GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2014
Final Exam

July 28, 2014

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section (otherwise you lose 3 points!):

	Mon	Tue
10 – 11:45am		L02 (Moore)
12 – 1:45pm		L03 (Moore)
2:40 – 3:50pm		L04 (Davis)
4 – 5:45pm	L01 (Barry)	

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
    - You must write your answer in the space provided on the exam paper itself.
      Only these answers will be graded. Circle your answers, or write them in the boxes provided.
      If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10+5	
No/Wrong Rec	–3	
Total		

PROB. SU14-Final-1. (The different parts of this problem are unrelated.)

(a) (Recall that the step response $s[n]$ of a system is its output when the unit step $u[n]$ is applied as input.)
If the step response of an LTI system is $s[n] = \delta[n] + 2\delta[n-2]$,
then it's impulse response $h[n]$ at times $n \in \{0,\ 1,\ 2,\ 3\}$ must be:

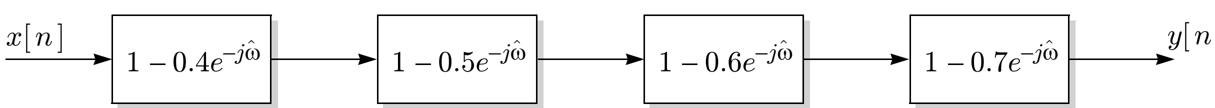
$h[0] =$ ☐

$h[1] =$ ☐

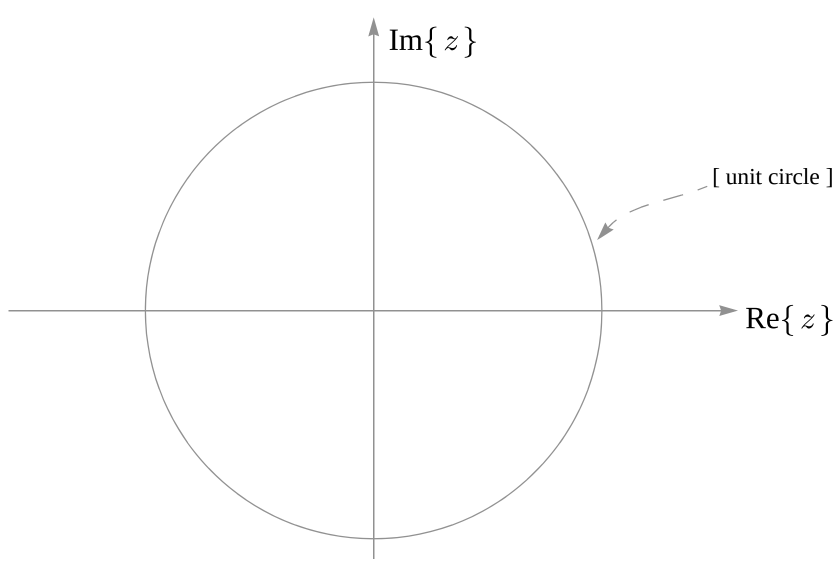
$h[2] =$ ☐

$h[3] =$ ☐

(b) Shown below is an *overall* system with input $x[n]$ and output $y[n]$ formed by connecting four FIR filters in cascade:



The frequency response of each FIR filter is as indicated in the figure.
In the space below, carefully sketch the pole-zero plot for the *overall* system
(be sure to show *all* zeros and *all* poles):



PROB. SU14-Final-2. (The different parts of this problem are unrelated.)

(a) If running the following MATLAB code:

```
soundsc(cos(2*pi*f0*(0:0.001:4)),fsamp);
```

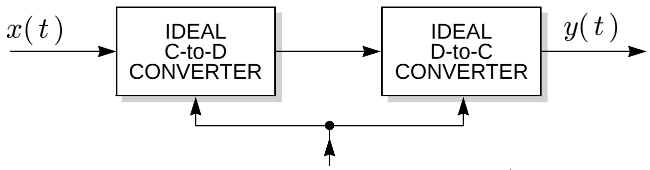
results in hearing a 220-Hz tone lasting two seconds from the computer speaker, the values of the unspecified parameters **f0** and **fsamp** must be:

f0 = ______ **fsamp** = ______ .

(b) If $X(z) = \dfrac{1 + 0.5z^{-2}}{1 - 0.5z^{-1}}$, then $x[n]$ at time 3 is $x[3] =$ ______ .

PROB. SU14-Final-3.

