GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2016
Final Exam

August 1, 2016

NAME: ANSWER KEY (FIRST) (LAST)   GT username: ________ (e.g., gtxyz123)

Circle your recitation section (otherwise you lose 3 points!):

	Tue
10 – 11:45am	L01 (Stuber)
12 – 1:45pm	L02 (Stuber)
2 – 3:45pm	L03 (Zhang)
4 – 5:45pm	L04 (Zhang)

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5” × 11”) of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
  - You must write your answer in the space provided on the exam paper itself.
    Only these answers will be graded. Circle your answers, or write them in the boxes provided.
    If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
No/Wrong Rec	–3	
Total		

PROB. SU16-Final-1.

Suppose that a sinusoidal signal $x(t) = 0.707\cos(100\pi t + 0.3\pi)$ is added to a delayed version of itself, resulting in the *zero* signal:

$$x(t) + x(t - t_0) = 0.$$

This equation does not uniquely specify the value of the delay t_0; there are many possible values it might take. Name any *three*. Make them all positive ($t_0 > 0$).

$t_0 = \boxed{0.01}$ sec, or $t_0 = \boxed{0.03}$ sec, or $t_0 = \boxed{0.05}$ sec.

Phasor addition: $1 + e^{-j100\pi t_0} = 0$

$\Rightarrow 100\pi t_0$ is an odd multiple of π

$\Rightarrow 100\pi t_0 = (2m + 1)\pi$

$\Rightarrow t_0 = (2m + 1)/100 \in \{0.01,\ 0.03,\ 0.05,\ 0.07,\ \dots\}$

PROB. SU16-Final-2.

Consider an LTI system with input $x[n]$ and output $y[n]$ defined by the difference equation:

$$y[n] = x[n] + \beta x[n-3] + x[n-6],$$

where β is a real constant to be determined. (It might take a different value in each part below.)

(a) If a constant input $x[n] = 7$ results in a constant output $y[n] = 3.5$, then it must be that $\beta = \boxed{-1.5}$.

DC gain $1 + \beta + 1 = 0.5$

(b) If a sinusoidal input $x[n] = \cos(\pi n/3)$ results in a *zero* output $y[n] = 0$, then it must be that $\beta = \boxed{2}$.

$$H(e^{j\pi/3}) = 1 + \beta e^{-j3\pi/3} + e^{-6j\pi/3} = 1 - \beta + 1 = 0$$

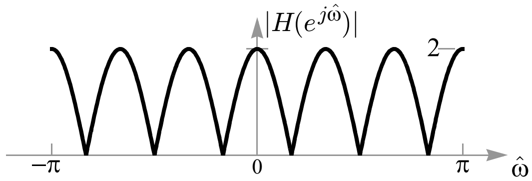
(c) Let $s[n]$ denote the *step* response (the output in response to unit step $u[n]$). If $s[5] = 2.6$, then it must be that $\beta = \boxed{1.6}$.

$$y[5] = x[n] + \beta x[n-3] + x[n-6],$$

$$\Rightarrow s[5] = u[5] + \beta u[5-3] + u[5-6]$$

$$= 1 + \beta = 2.6$$

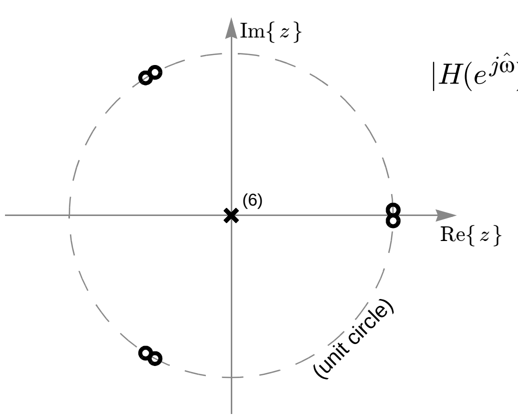
(d) If the magnitude response is as shown below,



then it must be that $\beta = \boxed{0}$.

