GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2024
Final Exam

July 29, 2024

NAME: ____________ (FIRST) _______________ (LAST) GT username: _______________ (e.g., gtxyz123)

Important Notes:

- Closed book, except for three double-sided pages (8.5” × 11”) of hand-written notes.
- No calculators or other electronics (no smartphones/readers/watches/tablets/laptops/etc.)
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
- Do not write on the backs of pages, only the fronts will be graded.

Problem	Points	Score
1	10	
2	10	
3	10	
4	15	
5	10	
6	15	
7	10	
8	10	
9	10	
TOTAL:	100	

θ	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

PROB. Su24-F.1. (The two parts of this problem are unrelated.)

(a) Find values for $A > 0$, $F > 0$, and $t_0 \in [0,\ 0.01)$ so that:

$$x(t) = A\cos(120\pi(t - t_0)) + 26\cos(2\pi F t)\cos(86\pi t)$$

is periodic with a fundamental frequency of 26 Hz.

$A =$ $\boxed{}_{>0}$,

$F =$ $\boxed{}_{>0}$ Hz,

$t_0 =$ $\boxed{}_{\in[0,\ 0.01)}$ sec.

(b) Find the smallest positive integer M and the corresponding value of B so that the following is true for all time t:

$$B\sin(\pi t) = \sum_{k=0}^{M} \cos\left(\pi\left(t - \frac{k}{4}\right)\right).$$

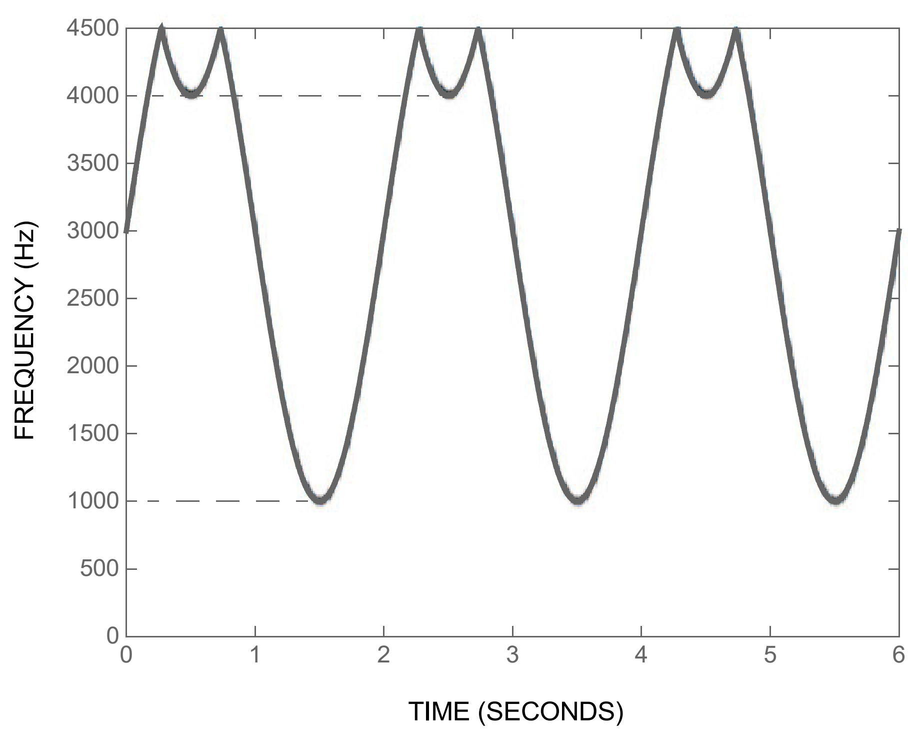
$M =$ $\boxed{}_{>0}$,

$B =$ $\boxed{}$.

PROB. Su24-F.2. Consider the MATLAB code:

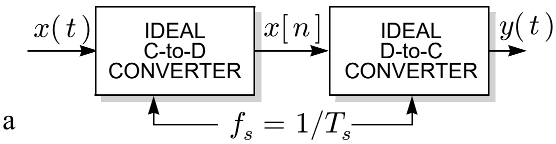
```
fsamp = [      ];
    T = [      ];
    D = [      ];
    E = [      ];
    F = [      ];
t = 0:(1/fsamp):T;
x = T*sin(D*t + E*cos(2*pi*F*t) + T);
spectrogram(x,300,[],1e4,fsamp,'yaxis');
```

Specify numerical values for the unspecified parameters so that running the above code produces the following spectrogram:



PROB. Su24-F.3. Consider the signal $x(t)$ whose spectrum is shown below:

Suppose we sample this signal with sampling rate f_s, and then feed the samples to an ideal D-to-C converter (with the same f_s parameter), producing a continuous-time output $y(t)$:



(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s >$ ☐ Hz.

(b) The largest sampling rate for which $y(t)$ is a *constant* is $f_s =$ ☐ Hz.