Suppose an ideal C-to-D converter is connected to an ideal D-to-C converter, as shown below, where both have the same sampling rate parameter $f_s = 800$ samples/s:



(a) If the input $x(t)$ is a sinusoid, is the ouput $y(t)$ necessarily a sinusoid? YES ☐ NO ☐

(b) If the output $y(t)$ is a sinusoid, is the input $x(t)$ necessarily a sinusoid? YES ☐ NO ☐

(c) Does there exist an input $x(t)$ such that the output is $y(t) = \cos(880\pi t)$? YES ☐ NO ☐

(d) If your answer to part (c) is YES, specify the input that does the job:

$x(t) =$ ☐ .

If your answer to part (c) is NO, explain why no such input exists.

(e) Suppose that a sinusoidal input of $x(t) = \cos(2\pi f_0 t)$ results in a sinusoidal output of $y(t) = \cos(440\pi t)$. This fact does not uniquely specify the the input frequency f_0; there are many possible values it might take. Name any *three*.

$f_0 =$ ☐ Hz, or $f_0 =$ ☐ Hz, or $f_0 =$ ☐ Hz.

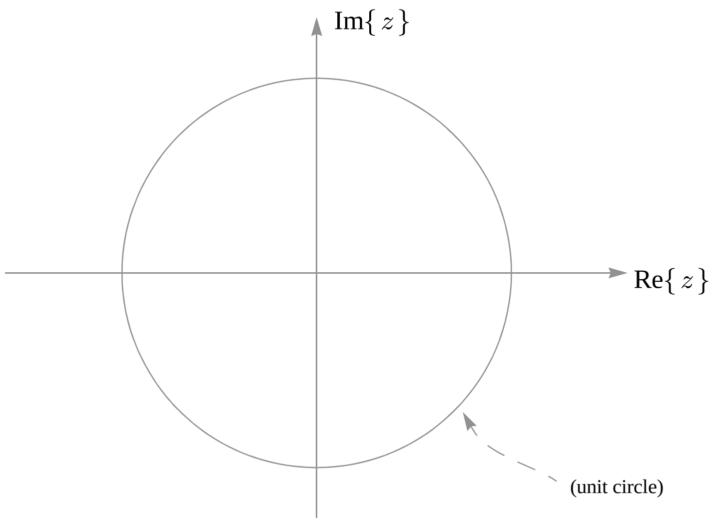
PROB. SU14-Final-4. Consider an initially at-rest LTI system whose difference equation is:

$$y[n] = -0.5y[n-1] + x[n] + x[n-1] + x[n-2].$$

(a) The system is [FIR][IIR] (circle one).

(b) The system is [stable][unstable] (circle one).

(c) Carefully sketch the pole-zero plot for this system:



(d) Let $s[n]$ denote the system's step response (*i.e.*, the output in response to a step input, $x[n] = u[n]$.) The step response $s[n]$ at times $n \in \{0, 1, 2\}$ is:

$s[0] =$ ☐

$s[1] =$ ☐

$s[2] =$ ☐

(e) In the limit as time $n \to \infty$, the step response approaches a *constant* value of $s[\infty] =$ ☐ .

PROB. SU14-Final-5.

Consider the signal: $x(t) = \cos(36\pi t) + \cos(90\pi t) - A_3\cos(2\pi f_3 t)$,

where the amplitude $A_3 > 0$ and frequency $f_3 > 0$ of the third sinusoid are not specified. (They may be different in each part below.)

(a) Sketch the two-sided spectrum in the space below, assuming that $A_3 = 1.1$ and $f_3 = 46$ Hz, taking care to **label** the frequency **and** complex amplitude (in polar form) of each line:

f (Hz)

(b) If $f_3 = 3$ Hz, then the fundamental fundamental frequency of $x(t)$ is $f_0 =$ ______ Hz.

(c) If $f_3 = 36$ Hz, then the fundamental fundamental frequency of $x(t)$ is $f_0 =$ ______ Hz.

(d) If $x(t)$ is periodic with a fundamental frequency of $f_0 = 18$ Hz then $f_3 =$ ______ Hz.