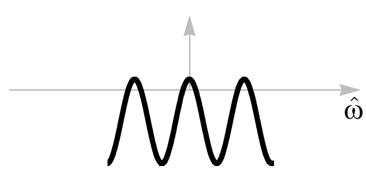
DC gain $1 + \beta + 1 = 2$

(e) If the pole-zero plot is as shown below, then the value of β is (circle one):



$$|H(e^{j\hat{\omega}})| = |1 + \beta e^{-3j\hat{\omega}} + e^{-6j\hat{\omega}}| = \beta + 2\cos(3\hat{\omega})$$

Shift down the sinusoid by $\beta = -1.99$
to get the two zeros near DC:



$\beta = -2.01$
$\beta = -1.99$
$\beta = -1.01$
$\beta = -0.99$
$\beta = -0.01$
$\beta = 0.01$
$\beta = 0.99$
$\beta = 1.01$
$\beta = 1.99$
$\beta = 2.01$

PROB. SU16-Final-3. Consider the six lines of MATLAB code shown below:

```
RAMBLIN = 4 ;
WRECK   = 1600 ;
fsamp   = 2400 ;
```

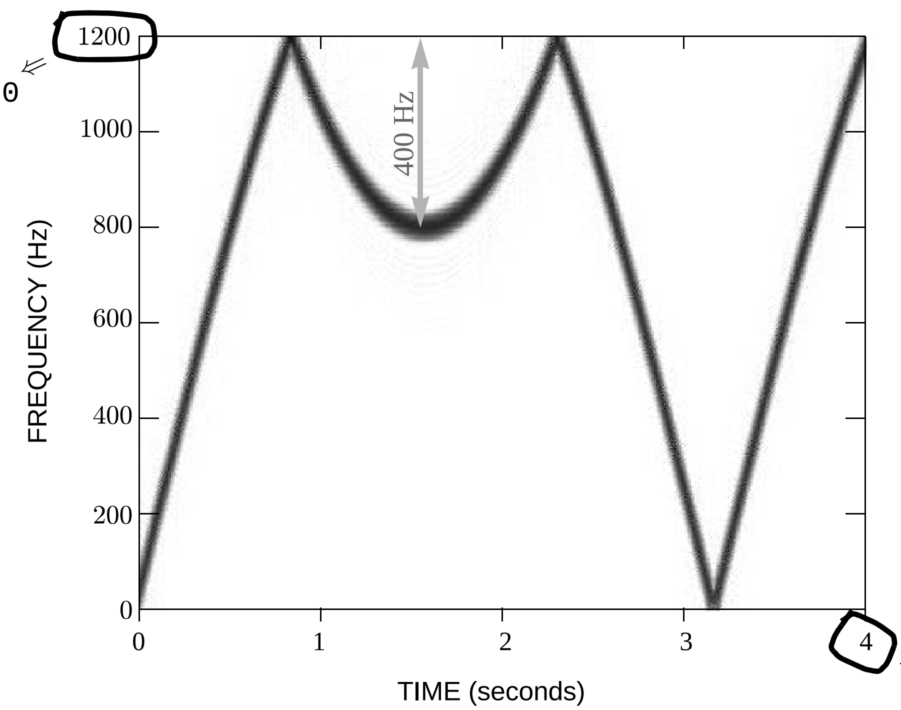
$x(t) = A\cos(\psi(t) + \theta)$, where
$\psi(t) = 2\pi W\cos(t)$
$\Rightarrow f_i(t) = \frac{1}{2\pi}\psi'(t) = -W\sin(t)$

If no aliasing, maximum instantaneous frequency would be:
$W = \texttt{WRECK} = 1200 + 400$
$= 1600$ Hz

```
tt = 0:(1/fsamp):RAMBLIN;
xx = real((20 + 26*j)*exp(j*2*pi*WRECK*cos(tt)));
spectrogram(xx, ... );   % see footnote1
```

Find numerical values for the three unspecified parameters `RAMBLIN`, `WRECK`, and `fsamp` so that running the above code produces the following spectrogram:

