GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2018
Final Exam

July 27, 2018

NAME: ______________________________

Important Notes:

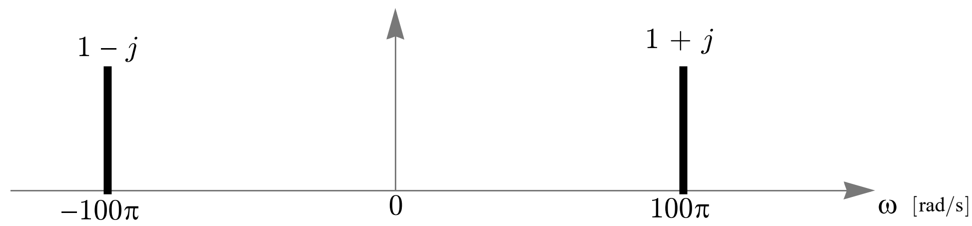
- Do not unstaple the test.
- Closed book and notes, except for three double-sided pages (8.5” $\times$ 11”) of hand-written notes.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Points	Score
1	10	
2	15	
3	15	
4	15	
5	15	
6	10	
7	20	
TOTAL:	100	

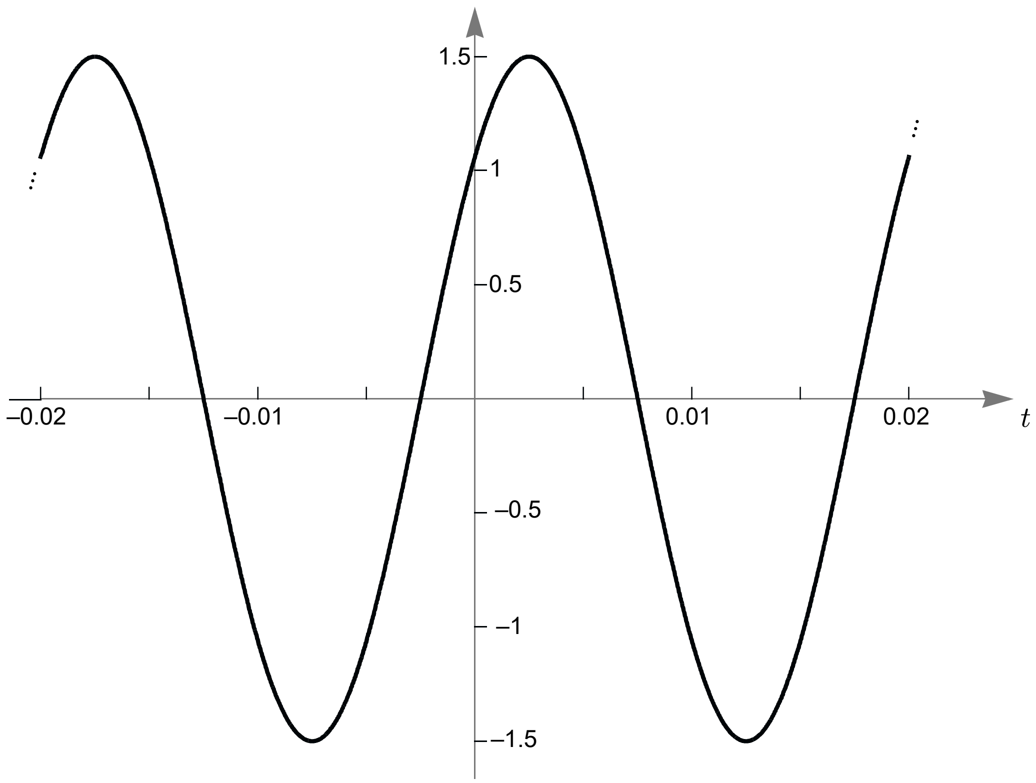
PROB. Su18-Fin.1.

Let $x_1(t) = 2\cos(100\pi(t - 0.002))$.

Let $x_2(t)$ be the signal whose spectrum is shown below:



Let $x_3(t)$ be the sinusoidal signal sketched below:

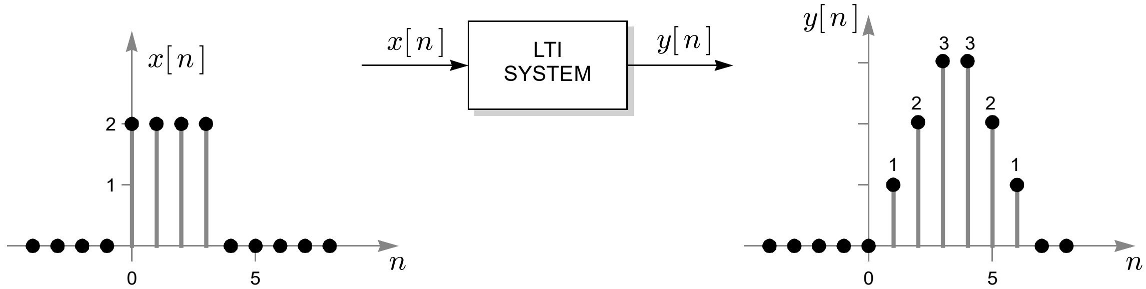


The *sum* of these three signals can be written as $x_1(t) + x_2(t) + x_3(t) = A\cos(100\pi t + \varphi)$,

where $A =$ ☐ and $\varphi =$ ☐.

PROB. Su18-Fin.2.