(c) When $f_s = 5$ Hz, the output has the form $y(t) = B + A\cos(2\pi f_0 t + \varphi)$ where (in standard form):

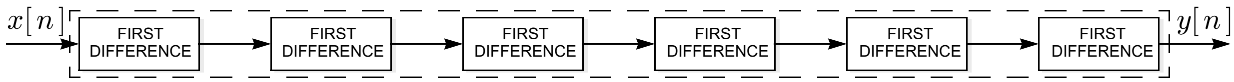
$B =$ ☐ ,

$A =$ ☐ > 0 ,

$f_0 =$ ☐ > 0 ,

$\varphi =$ ☐ $\in(-\pi, \pi]$,

PROB. Su24-F.4. Consider the following cascade of six first-difference filters:



(a) The dc gain of the *overall* system (indicated by the dashed box) is $\boxed{}$.

(b) Which of the following best describes the *overall* filter? [LPF][HPF][BPF][NOTCH][ALL-PASS].
(circle one)

(c) The impulse response of the *overall* system (indicated by the dashed box) satisfies:

$h[0] = \boxed{}$,

$h[1] = \boxed{}$,

$h[2] = \boxed{}$,

$h[3] = \boxed{}$,

$h[4] = \boxed{}$,

$h[5] = \boxed{}$,

$h[6] = \boxed{}$.

(d) If the output satisfies satisfies $y[n] = 0$ for $n < 0$ and $y[0] = y[1] = y[2] = 1$, then the *input* at time 2 must be:

$x[2] = \boxed{}$.

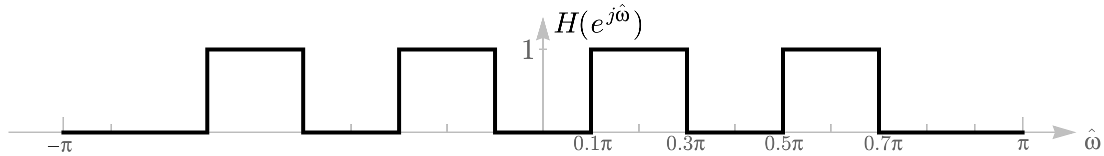
PROB. Su24-F.5. Consider an LTI filter defined by the difference equation:

$$y[n] = x[n] + x[n-1] + \beta x[n-2] + x[n-3] + x[n-4].$$

If the output in response to the sum of sinusoids $x[n] = \cos(0.5\pi n) + \cos(2\pi n/3)$ is $y[n] = 0$ (for all n), then it must be that:

$$\beta = \boxed{}.$$

PROB. Su24-F.6. Shown below is the real-valued frequency response of an LTI filter:



(a) TRUE FALSE ☐ ☐ The filter is FIR.

(b) Give an expression for the filter output $y[n]$ when the filter input is $x[n] = (\cos(0.2\pi n))^2$:

$y[n] =$ ☐ .

(simplify as much as possible)

Specify numerical values for the constants $\{A,\ B,\ ...\ Q\}$ so that the impulse response $h[n]$ can be written in any of the following four different ways:

(c) $h[n] = A\dfrac{\sin(B\pi n)}{\pi n}\big(\cos(C\pi n) + \cos(D\pi n)\big)$: $A =$ ☐ , $B =$ ☐ , $C =$ ☐ , $D =$ ☐ ,

(d) $h[n] = E\dfrac{\sin(F\pi n)}{\pi n}\cos(0.2\pi n)\cos(0.4\pi n)$: $E =$ ☐ , $F =$ ☐ ,

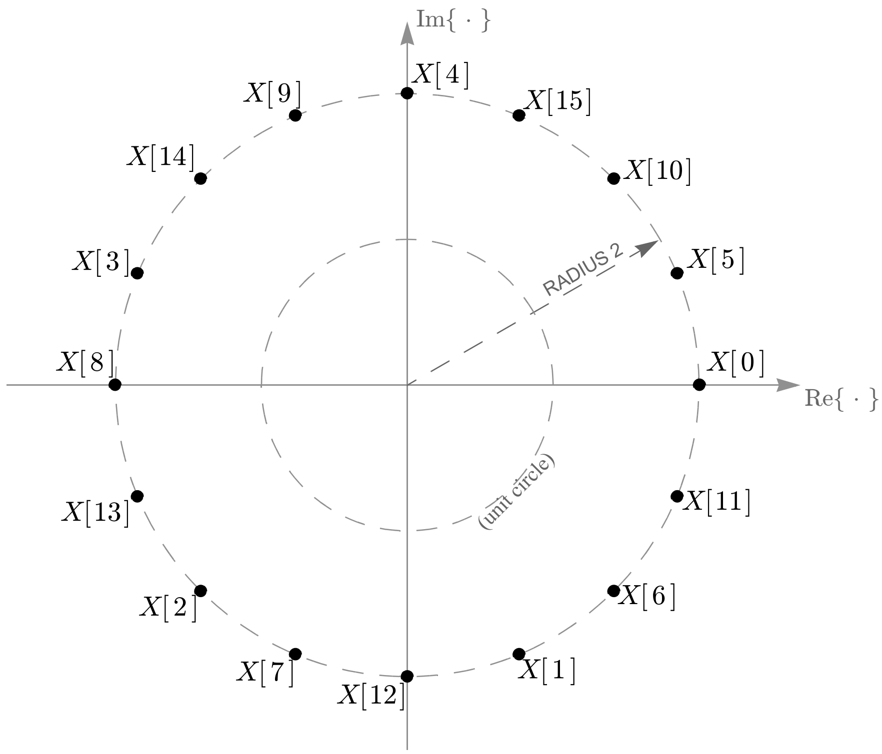
(e) $h[n] = \dfrac{\sin(G\pi n)}{\pi n} + \dfrac{\sin(H\pi n)}{\pi n} - \dfrac{\sin(J\pi n)}{\pi n} - \dfrac{\sin(K\pi n)}{\pi n}$: $G =$ ☐ , $H =$ ☐ , $J =$ ☐ , $K =$ ☐ ,

