix)		

PROB. Su26F.8. Consider an LTI filter whose system function $H(z)$ has the following pole-zero plot, with zeros at ± 2 and poles at $\pm 0.5j$:



(a) The filter is [FIR] [IIR]
(circle one)

poles not at origin

(b) If the difference equation for this filter has the form $y[n] = 5x[n] + Bx[n-2] + Ay[n-2]$,

then the unspecified filter coefficients must be:

$B = \boxed{-20}$,

$A = \boxed{-0.25}$.

$$H(z) = G\frac{(z+2)(z-2)}{(z+0.5j)(z-0.5j)}$$

$$= G\frac{z^2-4}{z^2+0.25}$$

$$= G\frac{1-4z^{-2}}{1+0.25z^{-2}}$$

Equate terms to solve for $A = a_2$ and $B = b_2$

$$= \frac{b_0 + b_2 z^{-2}}{1 - a_2 z^{-2}}$$

(General form of 2-nd order IIR like this)

$$= \frac{5 + Bz^{-2}}{1 - Az^{-2}}$$

But we are given $b_0 = 5 \Rightarrow G = 5$

$\Rightarrow -4G = b_2 = B = -20$

$\Rightarrow 0.25 = -a_2 = -A$