Suppose the output of a linear and time-invariant discrete-time filter in response to a finite-length input of $x[n] = \{\ldots\ 0\ 0\ \underline{2}\ 2\ 2\ 2\ 0\ 0\ \ldots\ \}$ is $y[n] = \{\ldots\ 0\ \underline{0}\ 1\ 2\ 3\ 3\ 2\ 1\ 0\ 0\ \ldots\ \}$, as shown below:

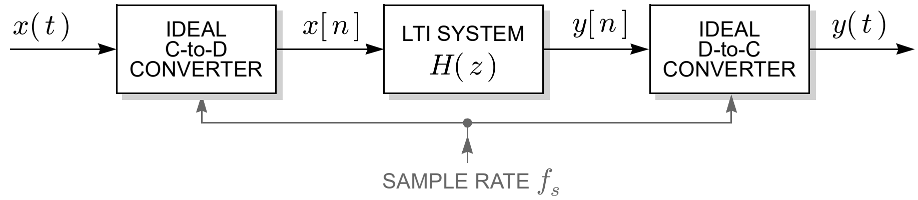


The filter's impulse response $h[n]$ has values:

$h[0] =$ ___ , $h[1] =$ ___ , $h[2] =$ ___ , $h[3] =$ ___ , $h[4] =$ ___

PROB. Su18-Fin.3.

Consider the following system in which a continuous-time input $x(t)$ is sampled, producing $x[n]$, then filtered, producing $y[n]$, and finally converted back to continuous time, yielding an overall output $y(t)$:



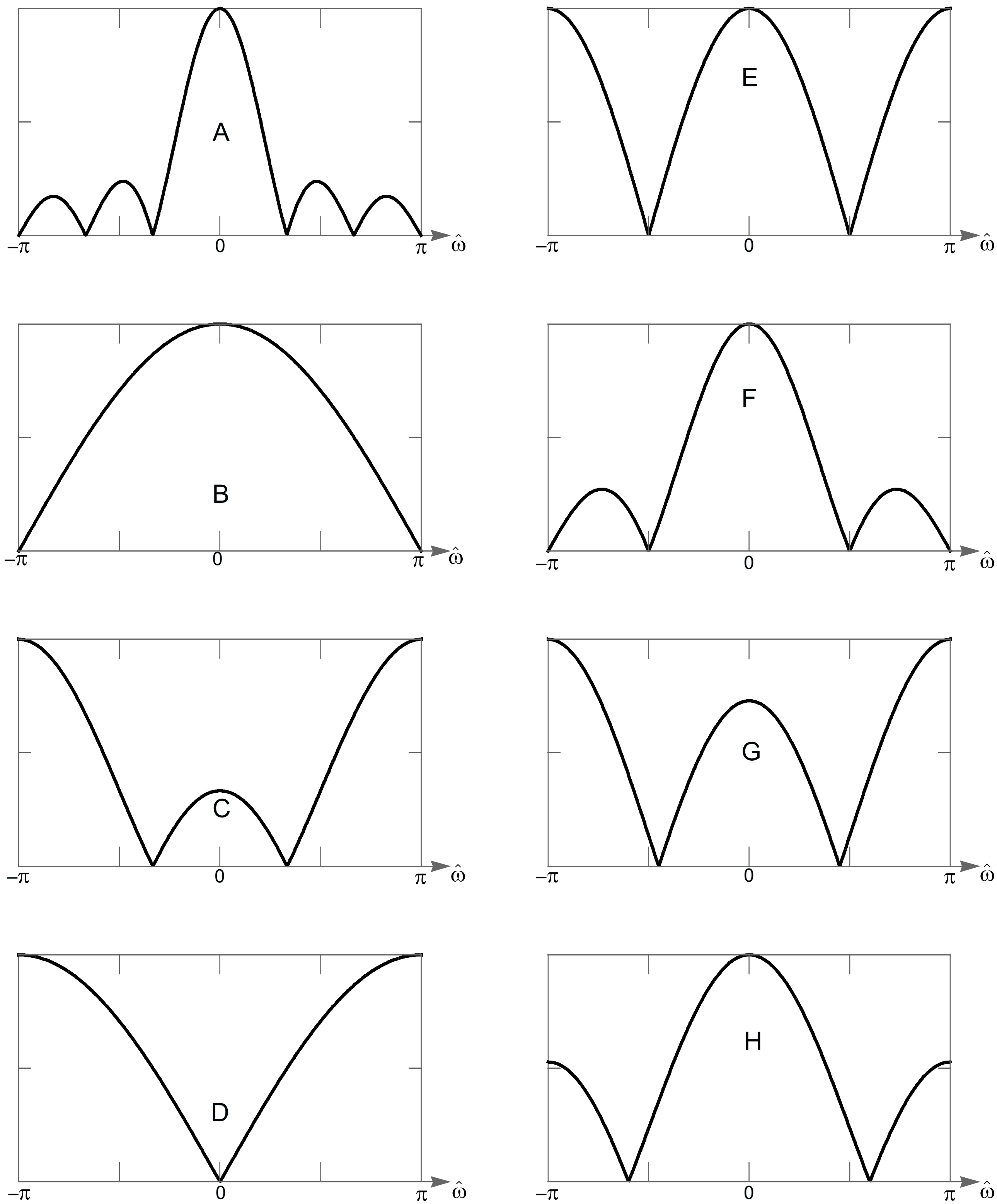
The system function of the discrete-time LTI system is $H(z) = 1 - z^{-1} + z^{-2}$.

(a) Which of the plots (A... H) on the next page shows the magnitude response of this filter? $\boxed{}$

(b) A *constant* input $x(t) = 1$ (for all t) will result in the *constant* output $y(t) = \boxed{}$ (for all t).

(c) Specify a sample rate $f_s = \boxed{}$ samples/s

so that a C-to-D input of $x(t) = \cos(10\pi t) + \cos(14\pi t)$
results in a *zero* output, $y(t) = 0$ (for all t).


A
B
C
D
E
F
G
H
−π
0
π
ω̂

PROB. Su18-Fin.4.

Consider an FIR filter whose difference equation is:

$$y[n] = (1 + \beta)x[n] + (1 - \beta)x[n - 1],$$

where β is an unspecified real constant.

If the output of this system in response to the sinusoidal input $x[n] = \cos(0.7\pi n)$

is $y[n] = \cos(0.7\pi n + \theta)$,

then it must be that $\beta = \boxed{}$ and $\theta = \boxed{}$.