(f) $h[n] = L\Big(\dfrac{\sin(P\pi n)}{\pi n} - \dfrac{\sin(Q\pi n)}{\pi n}\Big)\cos(0.4\pi n)$: $L =$ ☐ , $P =$ ☐ , $Q =$ ☐ .

PROB. Su24-F.7. Let $\{X[0], X[1], \ldots X[15]\}$ be the 16-point DFT of a length-four signal segment $\{x[0], x[1], x[2], x[3]\}$.

Find $x[0]$ through $x[3]$ if the locations of the DFT coefficients in the complex plane are as indicated below, all on a circle of radius 2, satisfying $X[k]^{16} = 2^{16}$ for all k:

Hint: Direct brute-force calculation not recommended without a calculator!



Hint: All are real, all are integers.

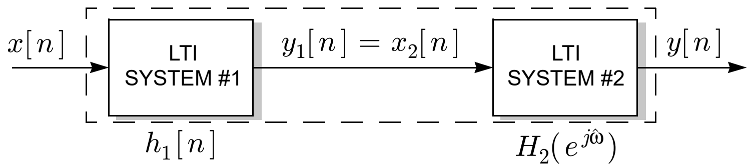
$x[0] =$ ______ ,

$x[1] =$ ______ ,

$x[2] =$ ______ ,

$x[3] =$ ______ .

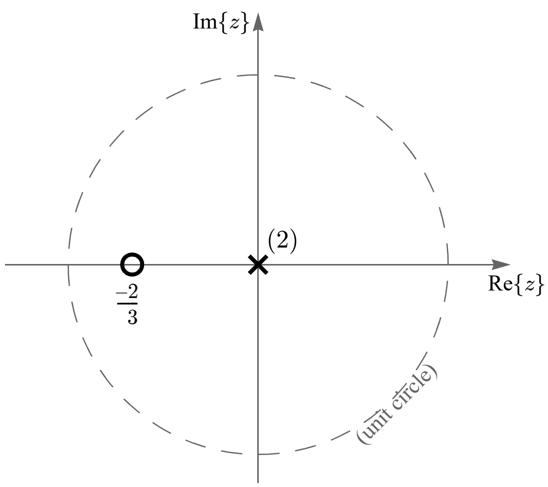
PROB. Su24-F.8. Consider the following serial cascade of a pair of LTI systems:



- The first system has impulse response $h_1[n] = 0.2^{n-1}u[n-1]$.
- The second system has frequency response $H_2(e^{j\hat{\omega}}) = 15 + Ae^{-j\hat{\omega}} + Be^{-2j\hat{\omega}}$, where A and B are real but otherwise unspecified.

Let $h[n]$ denote the impulse response of the *overall* system (dashed box), so that its Z transform $H(z)$ is the *overall* system function.