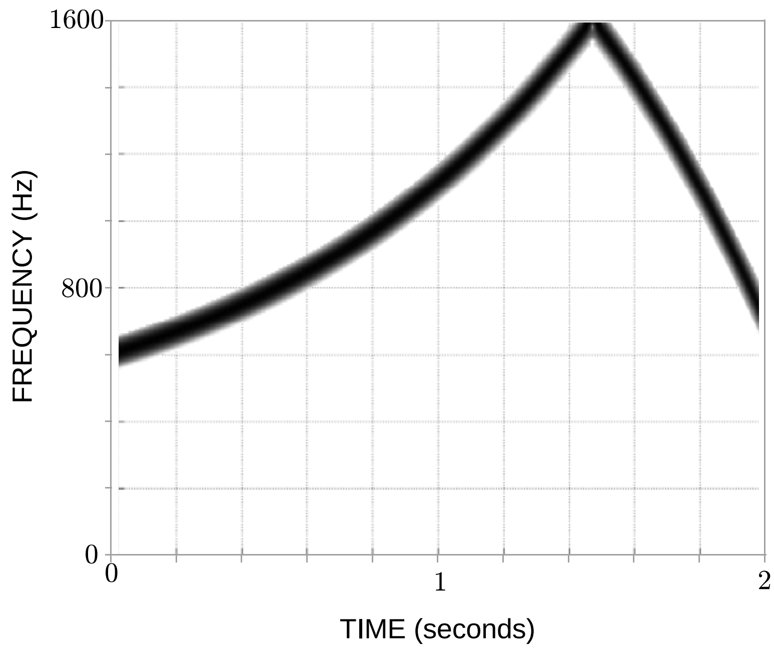
(e) If $x(t)$ is periodic with a fundamental frequency of $f_0 = 45$ Hz then $A_3 =$ ______ .

(f) An example of an f_3 for which $x(t)$ is *not* periodic is $f_3 =$ ______ Hz.

PROB. SU14-Final-6. Consider the six lines of MATLAB code shown below:

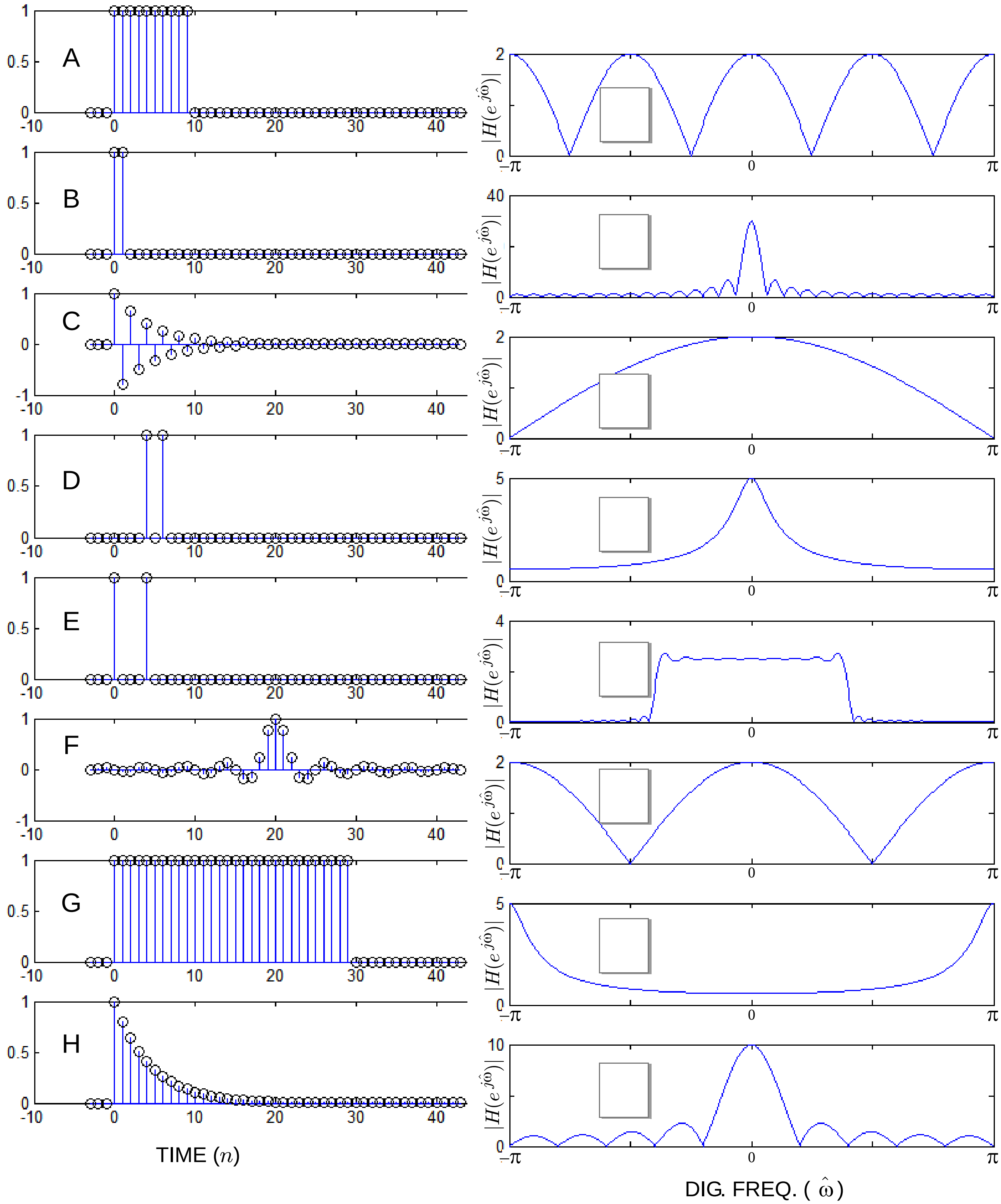
```
dur   =                ;

fsamp =                ;

ALPHA =                ;

tt = 0:(1/fsamp):dur;
xx = real((1 + 2*j)*exp(j*2*pi*ALPHA*(tt + exp(tt))));
spectrogram(xx, ... );   % see footnote[1]
```

