GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2023
Final Exam

July 28, 2023

NAME: ____________ ________________ GT username: ______________
(FIRST) (LAST) (e.g., gtxyz123)

Important Notes:

- Do not unstaple the test.
- Closed book, except for three (3) two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Points	Score
1	15	
2	15	
3	18	
4	10	
5	15	
6	15	
7	12	
TOTAL:	100	

PROB. Su23-F.1. (The two parts of this problem are unrelated.)

(a) There are an infinite number of positive integers $N > 0$ for which the following is true for all time t:

$$\sin(100\pi t) = \sum_{k=0}^{N} \cos\left(100\pi\left(t - \frac{k}{200}\right)\right).$$

Name any three:

$N =$ ☐ $> 0,$

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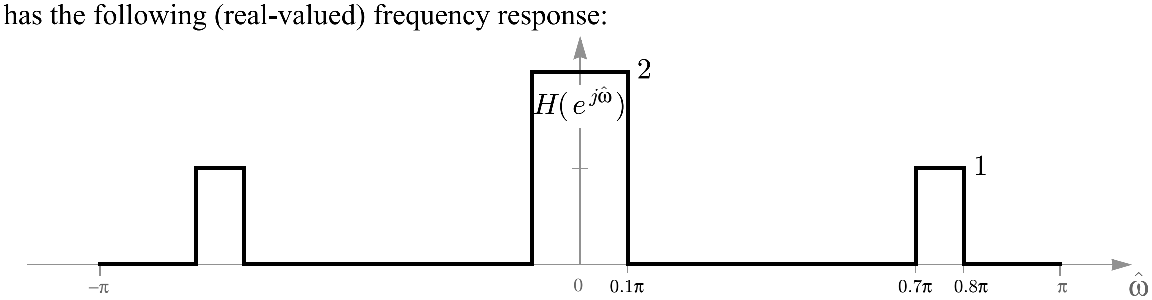
(b) Find positive numeric values for the constants A, B, and C so that a filter with impulse response:

$$h[n] = \frac{\sin(0.8\pi n) + \sin(A\pi n)}{\pi n} - B\cos(0.4\pi n)\frac{\sin(C\pi n)}{\pi n}:$$

has the following (real-valued) frequency response:



$A =$ ☐ $> 0,$

$B =$ ☐ $> 0,$

$C =$ ☐ $> 0.$

PROB. Su23-F.2. Consider an LTI filter whose impulse response is:

$$h[n] = \delta[n-2] + \delta[n-3] + \delta[n-4].$$

(a) The DC gain of this filter is $\boxed{}$.

(b) This is a nulling filter that nulls any input sinusoid whose digital frequency is $\hat{\omega} = \boxed{}$.

(c) If a filter input of the form $x[n] = A + B\cos(\frac{\pi}{3}n) + \cos(\frac{2\pi}{3}n)$ results in a filter output $y[n]$ that satisfies:

$$y[0] = 1,$$
$$y[1] = 20$$

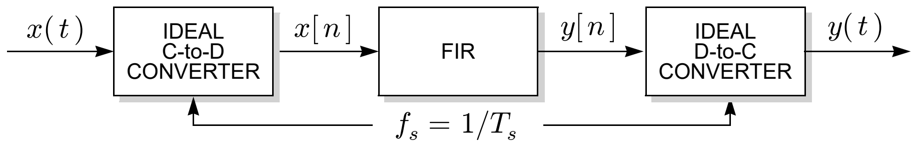
then the positive constants $(A, B > 0)$ must be:

$A = \boxed{}$

$B = \boxed{}$

PROB. Su23-F.3. Let $x(t)$ be a continuous-time signal whose spectrum is shown below:

Suppose we feed this signal into the ideal sampling-filtering-reconstruction system shown below, where the samples of $x(t)$ are filtered by an FIR filter, whose output $y[n]$ is fed to an ideal D-to-C converter, resulting in the continous-time output $y(t)$:



(The sampling rate f_s and the FIR filter parameters are unspecified and may be different in each part below.)

(a) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ ☐ Hz.

(b) At time zero, the input evaluates to $x(0) =$ ☐ .

(c) To avoid aliasing we need $f_s >$ ☐ Hz.