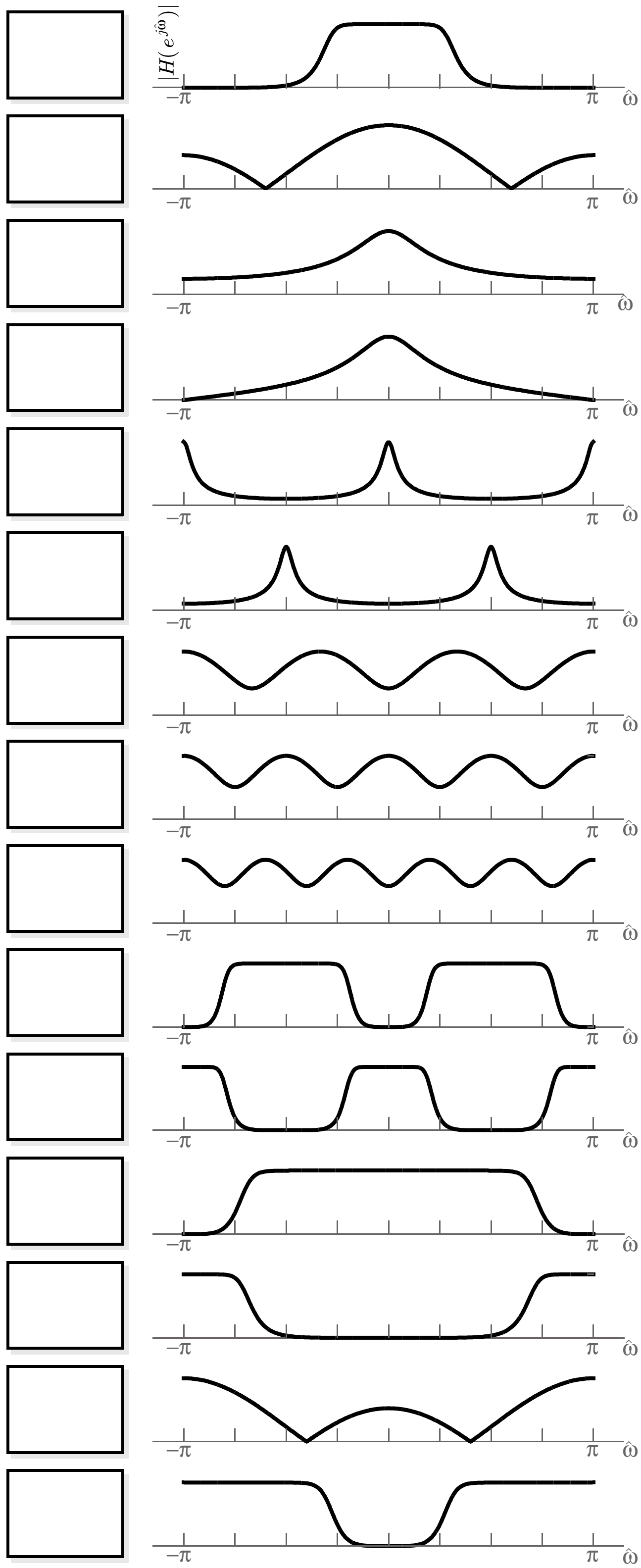
Find A and B so that the pole-zero plot for $H(z)$ is as shown below, with two poles at the origin and a single zero at $-2/3$:



$A =$ ______ ,

$B =$ ______ .

PROB. Su24-F.9. Shown on the right are the pole-zero plots for 15 LTI systems, labeled `A` through `P`. Shown on the left are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (`A` through `P`) in each answer box.



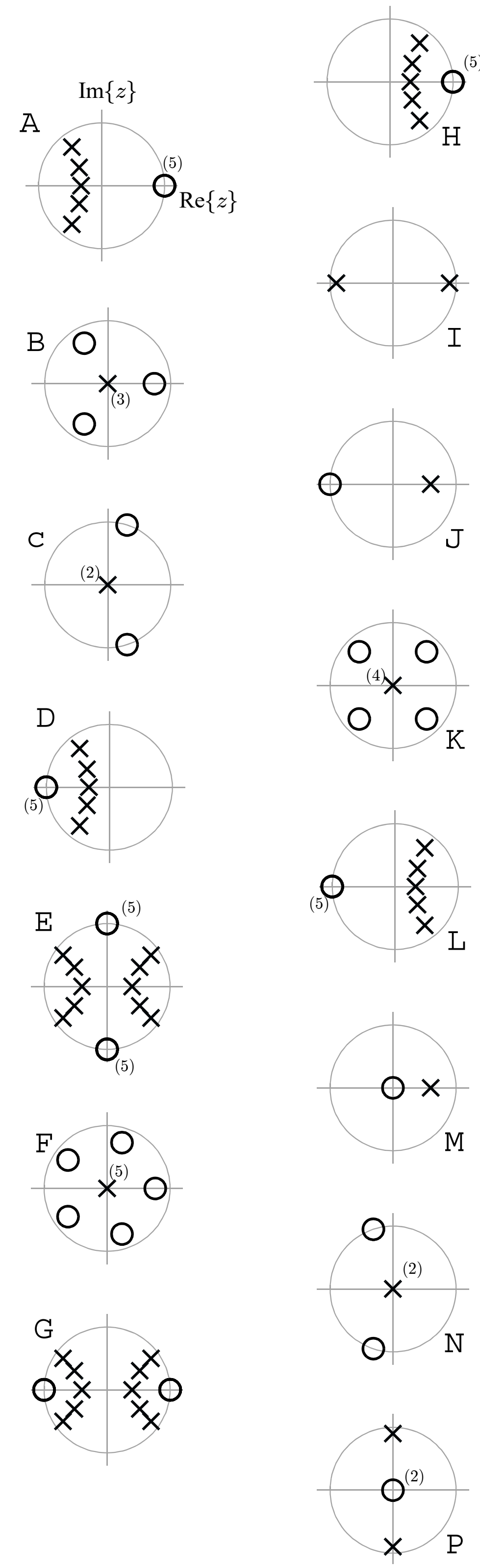


Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$u[n]-u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}e^{-j\hat{\omega}(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b)-u(\hat{\omega}-\hat{\omega}_b)=\begin{cases}1 & \lvert\hat{\omega}\rvert\le\hat{\omega}_b\\ 0 & \hat{\omega}_b<\lvert\hat{\omega}\rvert\le\pi\end{cases}$
$a^n u[n] \quad (\lvert a\rvert<1)$	$\dfrac{1}{1-ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)})=X(e^{j\hat{\omega}})$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(e^{j\hat{\omega}})+bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}})=X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (d=integer)	$x[n-d]$	$e^{-j\hat{\omega}d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)})+\frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n]*h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2$	$\displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n = 0, 1, 2, \ldots, N-1$	*Frequency-Domain:* $X[k], \quad k = 0, 1, 2, \ldots, N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\rvert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{-j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$u[n]-u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))} e^{-j(2\pi k/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))} e^{j(2\pi n/N)(L-1)/2}$	$N(u[k]-u[k-L])$, when $L \leq N$