Find numerical values for the three unspecified parameters `dur`, `fsamp`, and `ALPHA` so that running the above code produces the following spectrogram:

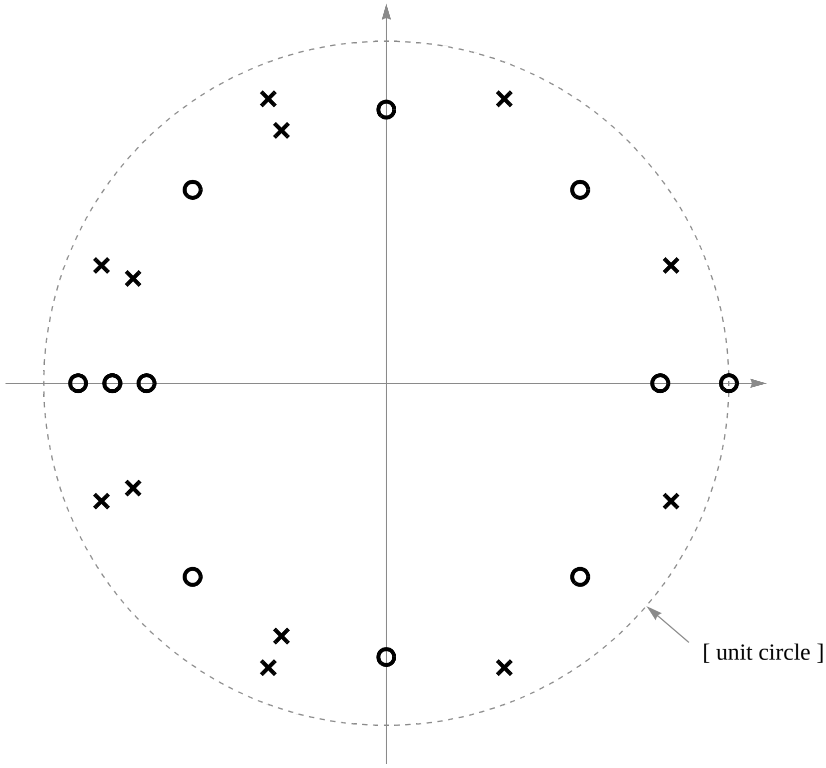


1. To avoid confusion, the remaining arguments of `spectrogram` are not shown. They are not relevant. If you are curious, however, the complete command is `spectrogram(xx,128,120,512,fsamp,'yaxis')`.