(d) When the sampling rate is $f_s = 840$ Hz, and when the difference equation for the FIR filter is

$$y[n] = x[n] + Ax[n-1] + Bx[n-2] + Ax[n-3] + x[n-4],$$

the output $y(t)$ will be identically *zero* for all time t when:

$A =$ [ ]

$B =$ [ ]

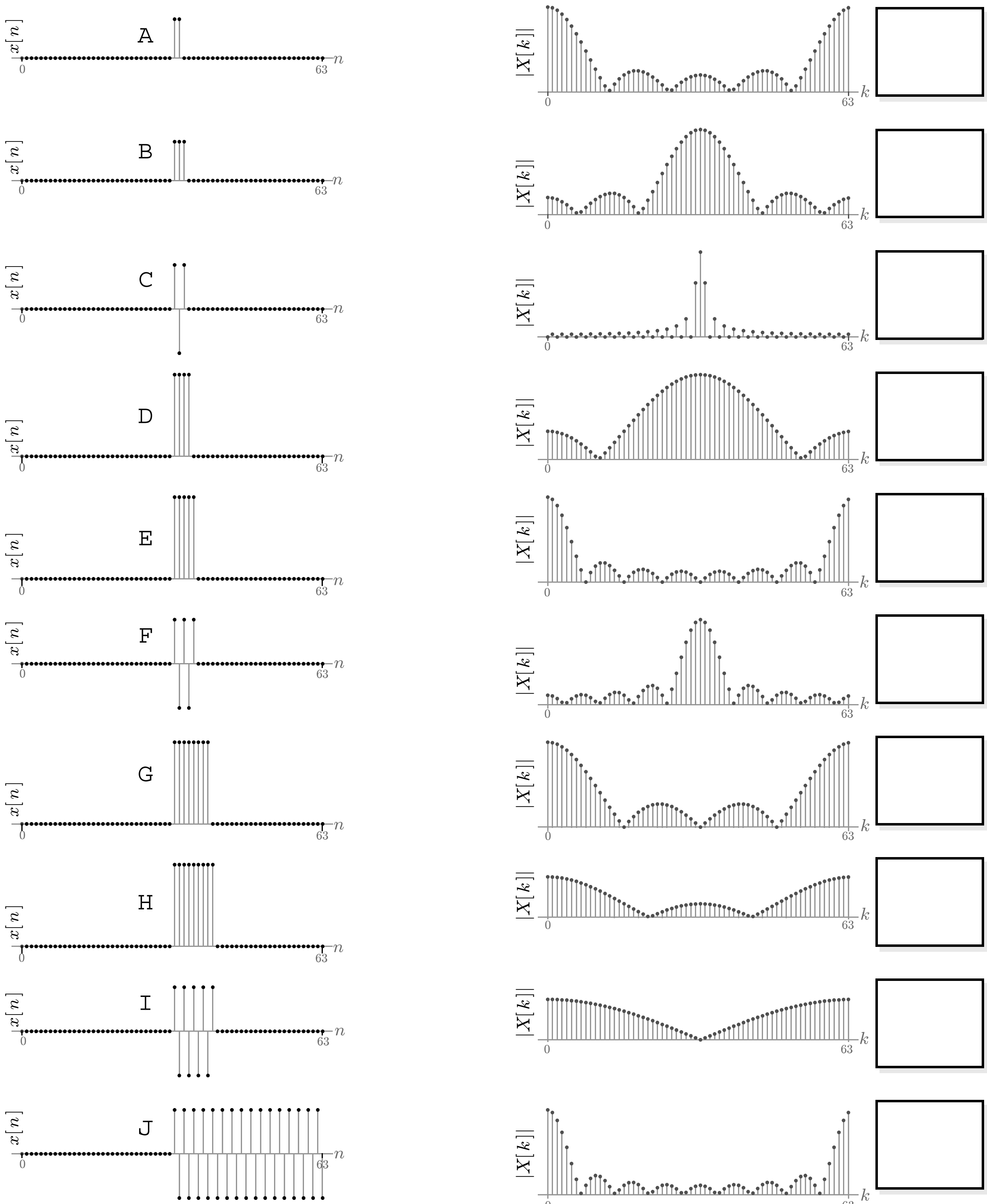
(e) When the sampling rate is $f_s = 350$ Hz, and when the difference equation for the FIR filter is

$$y[n] = x[n] + Cx[n-1] + x[n-2],$$

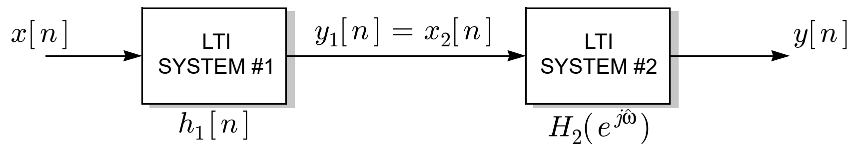
the output $y(t)$ will be identically *zero* for all time t when: $C =$ [ ] .