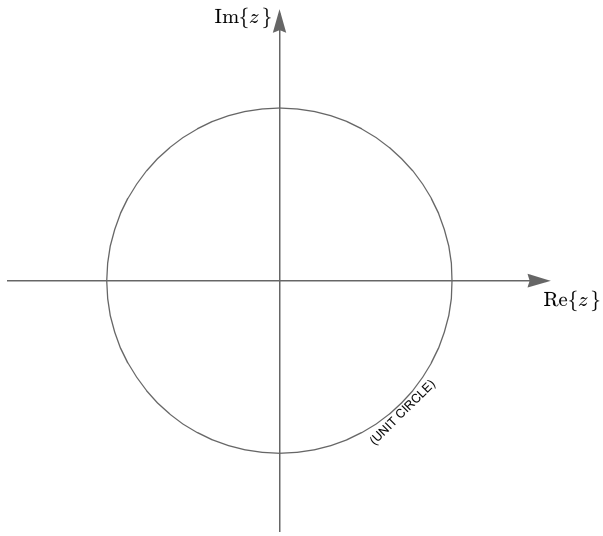
PROB. Su18-Fin.5. Consider an LTI system described by the recursive difference equation:

$$y[n] = x[n] - 0.2y[n-1].$$

(a) The system is [IIR][FIR] (circle one).

(b) The system transfer function is $H(z) = \boxed{}$.

(c) Sketch below the pole-zero plot for $H(z)$, carefully labeling the locations of all zeros and poles:



(d) If the output of this system in response to the input

$x[n] = 10\delta[n] + B\delta[n-1]$ is $y[n] = 10\delta[n]$

(or in other words, as lists of numbers, the input .... 0 <u>10</u> B 0 ... results in the output ... 0 <u>10</u> 0 ...),

then it must be that the constant B is

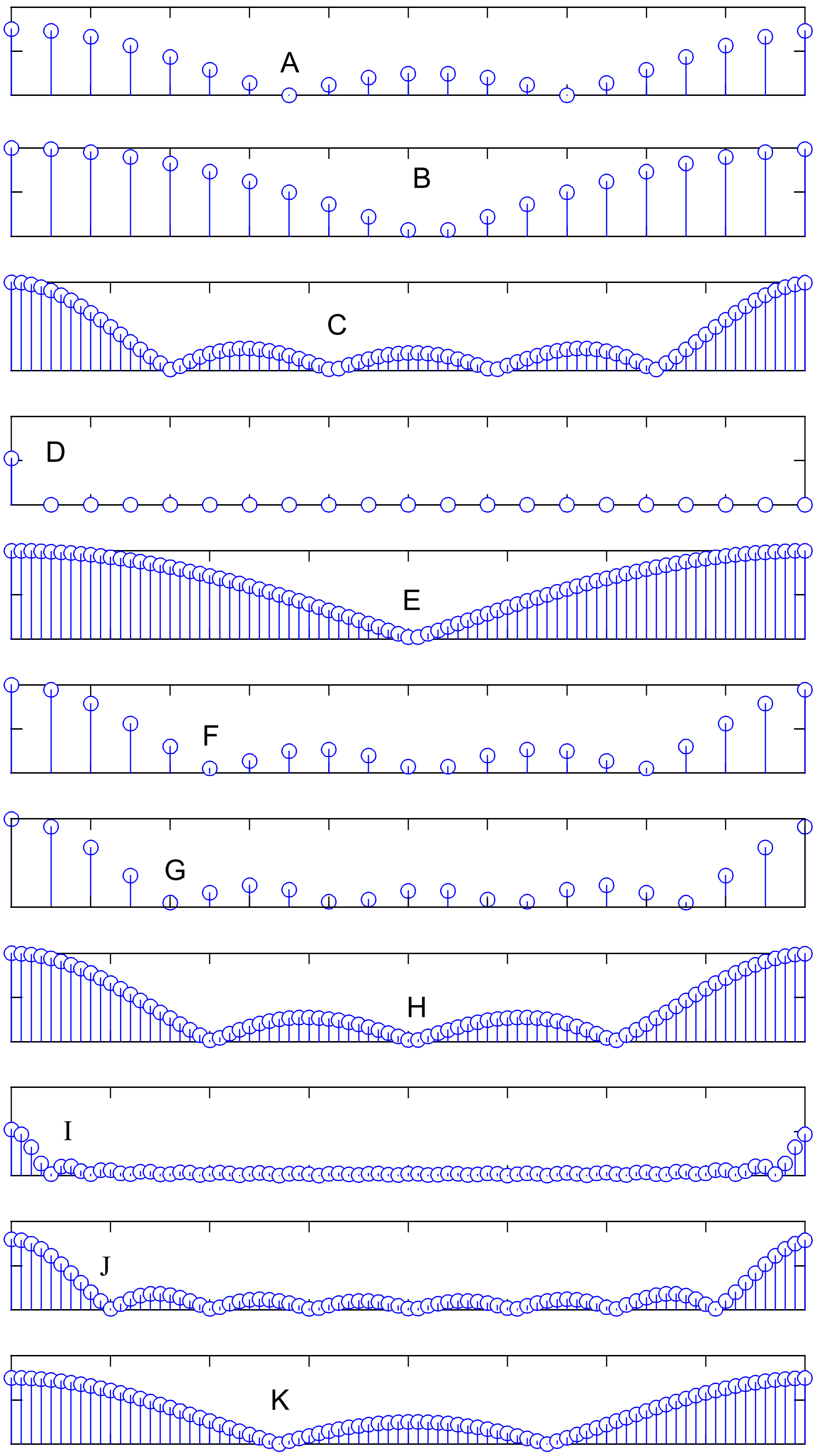
$B = \boxed{}$.

PROB. Su18-Fin.6.

Shown below are eleven different outcomes that result from executing the MATLAB code:

```
stem(abs(fft(ones(1,L),N)));
```

Match each plot with the corresponding values for the variables `N` and `L` by writing a letter (A through K) in each answer box.



L = 2, N = 21

L = 2, N = 81

L = 3, N = 21

L = 3, N = 81

L = 4, N = 21

L = 4, N = 81

L = 5, N = 21

L = 5, N = 81

L = 8, N = 81

L = 21, N = 21

L = 21, N = 81

PROB. Su18-Fin.7. (The different parts are unrelated.)