Table of z-Transform Pairs		
Signal Name	*Time-Domain:* $x[n]$	*z-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}, \quad \lvert a\rvert < 1$
Decaying cosine	$r^n \cos(\hat{\omega}_0 n) u[n]$	$\dfrac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$
Decaying sinusoid	$Ar^n \cos(\hat{\omega}_0 n+\varphi) u[n]$	$A\dfrac{\cos(\varphi)-r\cos(\hat{\omega}_0-\varphi)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties		
Property Name	*Time-Domain* $x[n]$	*z-Domain* $X(z)$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay (d=integer)	$x[n-d]$	$z^{-d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

Date: 28-April-2013

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NAME: ANSWER KEY ______ ______ GT username: ______
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4	15	
5	10	
6	15	
7	10	
8	10	
9	10	
TOTAL:	100	

θ	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

PROB. Su24-F.1. (The two parts of this problem are unrelated.)

(a) Find values for $A > 0$, $F > 0$, and $t_0 \in [0,\, 0.01)$ so that:

$$x(t) = A\cos(120\pi(t - t_0)) + 26\cos(2\pi F t)\cos(86\pi t)$$

is periodic with a fundamental frequency of 26 Hz.

$$x(t) = A\cos(2\pi(60)t - 120\pi t_0) + 13\cos(2\pi(43 - F)t)$$

$$+\, 13\cos(2\pi(43 + F)t)$$

cancel when $F = 17$ and $A = 13$ and $120\pi t_0 = \pi$,
leaving only a 26-Hz sinusoid

$A = \boxed{13}_{>0}$,

$F = \boxed{17}_{>0}$ Hz,

$t_0 = \boxed{\frac{1}{120}}_{\in[0,\, 0.01)}$ sec.

(b) Find the smallest positive integer M and the corresponding value of B so that the following is true for all time t:

$$B\sin(\pi t) = \sum_{k=0}^{M} \cos\left(\pi\left(t - \tfrac{k}{4}\right)\right).$$

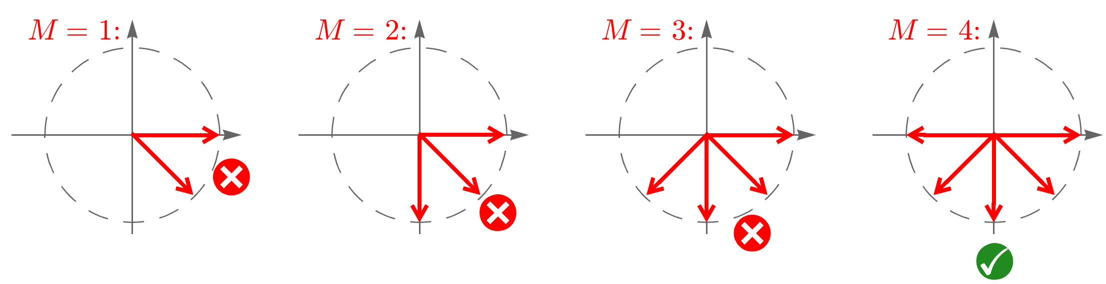
$M = \boxed{4}_{>0}$,

$B = \boxed{1 + \sqrt{2}}$.

Corresponding phasor equation:

$$-jB = \sum_{k=0}^{M} e^{-jk0.25\pi}$$

Since the LHS phasor for $-jB$ points *straight down*, we need $M = 4$ in order for the RHS sum of phasors to also point straight down:



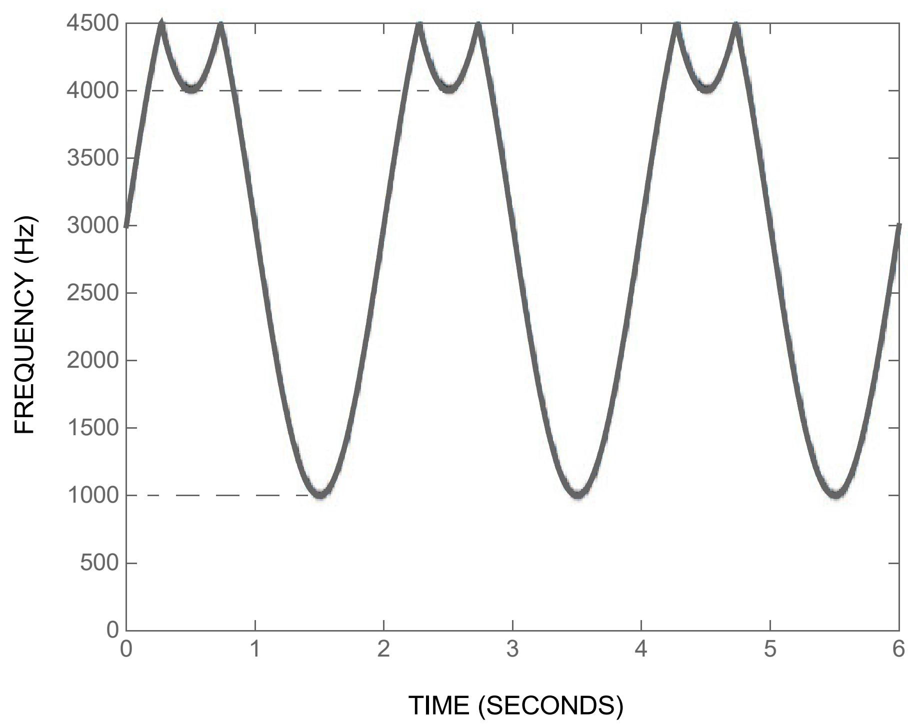
$$\Rightarrow -jB = 1 + \frac{1}{\sqrt{2}}(1 - j) - j + \frac{1}{\sqrt{2}}(-1 - j) - 1$$