PROB. SU14-Final-7. Shown on the left are impulse responses of eight different filters, labeled A through H. (Values not shown are zero.) Shown on the right are the magnitude responses for these filters, but the order is scrambled. Match each magntiude response to its corresponding impulse response by writing a letter (A through H) in each answer box:



PROB. SU14-Final-8. Below is shown the pole-zero plot for an IIR system:



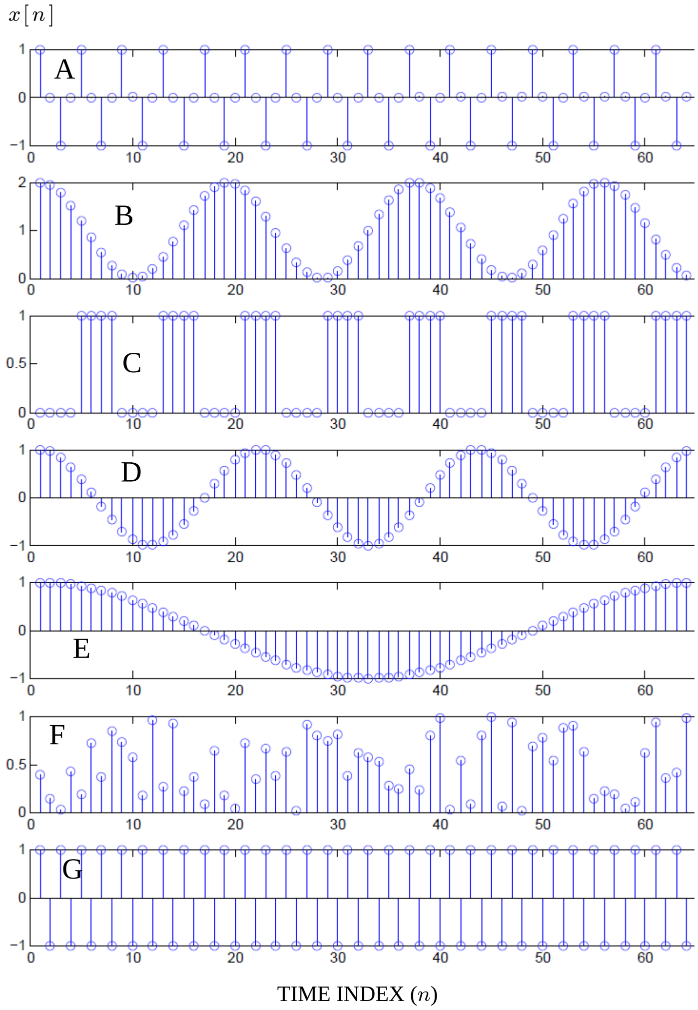
(a) The DC gain of the system is ☐ .

(b) *Estimate* (to within 20% or better) the value of $\hat{\omega}$ in the range $0 \le \hat{\omega} \le \pi$ that *maximizes* the system's magnitude response:

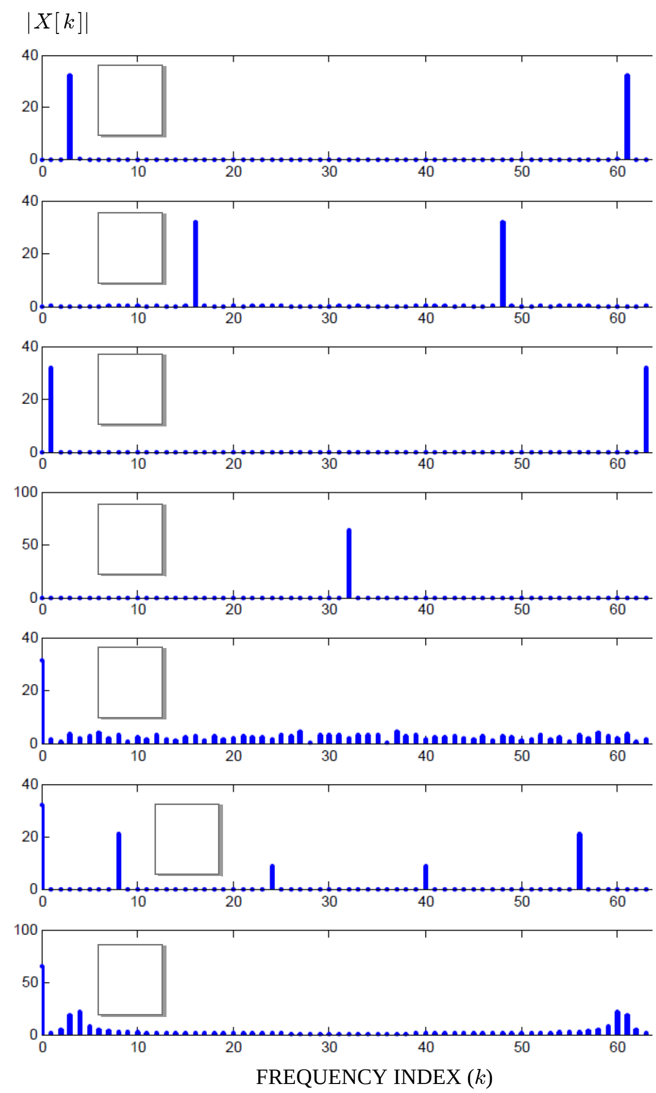
$|H(e^{j\hat{\omega}})|$ achieves a maximum at $\hat{\omega} \approx$ ☐ radians.