PROB. Su23-F.4. Shown below on the left are the plots of 10 different signal segments $[x[0], \ldots x[63]]$, labeled A through J, where each $x[n]$ is plotted versus $n \in \{0, 1, \ldots 63\}$. Let $[X[0], \ldots X[63]]$ be the $N = 64$-point DFT of $[x[0], \ldots x[63]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \ldots 63\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from A through J) into each of the 10 answer boxes.

(None of the y-axis scales are specified, they are not needed, only the shapes matter.)



PROB. Su23-F.5. Consider the following serial cascade of a pair of LTI systems:



As shown in the figure, an input sequence $x[n]$ is fed to a first LTI system, whose output is fed as an input to a second LTI system, producing an overall output sequence $y[n]$.

- The first sytem has impulse response $h_1[n] = \beta^n u[n]$, where the real parameter β is unspecified.
- The second system has frequency response $H_2(e^{j\hat{\omega}}) = 12 - 2e^{-j\hat{\omega}} - 4e^{-2j\hat{\omega}}$.

If the difference equation relating the overall output to the overall input is

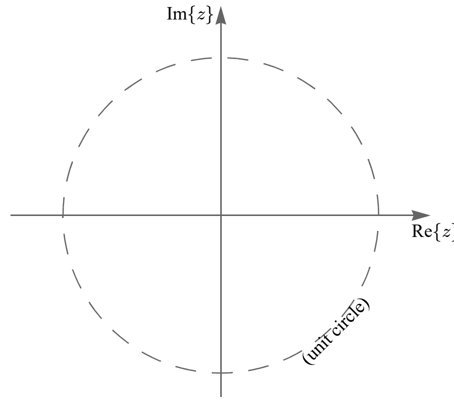
$$y[n] = 12x[n] + 6x[n-1],$$

then it must be that:

$$\beta = \boxed{}$$

PROB. Su23-F.6. Consider an IIR filter with system function $H(z) = \dfrac{9}{1-0.5z^{-1}} + \dfrac{3}{1+0.5z^{-1}}$.

(a) Sketch the pole-zero plot for this filter:



(b) The dc gain of this filter is $\boxed{}$.

(c) The impulse response evaluated at time zero is $h[0] = \boxed{}$.

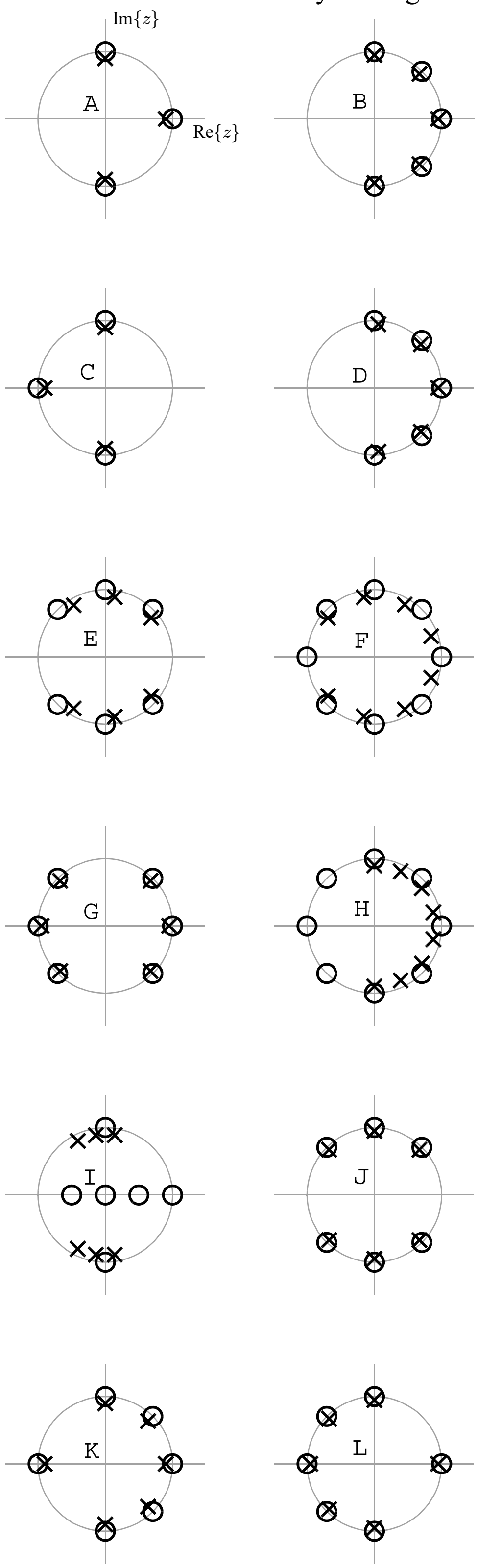
(d) The filter's difference equation has the form $y[n] = b_0x[n] + b_1x[n-1] + a_2y[n-2]$, where:

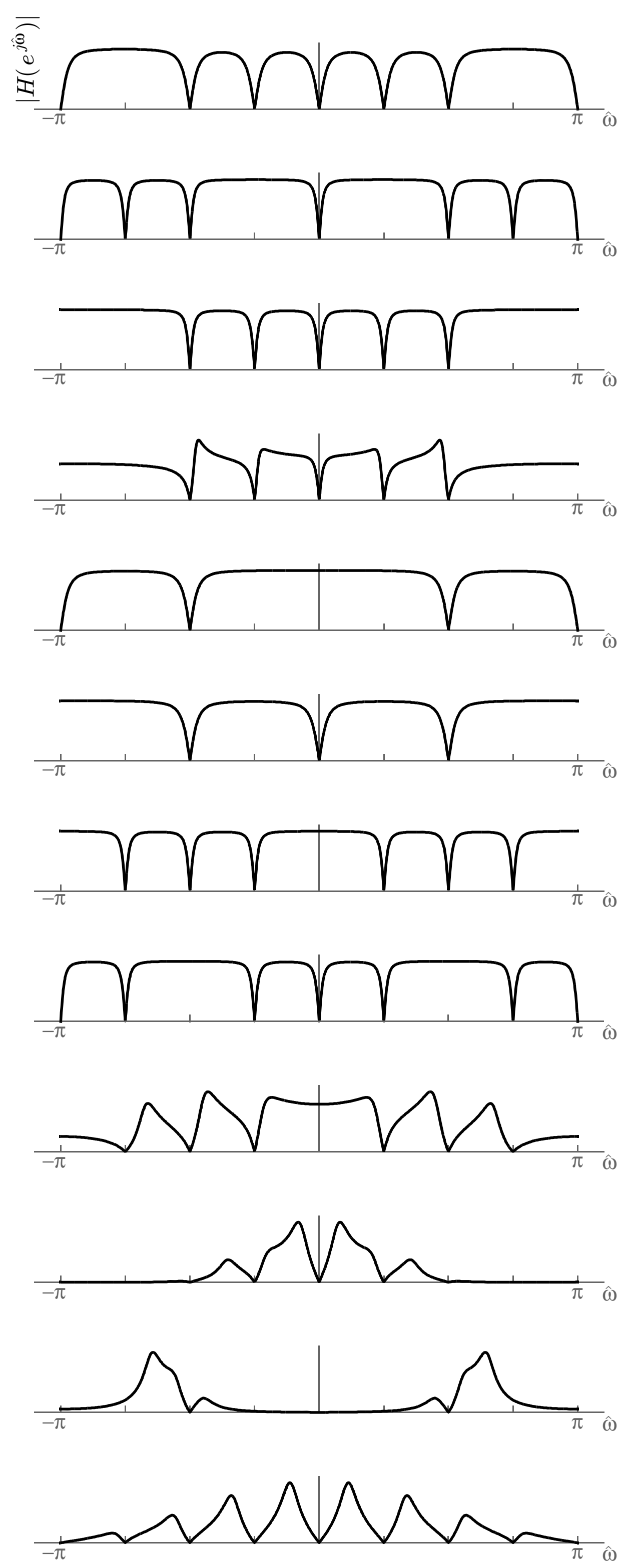
$b_0 = \boxed{}$

$b_1 = \boxed{}$

$a_2 = \boxed{}$

PROB. Su23-F.7. Shown on the left are the pole-zero plots for 12 LTI systems, labeled A through L. Shown on the right are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (A through L) in each answer box.