(a) There are an infinite number of values for the delay parameter t_0 below such that:

$$\cos(100\pi t) + \cos(100\pi(t - t_0)) + \cos(100\pi(t - 2t_0)) = 0, \qquad \text{for all time } t.$$

Name any two, both positive:

$t_0 = \boxed{}$, or $t_0 = \boxed{}$.

(b) Simplify $\sum_{k=-\infty}^{\infty} \frac{\sin(0.9\pi k)}{\pi k} \cdot \frac{\sin(0.75\pi(n-k))}{\pi(n-k)} = \underbrace{\boxed{}}_{\text{a function of } n}$.

(c) The signal $x(t) = \cos(88\pi t)\cos(8\pi t)) + \cos(240\pi t)$ is periodic with fundamental frequency $f_0 = \boxed{}$ Hz.

(d) If $\{X[0], \dots X[511]\}$ are the DFT coefficients that result from taking a 512-point DFT of the length-12 signal $x[n] = [1\ 1\ 1\ 2\ 2\ 2\ 3\ 3\ 3\ 4\ 4\ 4]$ for $n \in \{0, \dots 11\}$, then find numerical values for the following two DFT coefficients:

$X[0] = \boxed{}$,

$X[256] = \boxed{}$.

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$u[n]-u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}e^{-j\hat{\omega}(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b)-u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1-ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(e^{j\hat{\omega}})+bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (d=integer)	$x[n-d]$	$e^{-j\hat{\omega}d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)})+\frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n]*h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2$	$\displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Date: 28-Apr-2013

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n = 0, 1, 2, \ldots, N-1$	*Frequency-Domain:* $X[k], \quad k = 0, 1, 2, \ldots, N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{-j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$u[n]-u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))} e^{-j(2\pi k/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))} e^{j(2\pi n/N)(L-1)/2}$	$N(u[k]-u[k-L])$, when $L \leq N$

Table of z-Transform Pairs		
Signal Name	*Time-Domain:* $x[n]$	z*-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}, \quad \lvert a\rvert < 1$
Decaying cosine	$r^n \cos(\hat{\omega}_0 n)u[n]$	$\dfrac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$
Decaying sinusoid	$Ar^n \cos(\hat{\omega}_0 n+\varphi)u[n]$	$A\dfrac{\cos(\varphi)-r\cos(\hat{\omega}_0-\varphi)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties		
Property Name	*Time-Domain* $x[n]$	z*-Domain* $X(z)$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay (d=integer)	$x[n-d]$	$z^{-d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

Date: 28-April-2013

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ANSWER KEY

NAME: ______________________________

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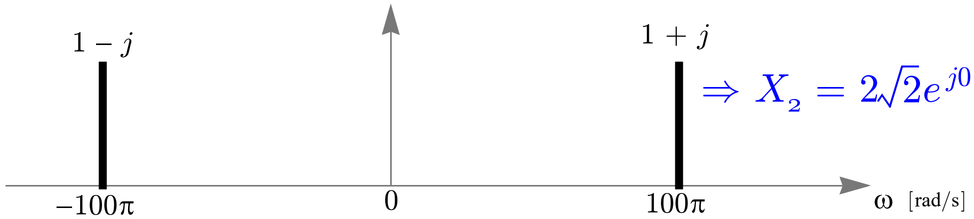
3	15	
4	15	
5	15	
6	10	
7	20	
TOTAL:	100	

PROB. Su18-Fin.1.

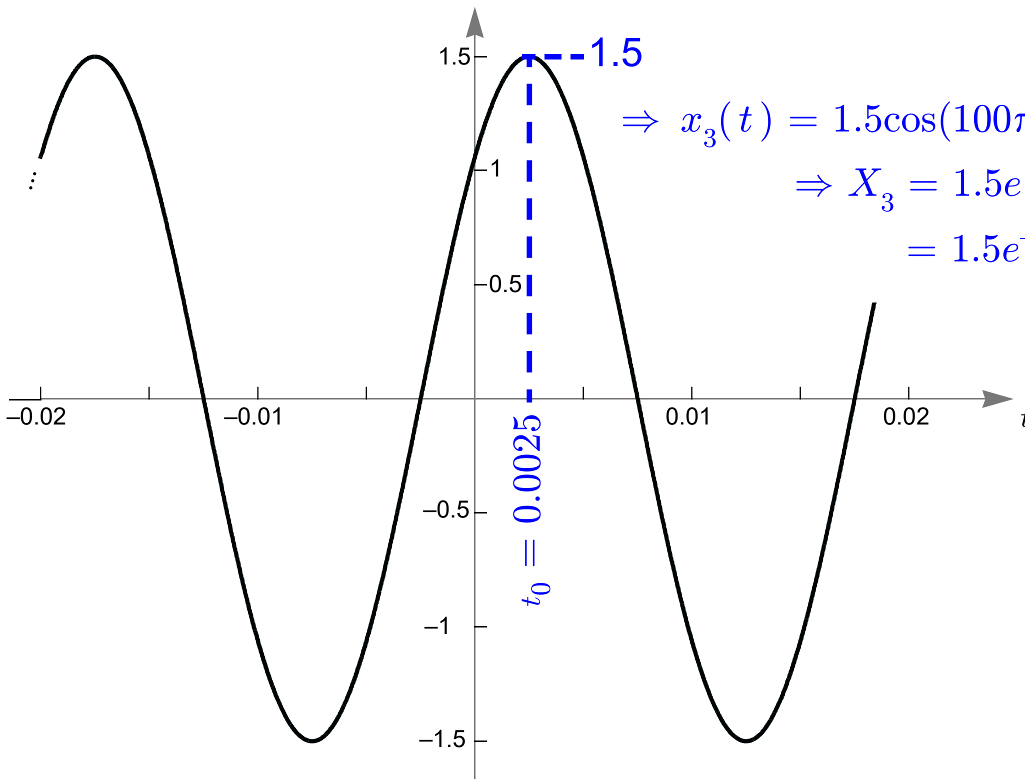
Let $x_1(t) = 2\cos(100\pi(t - 0.002))$. $\Rightarrow \quad X_1 = 2e^{-j0.2\pi}$