(You must *estimate* because there are too many zeros and poles to do an exact calculation, and further because you are not told the exact locations of these zeros and poles.)

PROB. SU14-Final-9. On the left below are stem plots of seven different length-64 signal vectors, labelled A through G. On the right below are sketches of the corresponding 64-point DFT magnitude spectrum. (The spectrum is represented by a stem plot of the coefficient magnitude $|X[k]|$ versus the frequency index $k \in \{0, 1, \ldots 63\}$.)

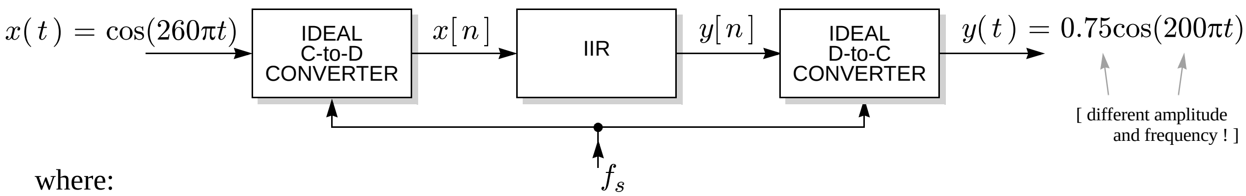


Match one of the seven signals $\{x[n]\}$ on the left to its corresponding DFT spectrum below by writing a letter (A through G) in each answer box:



PROB. SU14-Final-10.

Consider the following system for digitally filtering a continuous-time signal:



where:

- The input to the ideal C-to-D converter is $x(t) = \cos(260\pi t)$.
- The discrete-time IIR system is defined by the recursive difference equation:

$$y[n] = a_1 y[n-1] + x[n] + 0.25x[n-2].$$

- The sampling rate (common to both the C-to-D and D-to-C converters) satisfies $f_s > 200$ samples/s.

If the output of the ideal D-to-C converter is $y(t) = 0.75\cos(200\pi t)$,
then the unspecified parameters f_s and a_1 must be:

$f_s =$ ____________ samples/s

$a_1 =$ ____________ .

(5 BONUS POINTS
IF YOU GET THIS RIGHT!)

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$u[n]-u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}e^{-j\hat{\omega}(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b)-u(\hat{\omega}-\hat{\omega}_b)=\begin{cases}1 & \lvert\hat{\omega}\rvert\le\hat{\omega}_b\\ 0 & \hat{\omega}_b<\lvert\hat{\omega}\rvert\le\pi\end{cases}$
$a^n u[n] \quad (\lvert a\rvert<1)$	$\dfrac{1}{1-ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)})=X(e^{j\hat{\omega}})$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(e^{j\hat{\omega}})+bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}})=X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (d=integer)	$x[n-d]$	$e^{-j\hat{\omega}d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)})+\frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n]*h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2$	$\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n = 0, 1, 2, \ldots, N-1$	*Frequency-Domain:* $X[k], \quad k = 0, 1, 2, \ldots, N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{-j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$u[n]-u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))} e^{-j(2\pi k/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))} e^{j(2\pi n/N)(L-1)/2}$	$N(u[k]-u[k-L])$, when $L \leq N$

Table of z-Transform Pairs		
Signal Name	*Time-Domain:* $x[n]$	*z-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}, \quad \lvert a\rvert < 1$
Decaying cosine	$r^n \cos(\hat{\omega}_0 n)u[n]$	$\dfrac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$
Decaying sinusoid	$Ar^n \cos(\hat{\omega}_0 n+\varphi)u[n]$	$A\dfrac{\cos(\varphi)-r\cos(\hat{\omega}_0-\varphi)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties		
Property Name	*Time-Domain* $x[n]$	*z-Domain* $X(z)$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay (d=integer)	$x[n-d]$	$z^{-d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

Date: 28-April-2013

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NAME: ANSWER KEY
(FIRST) (LAST)

GT username: ______________
(e.g., gtxyz123)

Circle your recitation section (otherwise you lose 3 points!):

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Important Notes:

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2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10+5	
No/Wrong Rec	–3	
Total		

PROB. SU14-Final-1. (The different parts of this problem are unrelated.)