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PROB. Su23-F.1. (The two parts of this problem are unrelated.)

(a) There are an infinite number of positive integers $N > 0$ for which the following is true for all time t:

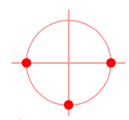
$$\sin(100\pi t) = \sum_{k=0}^{N} \cos\big(100\pi(t - \tfrac{k}{200})\big).$$

Name any three:

Corresponding phasor equation:

$$e^{-j0.5\pi} = \sum_{k=0}^{N} e^{-jk0.5\pi}$$

$N = 2$ works: $e^{-j0.5\pi} = 1 + e^{-j0.5\pi} - 1$ ✓



Adding any integer multiple of 4 will also work

$$\Rightarrow N = 2 + 4\ell \in \{2, 6, 10, 14, 18, 22, \ldots\}$$

$N =$ 2 > 0,

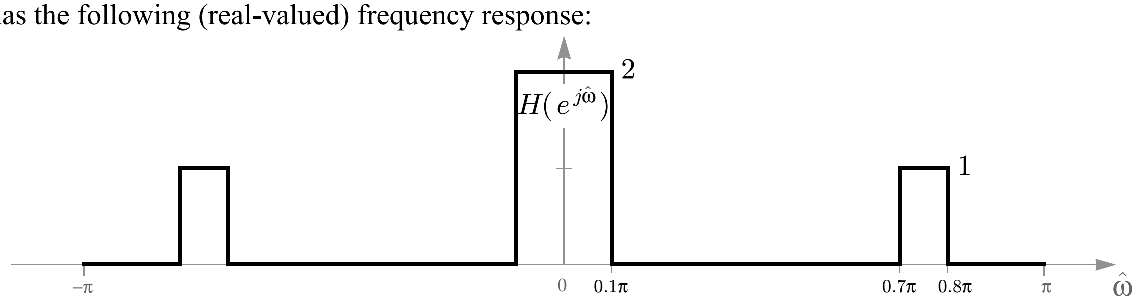
$N =$ 6 > 0,

$N =$ 10 > 0.

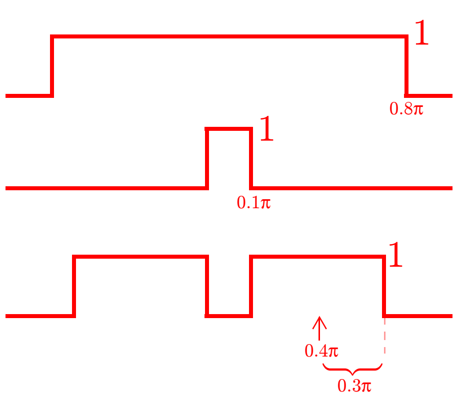
(b) Find positive numeric values for the constants A, B, and C so that a filter with impulse response:

$$h[n] = \frac{\sin(0.8\pi n) + \sin(A\pi n)}{\pi n} - B\cos(0.4\pi n)\frac{\sin(C\pi n)}{\pi n}:$$

has the following (real-valued) frequency response:



This shape can be achieved by subtracting the bottom BPF shape below from the sum of the first two rectangles:



$\Rightarrow A = 0.1$

$\Rightarrow B = 2,\ C = 0.3$

$A =$ 0.1 > 0,

$B =$ 2 > 0,

$C =$ 0.3 > 0.

PROB. Su23-F.2. Consider an LTI filter whose impulse response is:

$$h[n] = \delta[n-2] + \delta[n-3] + \delta[n-4].$$

(a) The DC gain of this filter is $\boxed{3}$.