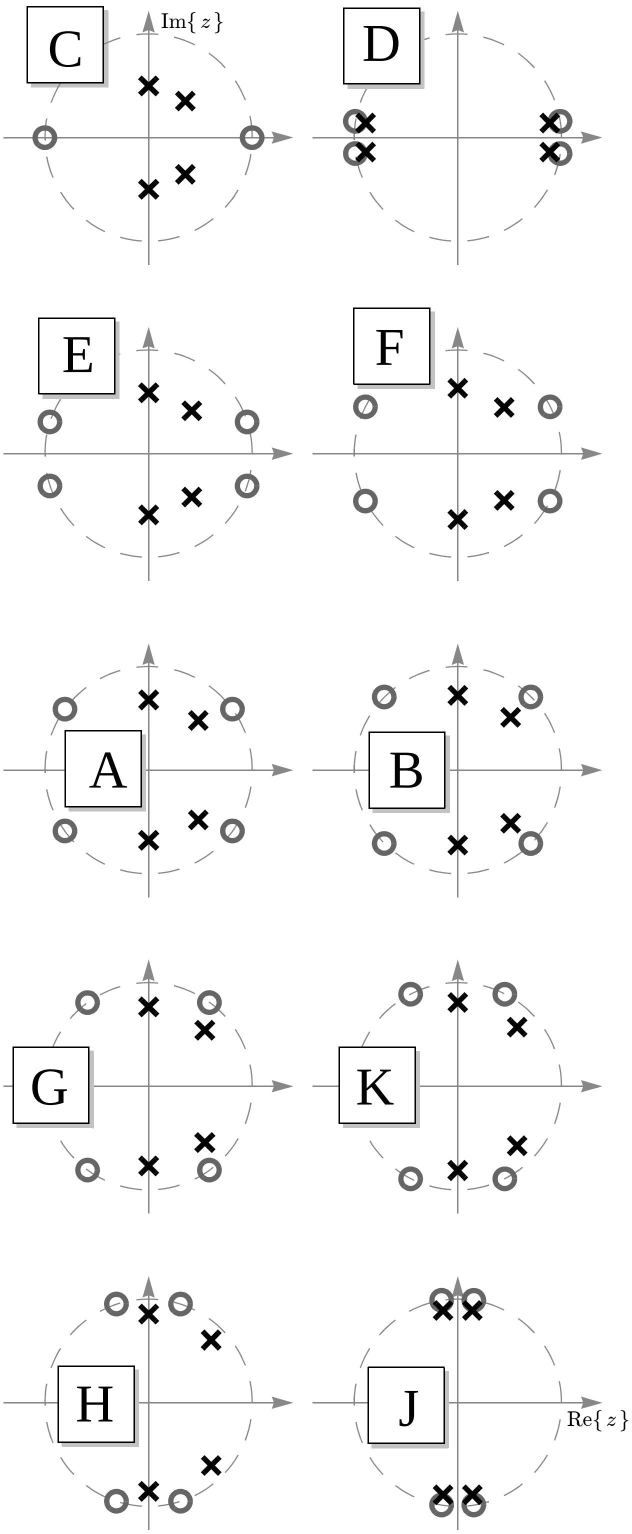
fsamp = 2400 $\Leftarrow$

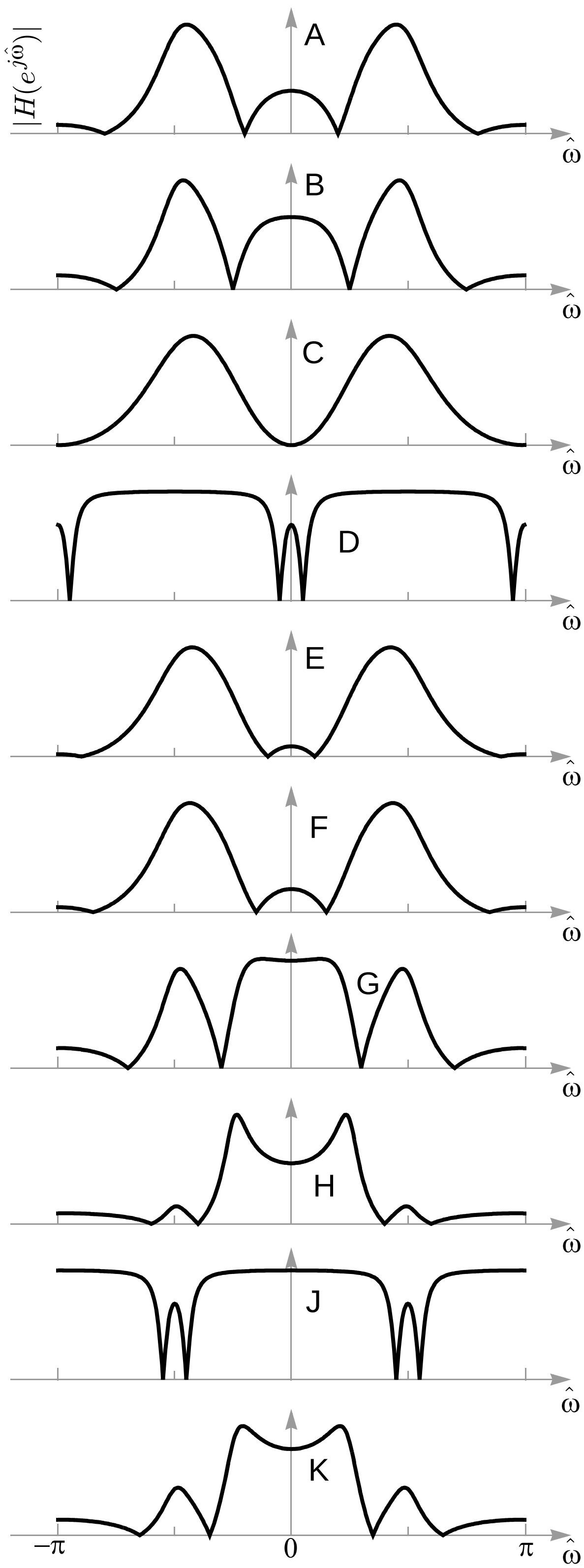


$\Rightarrow$ RAMBLIN = 4

1. To avoid confusion, the remaining arguments of `spectrogram` are not shown. They are not relevant. If you are curious, however, the complete command is `spectrogram(xx,128,120,512,fsamp,'yaxis')`.