Let $x_2(t)$ be the signal whose spectrum is shown below:



$\Rightarrow X_2 = 2\sqrt{2}e^{j0.25\pi}$

Let $x_3(t)$ be the sinusoidal signal sketched below:



$\Rightarrow x_3(t) = 1.5\cos(100\pi(t - t_0))$

$\Rightarrow X_3 = 1.5e^{-j100\pi t_0}$

$= 1.5e^{-j0.25\pi}$

The *sum* of these three signals can be written as $x_1(t) + x_2(t) + x_3(t) = A\cos(100\pi t + \varphi)$,

where $A = \boxed{4.68}$ and $\varphi = \boxed{-0.016\pi}$.

$(= -0.05 \text{ rads } = -2.89°)$

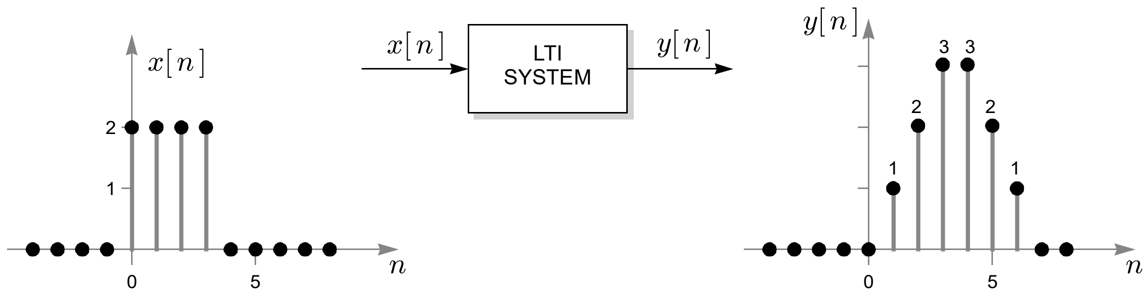
Phasor addition rule:

$$Ae^{j\varphi} = X_1 + X_2 + X_3$$

$$= 2e^{-j0.2\pi} + 2\sqrt{2}\,e^{j0.25\pi} + 1.5e^{-j0.25\pi} \quad = \quad 4.68e^{-j0.016\pi}$$

PROB. Su18-Fin.2.

Suppose the output of a linear and time-invariant discrete-time filter in response to a finite-length input of $x[n] = \{\ldots\ 0\ 0\ \underline{2}\ 2\ 2\ 2\ 0\ 0\ \ldots\ \}$ is $y[n] = \{\ldots\ 0\ \underline{0}\ 1\ 2\ 3\ 3\ 2\ 1\ 0\ 0\ \ldots\ \}$, as shown below:



The filter's impulse response $h[n]$ has values:

$h[0] = \boxed{0}$, $h[1] = \boxed{0.5}$, $h[2] = \boxed{0.5}$, $h[3] = \boxed{0.5}$, $h[4] = \boxed{0}$

$$y[n] = \Sigma_k h[k]x[n-k]$$

But $y[0] = 0 \Rightarrow 0 = h[0]x[\cancelto{2}{0}] = 2h[0] \qquad \Rightarrow h[0] = 0$

$$y[1] = 1 \Rightarrow 1 = h[\cancelto{0}{0}]x[\cancelto{2}{1}] + h[1]x[\cancelto{2}{0}] = 2h[1] \qquad \Rightarrow h[1] = 0.5$$

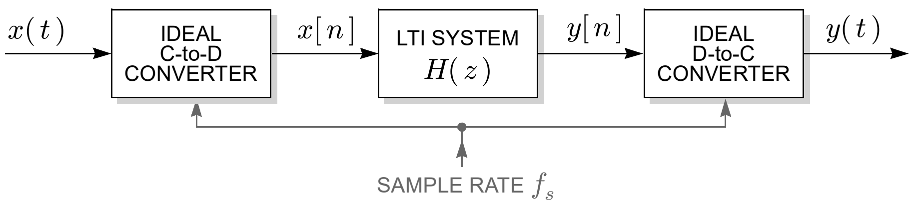
$$y[2] = 2 \Rightarrow 2 = h[\cancelto{0}{0}]x[2] + h[\cancelto{0.5}{1}]x[\cancelto{2}{1}] + h[2]x[\cancelto{2}{0}] = 1 + 2h[2]$$
$$\Rightarrow h[2] = 0.5$$

$$y[3] = 3 = h[\cancelto{0.5}{1}]x[\cancelto{2}{2}] + h[\cancelto{0.5}{2}]x[\cancelto{2}{1}] + h[3]x[\cancelto{2}{0}] = 2 + 2h[3]$$
$$\Rightarrow h[3] = 0.5$$

$$y[4] = 3 = h[\cancelto{0.5}{1}]x[\cancelto{2}{3}] + h[\cancelto{0.5}{2}]x[\cancelto{2}{2}] + h[\cancelto{0.5}{3}]x[\cancelto{2}{1}] + h[4]x[\cancelto{2}{0}]$$
$$\Rightarrow h[4] = 0$$

PROB. Su18-Fin.3.

Consider the following system in which a continuous-time input $x(t)$ is sampled, producing $x[n]$, then filtered, producing $y[n]$, and finally converted back to continuous time, yielding an overall output $y(t)$:



The system function of the discrete-time LTI system is $H(z) = 1 - z^{-1} + z^{-2}$.

(a) Which of the plots (A... H) on the next page shows the magnitude response of this filter? $\boxed{C}$

Recognize this as a *nulling filter*, where $-2\cos(\hat{\omega}) = -1$

$\Rightarrow$ look for nulls at $\hat{\omega} = \pm\pi/3$

(b) A *constant* input $x(t) = 1$ (for all t) will result in the *constant* output $y(t) = \boxed{1}$ (for all t).