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$
$$\Rightarrow H(e^{j0}) = 1 + 1 + 1$$

(b) This is a nulling filter that nulls any input sinusoid whose digital frequency is $\hat{\omega} = \boxed{\frac{2\pi}{3}}$.

$$\begin{aligned} H(e^{j\hat{\omega}}) &= e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} \\ &= e^{-j3\hat{\omega}}(1 + 2\cos(\hat{\omega})) \\ &= 0 \text{ when } \hat{\omega} = 2\pi/3 \end{aligned}$$

(c) If a filter input of the form $x[n] = A + B\cos(\frac{\pi}{3}n) + \cos(\frac{2\pi}{3}n)$ results in a filter output $y[n]$ that satisfies:

$$y[0] = 1,$$
$$y[1] = 20$$

then the positive constants ($A, B > 0$) must be:

$A = \boxed{13}$

$B = \boxed{19}$

$$H(e^{j0}) = 3$$
$$H(e^{j\pi/3}) = e^{-j3\pi/3}(1 + 2\cos(\pi/3)) = -2$$
$$H(e^{j2\pi/3}) = 0$$

$$\Rightarrow y[n] = 3A - 2B\cos(\pi n/3) + 0\cos(2\pi n/3)$$

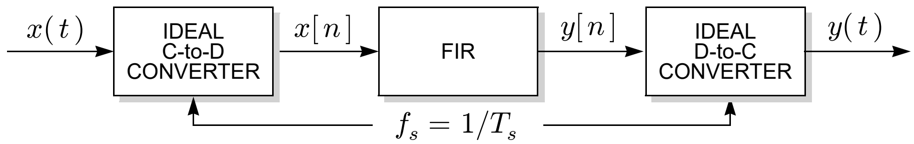
From this we get two equations, two unknowns:

$$\Rightarrow y[0] = 3A - 2B = 1$$
$$y[1] = 3A - B = (2B + 1) - B = 20 \Rightarrow B = 19$$

$$\Rightarrow A = (2B + 1)/3 = 13$$

PROB. Su23-F.3. Let $x(t)$ be a continuous-time signal whose spectrum is shown below:

Suppose we feed this signal into the ideal sampling-filtering-reconstruction system shown below, where the samples of $x(t)$ are filtered by an FIR filter, whose output $y[n]$ is fed to an ideal D-to-C converter, resulting in the continous-time output $y(t)$:



(The sampling rate f_s and the FIR filter parameters are unspecified and may be different in each part below.)

(a) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ 70 Hz.

gcd(140, 210)

(b) At time zero, the input evaluates to $x(0) =$ 4 .

Evaluate FS $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$ at time 0

$$\Rightarrow x(0) = \sum_{k=-\infty}^{\infty} a_k$$

$$= (1+j) + (1-j) + (1+j) + (1-j) = 4$$

(c) To avoid aliasing we need $f_s >$ 420 Hz.

$$f_s > 2f_{\max}$$

(d) When the sampling rate is $f_s = 840$ Hz, and when the difference equation for the FIR filter is

$$y[n] = x[n] + Ax[n-1] + Bx[n-2] + Ax[n-3] + x[n-4],$$

the output $y(t)$ will be identically *zero* for all time t when:

$A =$ -1

$B =$ 2

Concatenate filters of the form $[1, -2\cos(\hat{\omega}_0), 1]$

To null $\hat{\omega}_1 = \frac{140}{f_s/2}\pi = \frac{\pi}{3} \Rightarrow h_1 = [1, -1, 1]$

To null $\hat{\omega}_2 = \frac{210}{f_s/2}\pi = \frac{\pi}{2} \Rightarrow h_2 = [1, 0, 1]$

Convolve $h_1 * h_2$:

```
1   -1    1
     0    0    0
          1   -1    1
--------------------
1   -1    2   -1    1
    ↑     ↑
    A     B
```

(e) When the sampling rate is $f_s = 350$ Hz, and when the difference equation for the FIR filter is

$$y[n] = x[n] + Cx[n-1] + x[n-2],$$

the output $y(t)$ will be identically *zero* for all time t when $C =$ 1.618.

The second sinusoid *aliases* to the same digital frequency as the first:

$$\hat{\omega}_1 = \frac{140}{f_s/2}\pi = 0.8\pi$$

$$\hat{\omega}_2 = \frac{210}{f_s/2}\pi = 1.2\pi \rightarrow 0.8\pi$$

$\Rightarrow$ A single filter $[1, \underbrace{-2\cos(0.8\pi)}_{C}, 1] = [1, 1.618, 1]$ does the job

PROB. Su23-F.4. Shown below on the left are the plots of 10 different signal segments $[x[0], \ldots x[63]]$, labeled A through J, where each $x[n]$ is plotted versus $n \in \{0, 1, \ldots 63\}$. Let $[X[0], \ldots X[63]]$ be the $N = 64$-point DFT of $[x[0], \ldots x[63]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \ldots 63\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from A through J) into each of the 10 answer boxes.