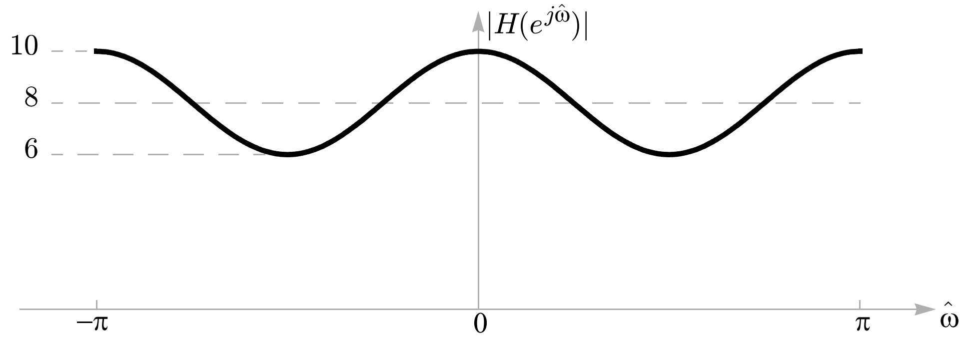
PROB. SU16-Final-4. Shown on the left are the pole-zero plots for ten LTI systems. Shown on the right are the corresponding magnitude responses, labeled A through K, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (A through K) in each answer box.





PROB. SU16-Final-5.