(a) (Recall that the step response $s[n]$ of a system is its output when the unit step $u[n]$ is applied as input.)
If the step response of an LTI system is $s[n] = \delta[n] + 2\delta[n-2]$,
then it's impulse response $h[n]$ at times $n \in \{0, 1, 2, 3\}$ must be:

$$S(z) = U(z)H(z) = \frac{H(z)}{1 - z^{-1}}$$

$$\Rightarrow H(z) = (1 - z^{-1})S(z)$$

$$= (1 - z^{-1})(1 + 2z^{-2})$$

$$= 1 - z^{-1} + 2z^{-2} - 2z^{-3}$$

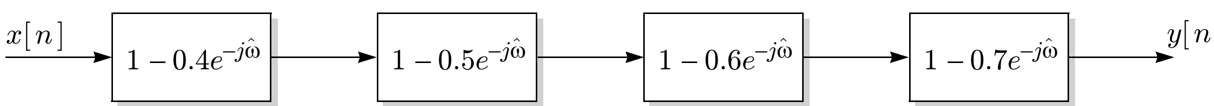
$h[0] =$ 1

$h[1] =$ –1

$h[2] =$ 2

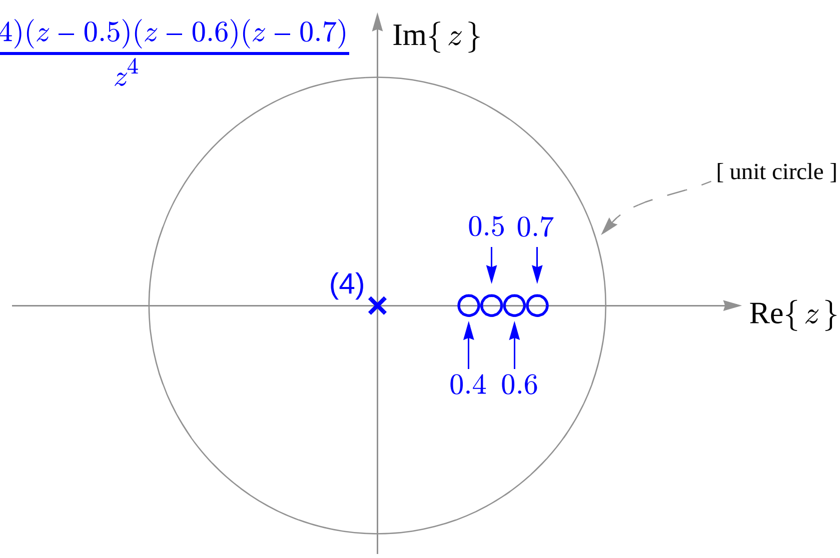
$h[3] =$ –2

(b) Shown below is an *overall* system with input $x[n]$ and output $y[n]$ formed by connecting four FIR filters in cascade:



The frequency response of each FIR filter is as indicated in the figure.
In the space below, carefully sketch the pole-zero plot for the *overall* system
(be sure to show *all* zeros and *all* poles):

$$H(z) = \frac{(z - 0.4)(z - 0.5)(z - 0.6)(z - 0.7)}{z^4}$$



PROB. SU14-Final-2. (The different parts of this problem are unrelated.)

(a) If running the following MATLAB code:

```
soundsc(cos(2*pi*f0*(0:0.001:4)),fsamp);
```

results in hearing a 220-Hz tone lasting two seconds from the computer speaker, the values of the unspecified parameters **`f0`** and **`fsamp`** must be:

`f0` = 110 `fsamp` = 2000 .

The input is 4-second tone sampled at 1000 Hz.

`soundsc` changes the duration from 4 seconds to 2 seconds

$\Rightarrow$ played twice as fast
$\Rightarrow$ sampling rate is doubled
$\Rightarrow$ heard frequency is doubled

(b) If $X(z) = \dfrac{1 + 0.5z^{-2}}{1 - 0.5z^{-1}}$, then $x[n]$ at time 3 is $x[3] = \dfrac{3}{8}$.

$$x[n] = (0.5)^n u[n] + 0.5(0.5)^{n-2} u[n-2]$$

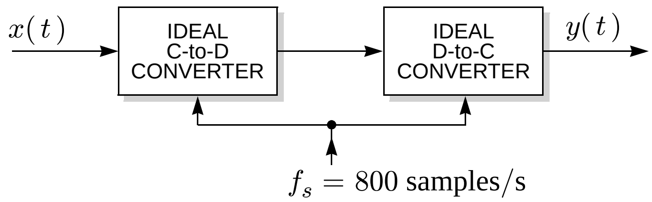
$$\Rightarrow x[3] = (0.5)^3 + 0.5(0.5)^{3-2}$$

$$= \frac{1}{8} + \frac{1}{4}$$

$$= \frac{3}{8}$$

PROB. SU14-Final-3.