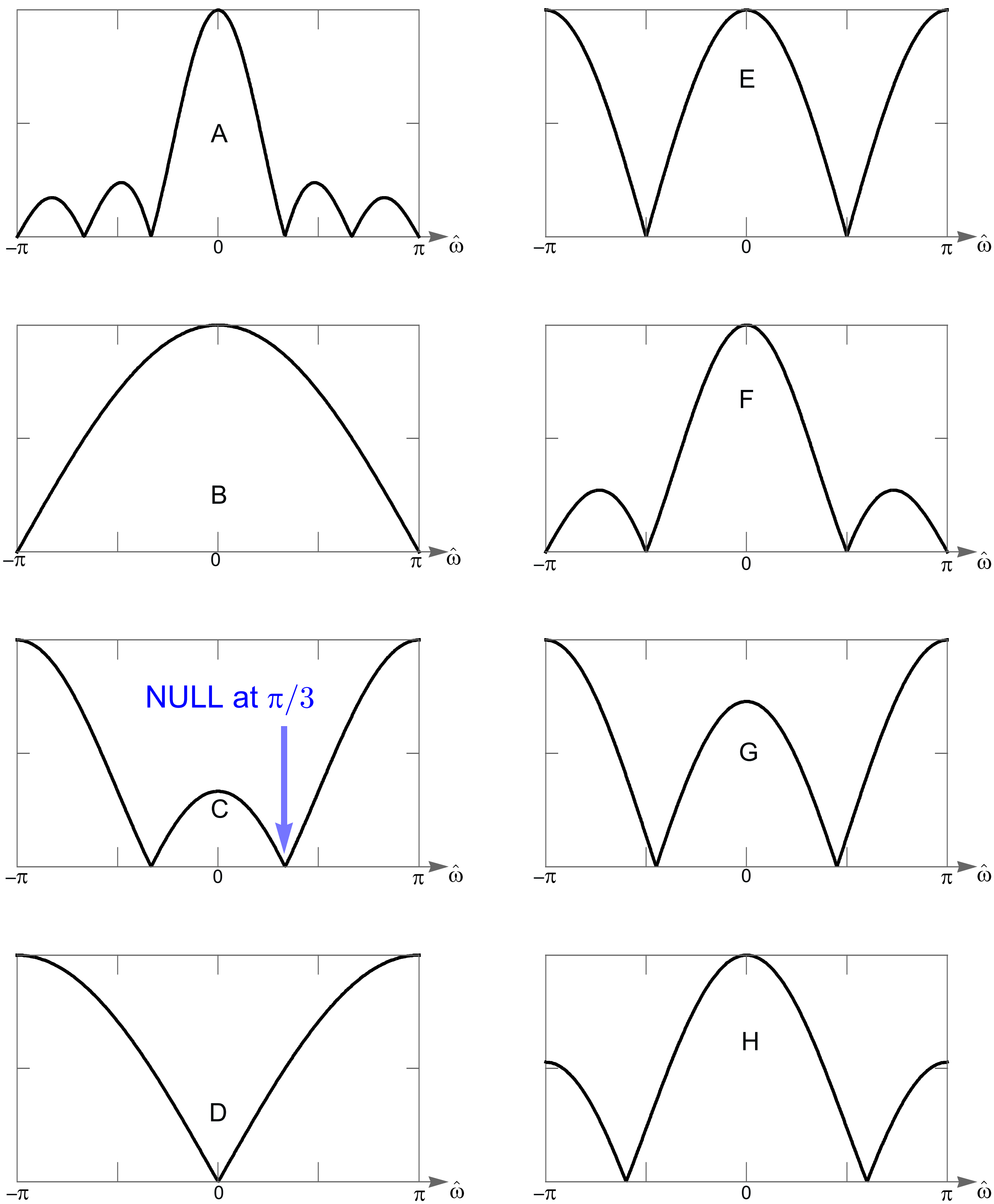
The DC gain is $b_0 + b_1 + b_2 = 1 - 1 + 1 = 1$

(c) Specify a sample rate $f_s = \boxed{6}$ samples/s

so that a C-to-D input of $x(t) = \cos(10\pi t) + \cos(14\pi t)$
results in a *zero* output, $y(t) = 0$ (for all t).

There are an infinite number of solutions, the largest being $f_s = 6$ Hz:

$$\left.\begin{aligned} &\Rightarrow \hat{\omega}_1 = \frac{10\pi}{f_s} = \frac{5\pi}{3} = \frac{-\pi}{3} + 2\pi \\ &\Rightarrow \hat{\omega}_2 = \frac{14\pi}{f_s} = \frac{7\pi}{3} = \frac{\pi}{3} + 2\pi \end{aligned}\right\} \text{both reduce to } \hat{\omega} = \pm\pi/3 \Rightarrow \text{both nulled}$$


A
B
C
NULL at $\pi/3$
D
E
F
G
H
$-\pi$
0
π
$\hat{\omega}$

PROB. Su18-Fin.4.

Consider an FIR filter whose difference equation is:

$$y[n] = (1+\beta)x[n] + (1-\beta)x[n-1],$$

where β is an unspecified real constant.

If the output of this system in response to the sinusoidal input $x[n] = \cos(0.7\pi n)$

is $y[n] = \cos(0.7\pi n + \theta)$,

then it must be that $\beta = \boxed{+0.2351}$ and $\theta = \boxed{-0.21\pi}$.