Shown below is the magnitude response of an FIR filter:



If the difference equation for this FIR system is:

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + b_4 x[n-4],$$

with $b_0 = 1$, then the remaining FIR filter coefficients are:

$b_1 = \boxed{0}$, $b_2 = \boxed{8}$, $b_3 = \boxed{0}$, $b_4 = \boxed{1}$.

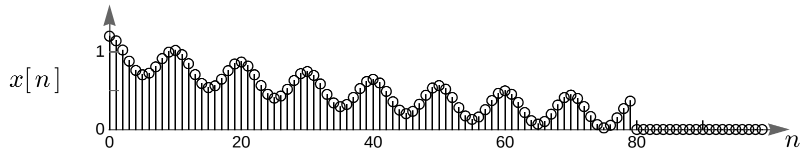
From the plot: $|H(e^{j\hat{\omega}})| = 8 + 2\cos(2\hat{\omega}) = 8 + e^{2j\hat{\omega}} + e^{-2j\hat{\omega}}$

which would arise from: $h[-2] = 1,\quad h[0] = 8,\quad h[2] = 1$

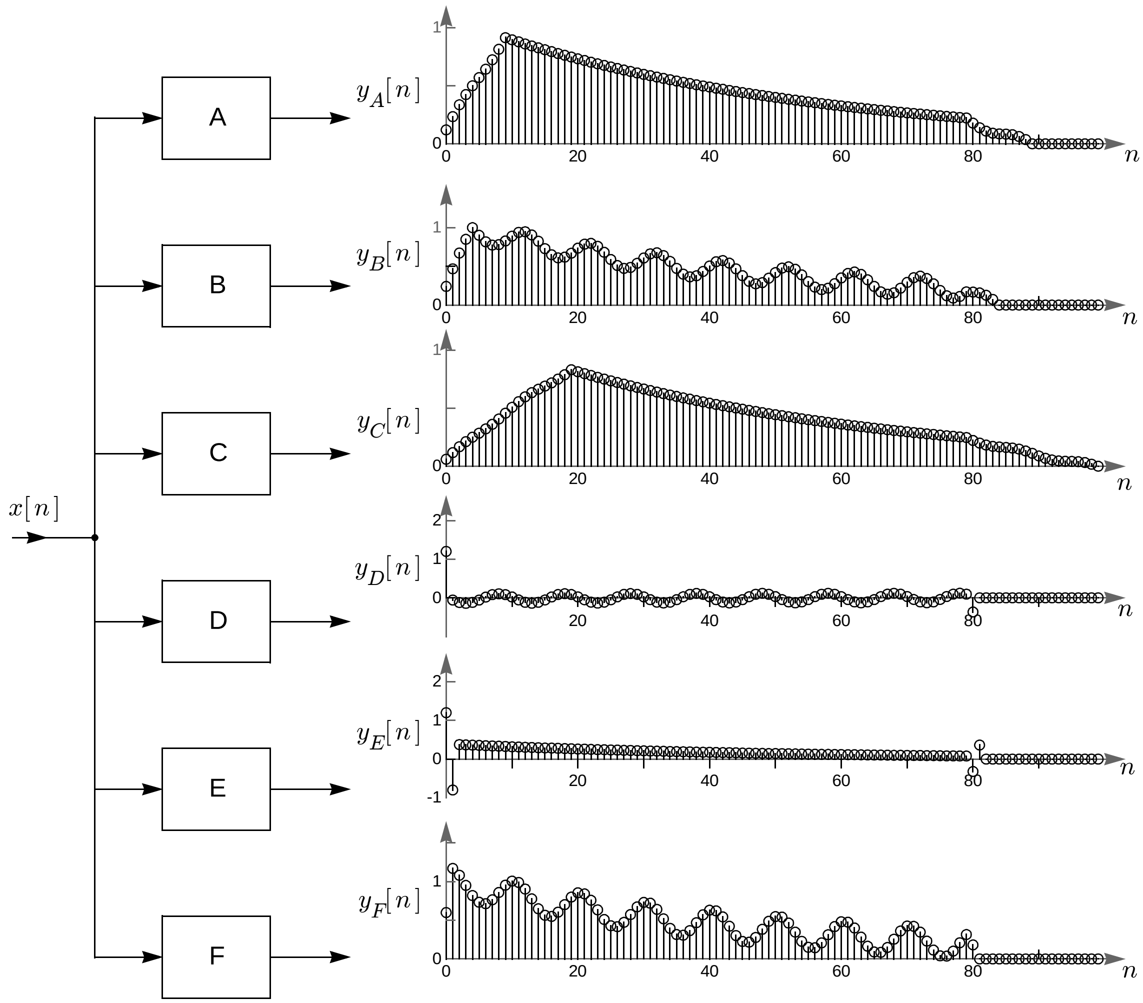
$\Rightarrow$ delay by 2 to get causal system: $h[0] = 1,\quad h[2] = 8,\quad h[4] = 1$