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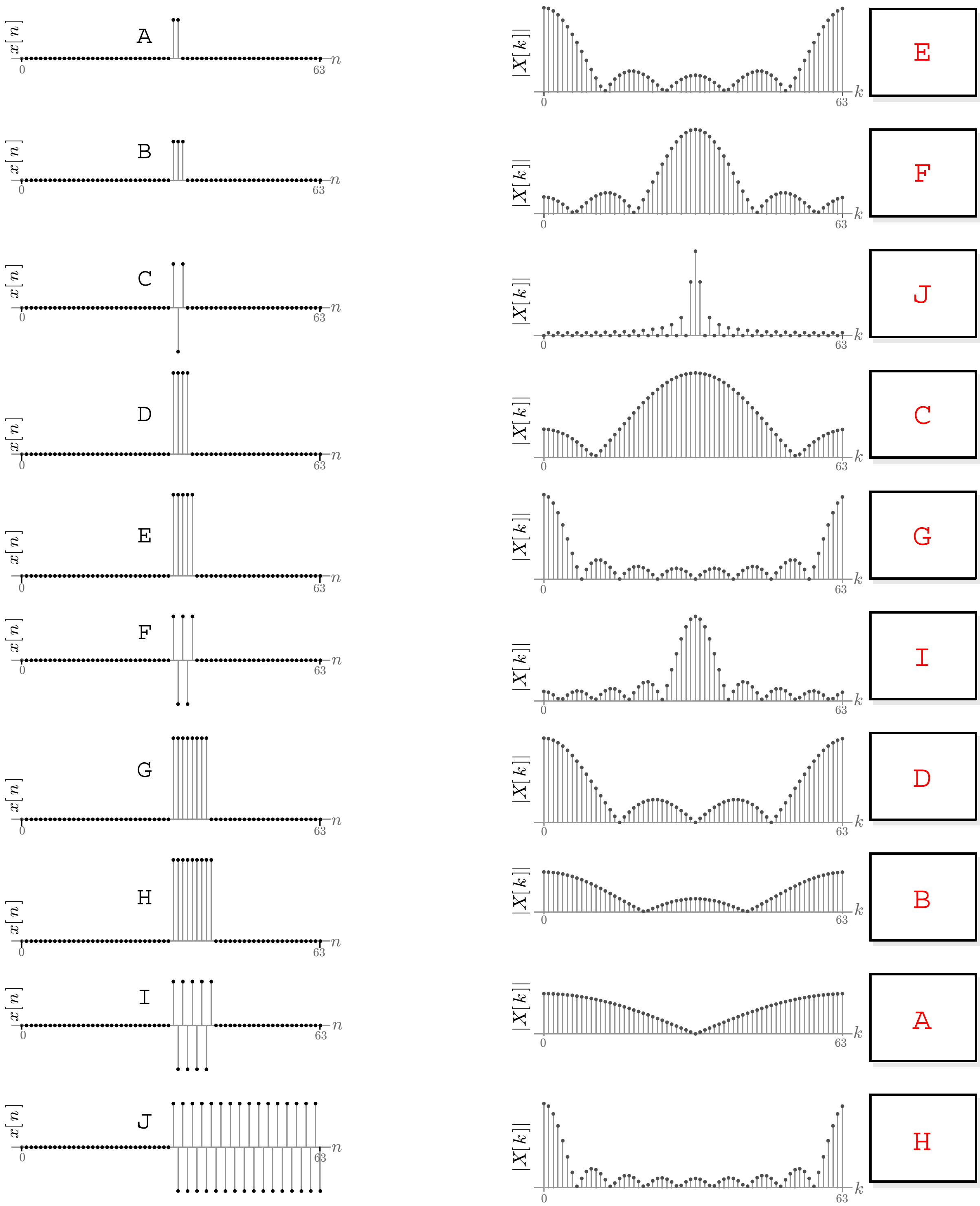
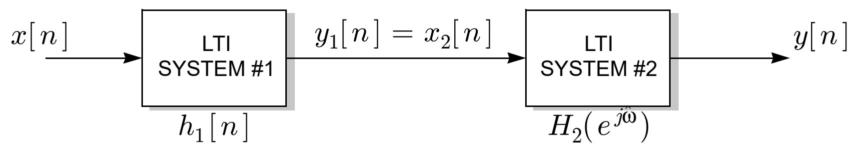


PROB. Su23-F.5. Consider the following serial cascade of a pair of LTI systems:



As shown in the figure, an input sequence $x[n]$ is fed to a first LTI system, whose output is fed as an input to a second LTI system, producing an overall output sequence $y[n]$.