$$= -(1 + \sqrt{2})j$$

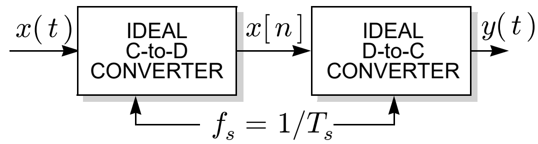
PROB. Su24-F.2. Consider the MATLAB code:

```
fsamp = 9000 ;
T = 6 ;
D = 6000π ;
E = −4000 ;
F = 0.5 ;
t = 0:(1/fsamp):T;
x = T*sin(D*t + E*cos(2*pi*F*t) + T);
spectrogram(x,300,[],1e4,fsamp,'yaxis');
```

Specify numerical values for the unspecified parameters so that running the above code produces the following spectrogram:



PROB. Su24-F.3. Consider the signal $x(t)$ whose spectrum is shown below:



Suppose we sample this signal with sampling rate f_s, and then feed the samples to an ideal D-to-C converter (with the same f_s parameter), producing a continuous-time output $y(t)$:

(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s >$ $\boxed{34}$ Hz.

$$f_{\text{max}} = 17 \text{ Hz}$$

(b) The largest sampling rate for which $y(t)$ is a *constant* is $f_s =$ $\boxed{1}$ Hz.

$$x\left(\frac{n}{f_s}\right) = \textit{constant} \Rightarrow \frac{1}{f_s} = \text{a period} = \frac{m}{f_0}$$

$$\Rightarrow \textit{largest } f_s \text{ is fundamental } f_s = f_0 = \gcd\{12,\ 15,\ 17\} = 1$$

(c) When $f_s = 5$ Hz, the output has the form $y(t) = B + A\cos(2\pi f_0 t + \varphi)$ where (in standard form):

$B =$ $\boxed{2}$,

$A =$ $\boxed{4}$ $_{>0}$,

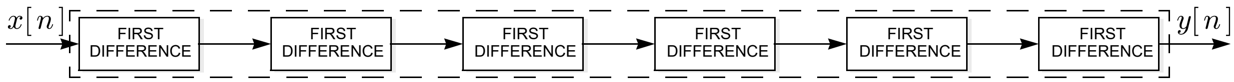
$f_0 =$ $\boxed{2}$ $_{>0}$,

$\varphi =$ $\boxed{0}$ $_{\in(-\pi,\ \pi]}$,

$$y(t) = 2\cos(2\pi(12 - (2)(5))t) + 2\cos(2\pi(15 - (3)(5))t) + 2\cos(2\pi(17 - (3)(5))t)$$

$$= 4\cos(2\pi(2)t) + 2$$

PROB. Su24-F.4. Consider the following cascade of six first-difference filters:



(a) The dc gain of the *overall* system (indicated by the dashed box) is $\boxed{0}$.

(b) Which of the following best describes the *overall* filter? [LPF][HPF][BPF][NOTCH][ALL-PASS].
(circle one)

(c) The impulse response of the *overall* system (indicated by the dashed box) satisfies:

```
1  -1
   -1   1
-----------
1  -2   1
   -1   2  -1
---------------
1  -3   3  -1
   -1   3  -3   1
--------------------
1  -4   6  -4   1
   -1   4  -6   4  -1
------------------------
1  -5  10 -10   5  -1
   -1   5 -10  10  -5   1
----------------------------
1  -6  15 -20  15  -6   1
```

$h[0] = \boxed{1}$,

$h[1] = \boxed{-6}$,

$h[2] = \boxed{15}$,

$h[3] = \boxed{-20}$,

$h[4] = \boxed{15}$,

$h[5] = \boxed{-6}$,

$h[6] = \boxed{1}$.