(delay does not change magnitude response)

PROB. SU16-Final-6. Consider the discrete-time signal $x[n]$ shown below (it is zero for all time $n < 0$):



As shown below, this signal is fed as an input to six different FIR systems (labeled A through F), producing the six different outputs $y_A[n]$ through $y_F[n]$ shown in the stem plots below:



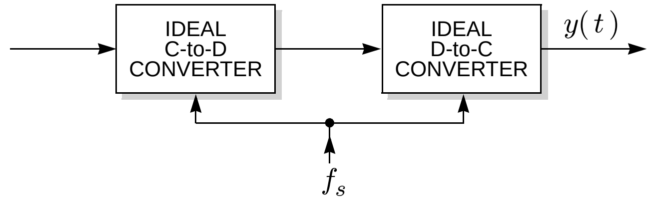
Match each system above to its description below by writing a letter (A through F) in each answer box:

D	first difference filter	B	5-point running average filter
E	$y[n] = x[n] - 2\cos(0.2\pi)x[n-1] + x[n-2]$	A	10-point running average filter
F	2-point running average filter	C	20-point running average filter

PROB. SU16-Final-7. The signal $x(t) = \cos(240\pi t) + \cos(250\pi t)$ is periodic with fundamental frequency $f_0 = \boxed{5}$ Hz.

Suppose we pass this signal through the cascade of an ideal C-to-D converter and an ideal D-to-C converter, as shown below, where both have the same (unspecified) sampling rate parameter f_s:

$x(t) = \cos(240\pi t) + \cos(250\pi t)$



(a) Specify a value for f_s <u>in the range</u> $240 < f_s < 250$ Hz so that $y(t)$ is periodic with fundamental frequency $f_0 = 120$ Hz: $f_s = \boxed{245}$ samples/sec.

(b) Specify a value for f_s <u>in the range</u> $240 < f_s < 250$ Hz so that $y(t)$ is periodic with fundamental frequency $f_0 = 4$ Hz: $f_s = \boxed{249}$ samples/sec.

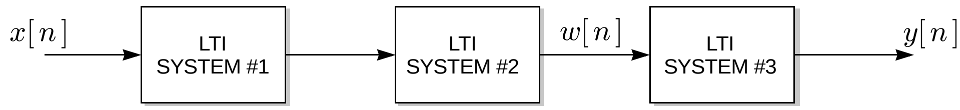
(c) Specify a value for f_s <u>in the range</u> $240 < f_s < 250$ Hz so that $y(t)$ is periodic with fundamental frequency $f_0 = 3$ Hz: $f_s = \boxed{248}$ samples/sec.

(d) Specify a value for f_s <u>in the range</u> $240 < f_s < 250$ Hz so that $y(t)$ is periodic with fundamental frequency $f_0 = 2$ Hz: $f_s = \boxed{247}$ samples/sec.

(e) Specify a value for f_s <u>in the range</u> $240 < f_s < 250$ Hz so that $y(t)$ is periodic with fundamental frequency $f_0 = 1$ Hz: $f_s = \boxed{246}$ samples/sec.

PROB. SU16-Final-8.