-0.2351 (alternative answer) -0.49π

The frequency response must have magnitude 1 at 0.7π

$$\begin{aligned} \Rightarrow \quad 1 &= |H(e^{j0.7\pi})|^2 = |(1+\beta) + (1-\beta)e^{-j0.7\pi}|^2 \\ &= (1+\beta)^2 + (1-\beta)^2 + 2(1+\beta)(1-\beta)\cos(0.7\pi) \\ &= 2 + 2\beta^2 + 2(1-\beta^2)\cos(0.7\pi) \end{aligned}$$

$$\Rightarrow \beta^2 = \frac{-1-2\cos(0.7\pi)}{2-2\cos(0.7\pi)} = 0.0553 \qquad \Rightarrow \beta = \pm 0.2351$$

With $\beta = +0.2351$, the frequency response simplifies to

$$\begin{aligned} H(e^{j0.7\pi}) &= (1+\beta) + (1-\beta)e^{-j0.7\pi} \\ &= 1.2351 + 0.7649e^{-j0.7\pi} = e^{-j0.2124\pi} \end{aligned}$$

Alternative answer: With $\beta = -0.2351$, the frequency response simplifies to

$$\begin{aligned} H(e^{j0.7\pi}) &= (1+\beta) + (1-\beta)e^{-j0.7\pi} \\ &= 0.7649 - 1.2351e^{-j0.7\pi} = e^{-j0.4876\pi} \end{aligned}$$

PROB. Su18-Fin.5. Consider an LTI system described by the recursive difference equation:

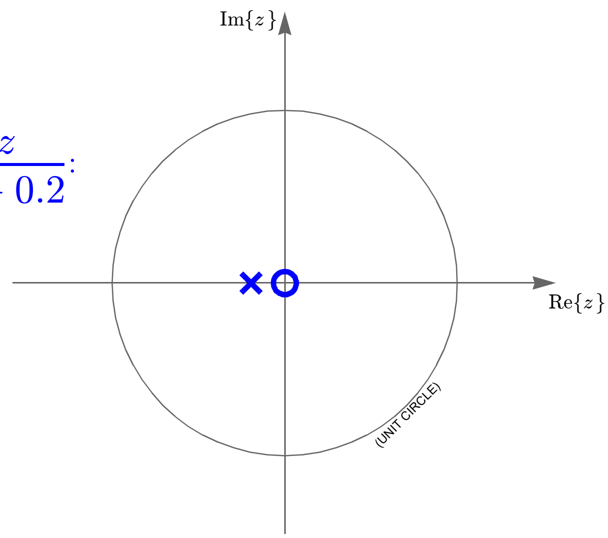
$$y[n] = x[n] - 0.2y[n-1].$$

(a) The system is [IIR] [FIR] (circle one).

(b) The system transfer function is $H(z) = \boxed{\dfrac{1}{1+0.2z^{-1}}}$.

(c) Sketch below the pole-zero plot for $H(z)$, carefully labeling the locations of all zeros and poles:

$H(z) = \dfrac{z}{z+0.2}$:



(d) If the output of this system in response to the input

$x[n] = 10\delta[n] + B\delta[n-1]$ is $y[n] = 10\delta[n]$

(or in other words, as lists of numbers, the input .... 0 $\underline{10}$ B 0 ... results in the output ... 0 $\underline{10}$ 0 ...),

then it must be that the constant B is $B = \boxed{2}$.

$$Y(z) = X(z)H(z) = (10 + Bz^{-1})\frac{1}{1+0.2z^{-1}} = 10\left(\frac{1+\frac{B}{10}z^{-1}}{1+0.2z^{-1}}\right)$$

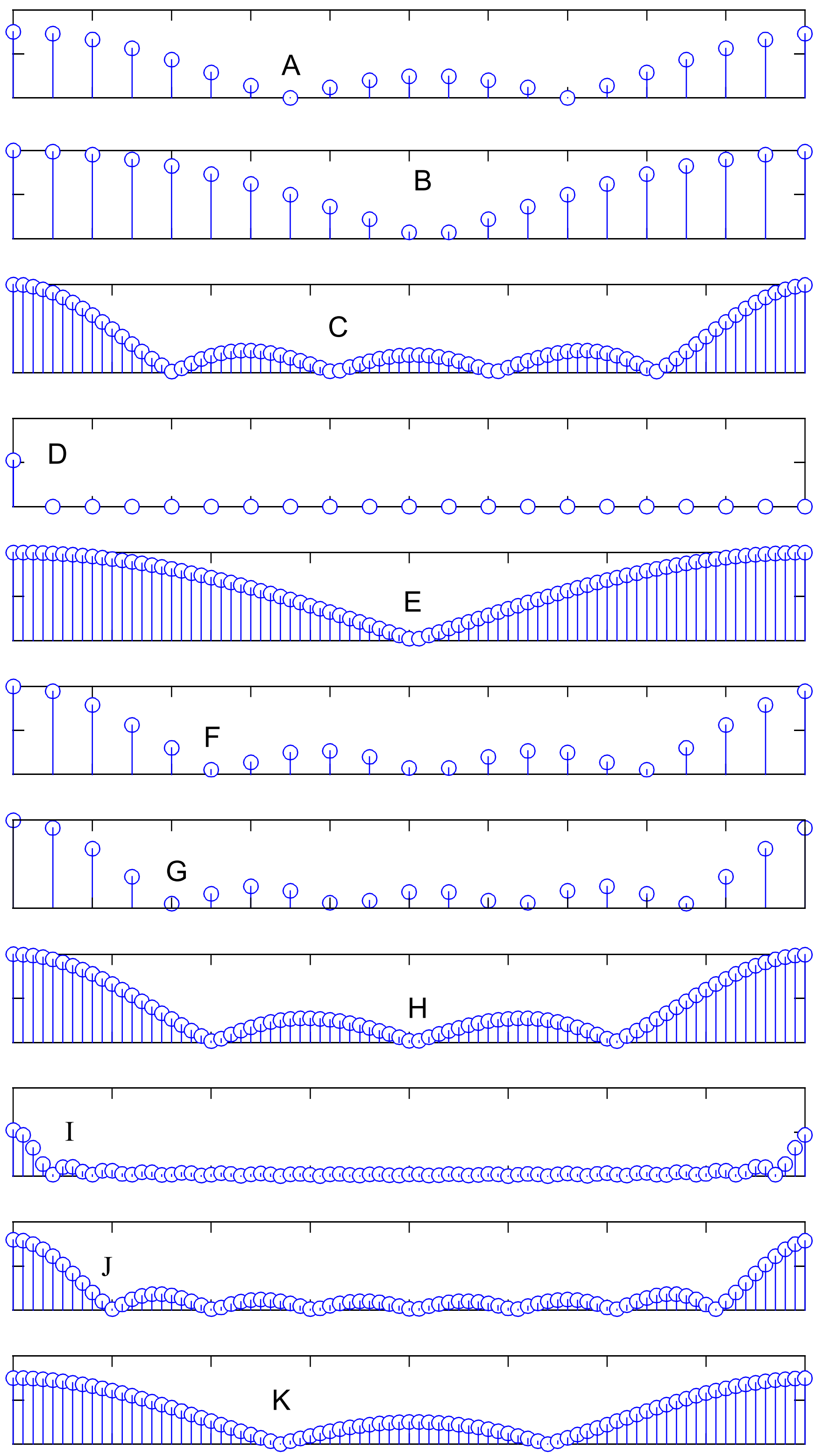
But $Y(z) = 10 \quad \Rightarrow \quad \dfrac{B}{10} = 0.2 \quad \Rightarrow \quad B = 2$