(d) If the output satisfies satisfies $y[n] = 0$ for $n < 0$ and $y[0] = y[1] = y[2] = 1$, then the *input* at time 2 must be:

Convolving $\{x[0]\ x[1]\ x[2]\ \ldots\} = \{a\ b\ c\ \ldots\}$ with $h[n]$ yields $\{1\ 1\ 1\ \ldots\}$:

$x[2] = \boxed{28}$.

$$
\begin{array}{r|ccc}
a\,(& 1 & -6 & 15 \\
b\,(& & 1 & -6 \\
c\,(& & & 1 \\
\hline
 & 1 & 1 & 1
\end{array}
$$

$\Rightarrow a = 1$

$\Rightarrow 1 = b - 6a$

$\Rightarrow b = 7$

$\Rightarrow 1 = c - 6b + 15a$

$= c - 42 + 15 \quad \Rightarrow c = 28$

PROB. Su24-F.5. Consider an LTI filter defined by the difference equation:

$$y[n] = x[n] + x[n-1] + \beta x[n-2] + x[n-3] + x[n-4].$$

If the output in response to the sum of sinusoids $x[n] = \cos(0.5\pi n) + \cos(2\pi n/3)$ is $y[n] = 0$ (for all n), then it must be that:

$$\beta = \boxed{2}.$$

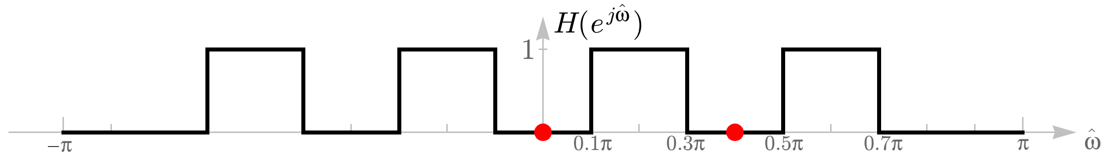
Null 0.5π $\Rightarrow$ $[\,1 \quad -2\cos(0.5\pi) \quad 1\,] = [\,1 \quad 0 \quad 1\,]$

Null $2\pi/3$ $\Rightarrow$ $[\,1 \quad -2\cos(2\pi/3) \quad 1\,] = [\,1 \quad 1 \quad 1\,]$

Null both $\Rightarrow$ convolve:

$$\begin{array}{ccccc} 1 & 0 & 1 & & \\ & 1 & 0 & 1 & \\ & & 1 & 0 & 1 \\ \hline 1 & 1 & 2 & 1 & 1 \end{array}$$

PROB. Su24-F.6. Shown below is the real-valued frequency response of an LTI filter:



(a) TRUE ☐ FALSE ☒ The filter is FIR.

Both dc and 0.4π are nulled: $0.5 + 0.5\cos(0.4\pi n)$

(b) Give an expression for the filter output $y[n]$ when the filter input is $x[n] = (\cos(0.2\pi n))^2$:

$y[n] =$ $\boxed{0}$.

(simplify as much as possible)

Specify numerical values for the constants $\{A,\ B,\ \dots\ Q\}$ so that the impulse response $h[n]$ can be written in any of the following four different ways:

(c) $h[n] = A\dfrac{\sin(B\pi n)}{\pi n}\big(\cos(C\pi n) + \cos(D\pi n)\big)$: $A = \boxed{2}$, $B = \boxed{0.1}$, $C = \boxed{0.2}$, $D = \boxed{0.6}$,

(SWAP OK)

(d) $h[n] = E\dfrac{\sin(F\pi n)}{\pi n}\cos(0.2\pi n)\cos(0.4\pi n)$: $E = \boxed{4}$, $F = \boxed{0.1}$,

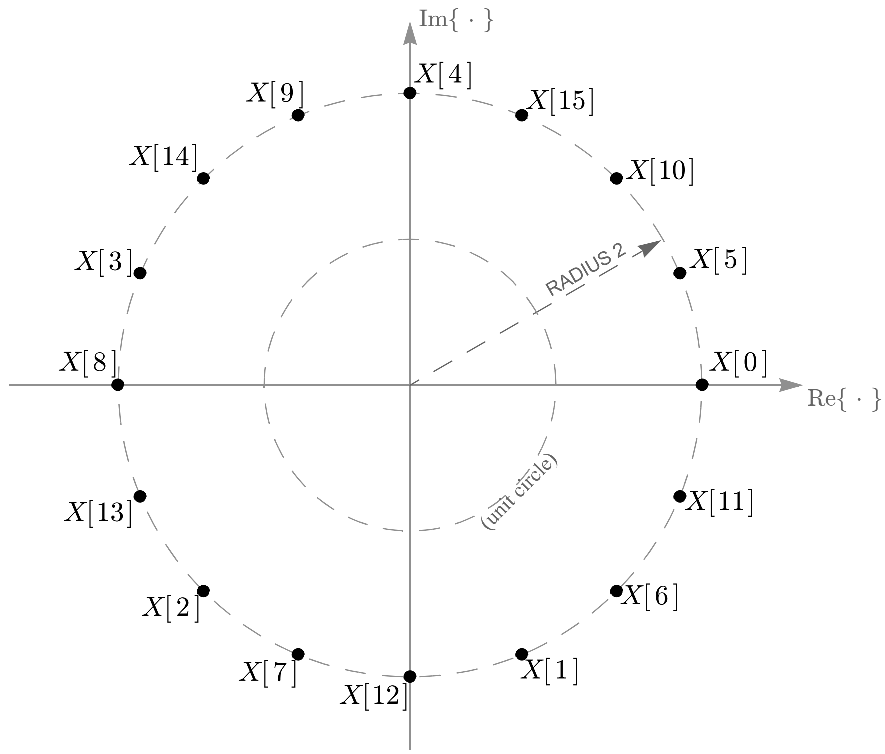
(e) $h[n] = \dfrac{\sin(G\pi n)}{\pi n} + \dfrac{\sin(H\pi n)}{\pi n} - \dfrac{\sin(J\pi n)}{\pi n} - \dfrac{\sin(K\pi n)}{\pi n}$: $G = \boxed{0.7}$, $H = \boxed{0.3}$, $J = \boxed{0.5}$, $K = \boxed{0.1}$,