Shown below is the serial cascade of three LTI systems:



- The first system is defined by an impulse response whose Z transform is $H_1(z) = 2 - 0.4z^{-1}$.
- The second system is defined by the frequency response $H_2(e^{j\hat{\omega}}) = 5 - 1.5e^{-j\hat{\omega}}$.
- The third system (with input $w[n]$ and output $y[n]$) is defined by the difference equation:

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 w[n] + b_1 w[n-1].$$

If the third system *inverts* the previous two, so that its output $y[n]$ is the same as the original input $x[n]$ (satisfying $y[n] = x[n]$), then its coefficients must be:

$$a_1 = \boxed{0.5}, \; a_2 = \boxed{-0.06}, \; b_0 = \boxed{0.1}, \; b_1 = \boxed{0}.$$

First two systems: $H_{12}(z) = (2 - 0.4z^{-1})(5 - 1.5z^{-1})$

$$= 10 - 5z^{-1} + 0.6z^{-2}$$

$$= 10(1 - 0.5z^{-1} + 0.06z^{-2})$$

Third system: $H_3(z) = \dfrac{1}{H_{12}(z)} = \dfrac{0.1}{1 - 0.5z^{-1} + 0.06z^{-2}} = \dfrac{Y(z)}{W(z)}$

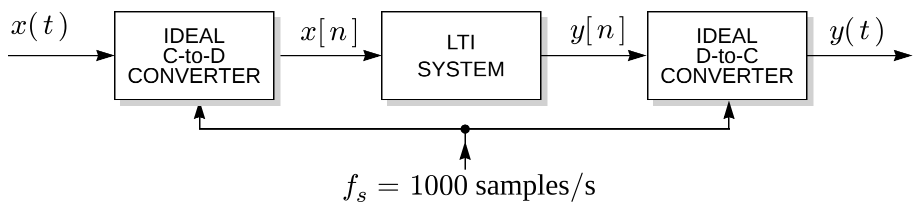
$$\Rightarrow \quad (1 - 0.5z^{-1} + 0.06z^{-2})Y(z) = 0.1W(z)$$

$$\Rightarrow \quad y[n] - 0.5y[n-1] + 0.06y[n-2] = 0.1w[n]$$

$$\Rightarrow \quad y[n] = \underbrace{0.5}_{a_1} y[n-1] \underbrace{-\,0.06}_{a_2} y[n-2] + \underbrace{0.1}_{b_0} w[n]$$

PROB. SU16-Final-9.

Consider the following system with input $x(t)$ and output $y(t)$:



The sampling rate for both the C-to-D and D-to-C is $f_s = 1000$ samples/s. The LTI system is defined by the difference equation:

$$y[n] = 0.5x[n] + 4x[n-1] + 0.5x[n-2].$$

If the input is $x(t) = 2\cos(400\pi t) + 4\cos(500\pi t)$, then the output is:

$$y(t) = A_1\cos(2\pi f_1 t + \theta_1) + A_2\cos(2\pi f_2 t + \theta_2), \text{ where:}$$

$A_1 = \boxed{8.618}$, $f_1 = \boxed{200}$ Hz, $\theta_1 = \boxed{-0.4\pi}$ rads,

$A_2 = \boxed{16}$, $f_2 = \boxed{250}$ Hz, $\theta_2 = \boxed{-0.5\pi}$ rads.

Sampling rate high enough to avoid aliasing

$\Rightarrow$ output frequencies same as input frequencies, 200 and 250 Hz

Digital frequencies after sampling:

$$\hat{\omega}_1 = 400\pi/1000 = 0.4\pi$$
$$\hat{\omega}_2 = 500\pi/1000 = 0.5\pi$$

Evaluating frequency response $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(4 + \cos(\hat{\omega}))$ at these frequencies yields:

$$H(e^{j\hat{\omega}_1}) = e^{-j0.4\pi}(4 + \cos(0.4\pi)) = 4.39e^{-j0.4\pi} \quad \Rightarrow A_1 e^{j\theta_1} = 2(\cdot) = 8.618e^{-j0.4\pi}$$