- The first sytem has impulse response $h_1[n] = \beta^n u[n]$, where the real parameter β is unspecified.
- The second system has frequency response $H_2(e^{j\hat{\omega}}) = 12 - 2e^{-j\hat{\omega}} - 4e^{-2j\hat{\omega}}$.

If the difference equation relating the overall output to the overall input is

$$y[n] = 12x[n] + 6x[n-1],$$

then it must be that:

$$\beta = \boxed{\frac{2}{3}}$$

The overall system function is

$$12 + 6z^{-1} = H_1(z)H_2(z)$$

$$= \left(\frac{1}{1-\beta z^{-1}}\right)(12 - 2z^{-1} - 4z^{-2})$$

$$\Rightarrow 12 - 2z^{-1} - 4z^{-2} = (12 + 6z^{-1})(1 - \beta z^{-1})$$

$$= 12 - \underbrace{(12\beta - 6)}_{2} - \underbrace{6\beta}_{4} z^{-2}$$

Solve either one $\Rightarrow \beta = 2/3$

PROB. Su23-F.6. Consider an IIR filter with system function $H(z) = \dfrac{9}{1 - 0.5z^{-1}} + \dfrac{3}{1 + 0.5z^{-1}}$.

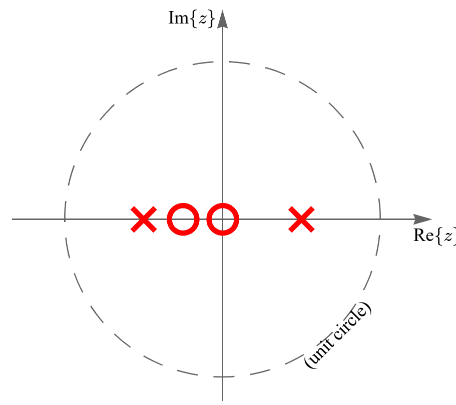
(a) Sketch the pole-zero plot for this filter:

$$H(z) = \frac{12 + 3z^{-1}}{(1 + 0.5z^{-1})(1 - 0.5z^{-1})}$$

$$= \frac{12z(z + 0.25)}{(z + 0.5)(z - 0.5)}$$

roots → zeros at $0, -0.25$

roots → poles at ± 0.5



(b) The dc gain of this filter is **20**.

$$H(e^{j0}) = H(1) = \frac{9}{1 - 0.5} + \frac{3}{1 + 0.5} = 18 + 2 = 20$$

(c) The impulse response evaluated at time zero is $h[0] =$ **12**.

Inverse Z transform via Table:

$$h[n] = 9(0.5)^n u[n] + 3(-0.5)^n u[n]$$

$$\Rightarrow h[0] = 9 + 3$$

(d) The filter's difference equation has the form $y[n] = b_0 x[n] + b_1 x[n-1] + a_2 y[n-2]$, where:

Equate $\dfrac{12 + 3z^{-1}}{(1 + 0.5z^{-1})(1 - 0.5z^{-1})} = \dfrac{Y(z)}{X(z)}$

Cross multiply

$$\Rightarrow Y(z) - 0.25z^{-2}Y(z) = 12X(z) + 3z^{-1}X(z)$$

Inverse transform

$$\Rightarrow y[n] - 0.25y[n-2] = 12x[n] + 3x[n-1]$$

$$\Rightarrow y[n] = \underbrace{12}_{b_0}x[n] + \underbrace{3}_{b_1}x[n-1] + \underbrace{0.25}_{a_2}y[n-2]$$

$b_0 =$ **12**

$b_1 =$ **3**

$a_2 =$ **0.25**

PROB. Su23-F.7. Shown on the left are the pole-zero plots for 12 LTI systems, labeled `A` through `L`. Shown on the right are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (`A` through `L`) in each answer box.