(f) $h[n] = L\Big(\dfrac{\sin(P\pi n)}{\pi n} - \dfrac{\sin(Q\pi n)}{\pi n}\Big)\cos(0.4\pi n)$: $L = \boxed{2}$, $P = \boxed{0.3}$, $Q = \boxed{0.1}$.

PROB. Su24-F.7. Let $\{X[0], X[1], \ldots X[15]\}$ be the 16-point DFT of a length-four signal segment $\{x[0], x[1], x[2], x[3]\}$.

Find $x[0]$ through $x[3]$ if the locations of the DFT coefficients in the complex plane are as indicated below, all on a circle of radius 2, satisfying $X[k]^{16} = 2^{16}$ for all k:

Hint: Direct brute-force calculation not recommended without a calculator!



$$X[k] = 2e^{-j(2\pi k/N)(3)} \text{ where } N = 16$$

Hint: All are real, all are integers.

From DFT table:

$\delta[n - n_0]$	$e^{-j(2\pi k/N)n_0}$

$$\Rightarrow x[n] = 2\delta[n-3]$$

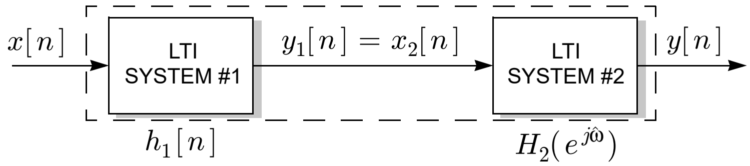
$x[0] = \boxed{0}$,

$x[1] = \boxed{0}$,

$x[2] = \boxed{0}$,

$x[3] = \boxed{2}$.

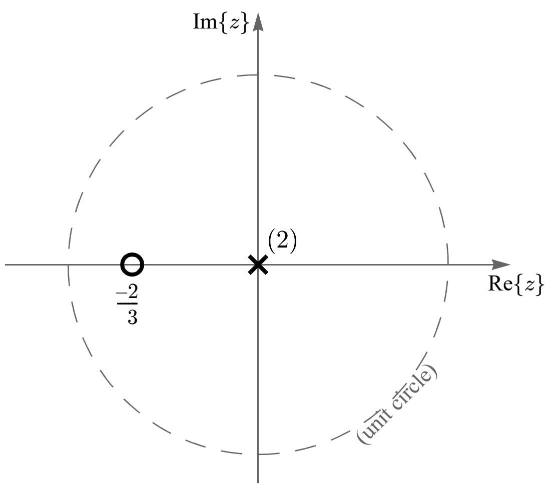
PROB. Su24-F.8. Consider the following serial cascade of a pair of LTI systems:



- The first system has impulse response $h_1[n] = 0.2^{n-1}u[n-1]$. $\Rightarrow H_1(z) = \frac{z^{-1}}{1-0.2z^{-1}}$
- The second system has frequency response $H_2(e^{j\hat{\omega}}) = 15 + Ae^{-j\hat{\omega}} + Be^{-2j\hat{\omega}}$, where A and B are real but otherwise unspecified. $\Rightarrow H_2(z) = 15 + Az^{-1} + Bz^{-2}$

Let $h[n]$ denote the impulse response of the *overall* system (dashed box), so that its Z transform $H(z)$ is the *overall* system function.

Find A and B so that the pole-zero plot for $H(z)$ is as shown below, with two poles at the origin and a single zero at $-2/3$:



$\Rightarrow H(z) = G\left(\frac{z+2/3}{z^2}\right)$

But $H(z) = H_1(z)H_2(z)$

$$\Rightarrow G\left(\frac{z+2/3}{z^2}\right) = \frac{z^{-1}}{1-0.2z^{-1}}(15 + Az^{-1} + Bz^{-2})$$

$$\Rightarrow 15z^2 + Az + B = G(z + 2/3)(z - 0.2)$$
$$= 15(z + 2/3)(z - 0.2)$$
$$= 15z^2 + 7z - 2$$

$A = \boxed{7}$,

$B = \boxed{-2}$.

PROB. Su24-F.9. Shown on the right are the pole-zero plots for 15 LTI systems, labeled `A` through `P`. Shown on the left are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (`A` through `P`) in each answer box.