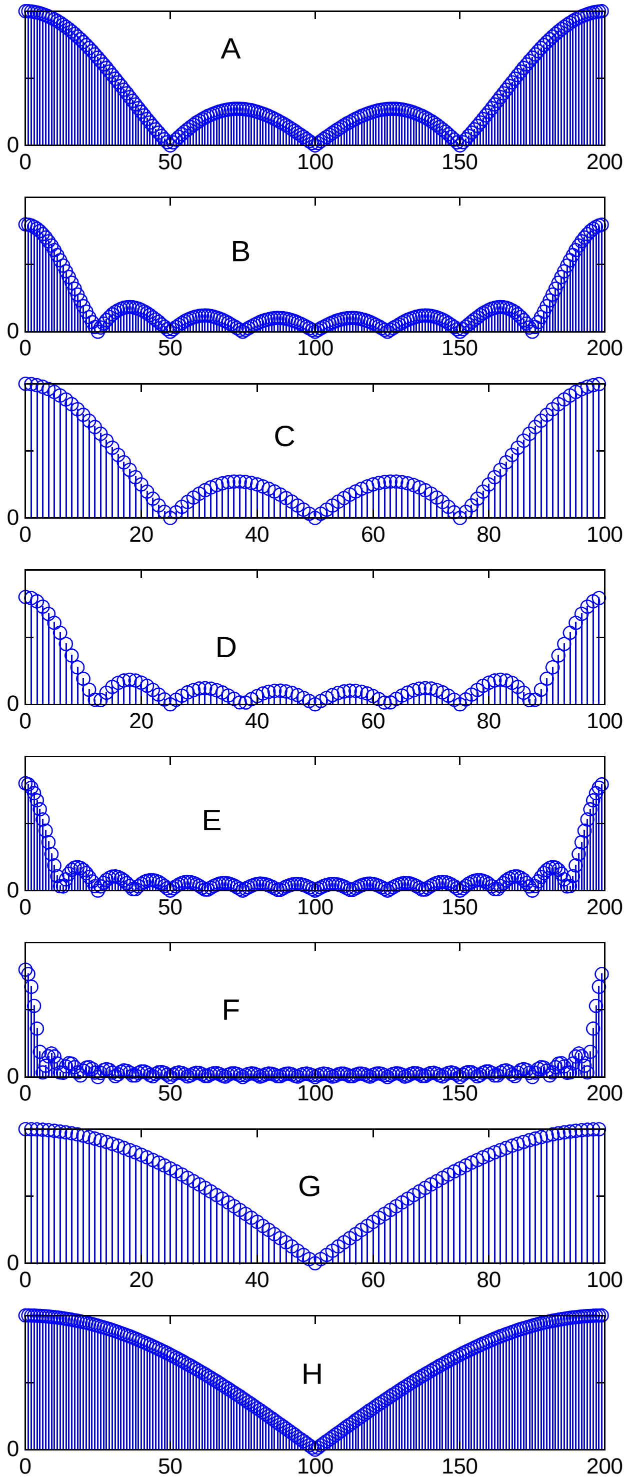
$$H(e^{j\hat{\omega}_2}) = e^{-j0.5\pi}(4 + \cos(0.5\pi)) = 4e^{-j0.5\pi} \quad \Rightarrow A_2 e^{j\theta_2} = 4(\cdot) = 16e^{-j0.5\pi}$$

PROB. SU16-Final-10.

Shown below are eight different outcomes that result from executing the MATLAB code:

```
stem(abs(fft(ones(1,L),N)));
```

Match each plot with the corresponding values for the variables N and L by writing a letter (A through H) in each answer box.



`L = 2, N = 100`	G
`L = 2, N = 200`	H
`L = 4, N = 100`	C
`L = 4, N = 200`	A
`L = 8, N = 100`	D
`L = 8, N = 200`	B
`L = 16, N = 200`	E
`L = 32, N = 200`	F