PROB. Su18-Fin.6.

Shown below are eleven different outcomes that result from executing the MATLAB code:

```
stem(abs(fft(ones(1,L),N)));
```

Match each plot with the corresponding values for the variables N and L by writing a letter (A through K) in each answer box.



L = 2, N = 21	B
L = 2, N = 81	E
L = 3, N = 21	A
L = 3, N = 81	K
L = 4, N = 21	F
L = 4, N = 81	H
L = 5, N = 21	G
L = 5, N = 81	C
L = 8, N = 81	J
L = 21, N = 21	D
L = 21, N = 81	I

PROB. Su18-Fin.7. (The different parts are unrelated.)

(a) There are an infinite number of values for the delay parameter t_0 below such that:

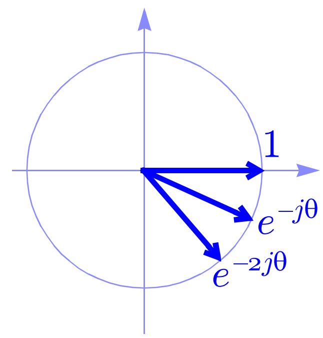
$$\cos(100\pi t) + \cos(100\pi(t - t_0)) + \cos(100\pi(t - 2t_0)) = 0, \qquad \text{for all time } t.$$

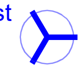
Name any two, both positive:

$t_0 = \boxed{\dfrac{1}{150}}$, or $t_0 = \boxed{\dfrac{1}{75}}$.

Phasor addition rule, where $\theta = 100\pi t_0$:

$$0 = 1 + e^{-j\theta} + e^{-j2\theta}$$



In order for these phasors to add to zero, they must form a "peace sign", with equally spaced angles 

$\Rightarrow$ add to zero when either

$$\theta = \frac{2\pi}{3} + m2\pi \qquad \Rightarrow t_0 = \frac{1+3m}{150} \in \left\{\frac{1}{150}, \frac{4}{150}, \frac{7}{150}, \frac{10}{150}, \ldots\right\}$$

or

$$\theta = \frac{4\pi}{3} + k2\pi \qquad \Rightarrow t_0 = \frac{2+3k}{150} \in \left\{\frac{2}{150}, \frac{5}{150}, \frac{8}{150}, \frac{11}{150}, \ldots\right\}$$

(b) Simplify

$$\sum_{k=-\infty}^{\infty} \frac{\sin(0.9\pi k)}{\pi k} \cdot \frac{\sin(0.75\pi(n-k))}{\pi(n-k)} = \boxed{\frac{\sin(0.75\pi n)}{\pi n}}.$$

a function of n

Recognize as the *convolution* of

$$h[n] = \frac{\sin(0.9\pi n)}{\pi n} \qquad \text{with} \qquad x[n] = \frac{\sin(0.75\pi n)}{\pi n}:$$

$$y[n] = x[n] * h[n] \Rightarrow \quad Y(e^{j\hat{\omega}}) = \underbrace{X(e^{j\hat{\omega}})}_{\text{(narrow)}}\underbrace{H(e^{j\hat{\omega}})}_{\text{(wide)}} = X(e^{j\hat{\omega}})$$

(multiplying a narrow rectangle by a wide rectangle yields a narrow rectangle) $\Rightarrow \quad y[n] = x[n].$

(c) The signal $x(t) = \cos(88\pi t)\cos(8\pi t)) + \cos(240\pi t)$ is periodic with fundamental frequency $f_0 = \boxed{8}$ Hz.

Multiplying the 44-Hz sinusoid by the 4-Hz sinusoid leads to the sum of two sinusoids, one with frequency 40 Hz and the other with frequency 48 Hz, so overall $x(t)$ has contributations at three frequencies:

$$\begin{aligned} 40 \text{ Hz} &= (8)(5) \\ 48 \text{ Hz} &= (8)(6) \\ 120 \text{ Hz} &= (8)(15) \qquad \Rightarrow \text{greatest common factor is } 8 \end{aligned}$$

(d) If $\{X[0], \ldots X[511]\}$ are the DFT coefficients that result from taking a 512-point DFT of the length-12 signal $x[n] = [1\ 1\ 1\ 2\ 2\ 2\ 3\ 3\ 3\ 4\ 4\ 4]$ for $n \in \{0, \ldots 11\}$, then find numerical values for the following two DFT coefficients:

$X[0] = \boxed{30}$,

$$X[k] = \sum_n x[n] e^{-jk2\pi n/N}$$

$X[256] = \boxed{-2}$.

$$\Rightarrow X[0] = \sum_n x[n] = 1 + 1 + 1 + 2 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 4 = 30$$

$$\Rightarrow X[256] = \sum_n x[n] e^{-j\pi n} = 1 - 1 + 1 - 2 + 2 - 2 + 3 - 3 + 3 - 4 + 4 - 4 = -2$$