Suppose an ideal C-to-D converter is connected to an ideal D-to-C converter, as shown below, where both have the same sampling rate parameter $f_s = 800$ samples/s:



(a) If the input $x(t)$ is a sinusoid, is the ouput $y(t)$ necessarily a sinusoid? YES [X] NO [ ]

(b) If the output $y(t)$ is a sinusoid, is the input $x(t)$ necessarily a sinusoid? YES [ ] NO [X] (crazy things might happen between samples)

(c) Does there exist an input $x(t)$ such that the output is $y(t) = \cos(880\pi t)$? YES [ ] NO [X]

(d) If your answer to part (c) is YES,
specify the input that does the job:

$x(t) =$ [ ].

If your answer to part (c) is NO, explain why no such input exists.

The maximum frequency out of an ideal D-C converter is $f_s/2 = 400$ Hz

(e) Suppose that a sinusoidal input of $x(t) = \cos(2\pi f_0 t)$
results in a sinusoidal output of $y(t) = \cos(440\pi t)$.
This fact does not uniquely specify the the input frequency f_0;
there are many possible values it might take. Name any *three*.

$f_0 =$ [220] Hz, or $f_0 =$ [1020] Hz, or $f_0 =$ [1820] Hz.

All answers are $220 + kf_s \in \{220,\ 1020,\ -580,\ 1820,\ -1380\ \ldots\}$

(any integer k): $k = 0$ $k = 1$ $k = -1$ $k = 2$ $k = -2$ ...

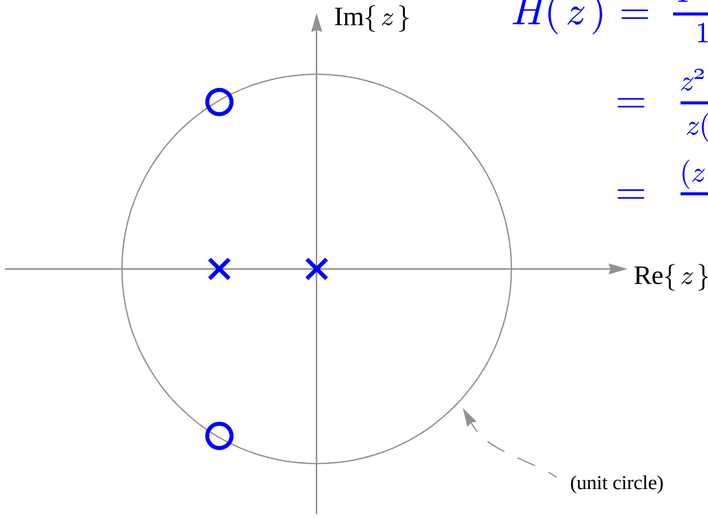
PROB. SU14-Final-4. Consider an initially at-rest LTI system whose difference equation is:

$$y[n] = -0.5y[n-1] + x[n] + x[n-1] + x[n-2].$$

(a) The system is [FIR] [IIR] (circle one).

(b) The system is [stable] [unstable] (circle one).

(c) Carefully sketch the pole-zero plot for this system:



$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 + 0.5z^{-1}} = \frac{z^2 + z + 1}{z(z + 0.5)} = \frac{(z - e^{j2\pi/3})(z - e^{-j2\pi/3})}{z(z + 0.5)}$$

(d) Let $s[n]$ denote the system's step response (*i.e.*, the output in response to a step input, $x[n] = u[n]$.) The step response $s[n]$ at times $n \in \{0, 1, 2\}$ is:

$$s[0] = -0.5(0) + 1 + 0 + 0 = 1$$

$$s[1] = -0.5(1) + 1 + 1 + 0 = 1.5$$

$$s[2] = -0.5(1.5) + 1 + 1 + 1 = 2.25$$

$s[0] =$ 1

$s[1] =$ 1.5

$s[2] =$ 2.25

(e) In the limit as time $n \to \infty$, the step response approaches a *constant* value of $s[\infty] =$ 2.

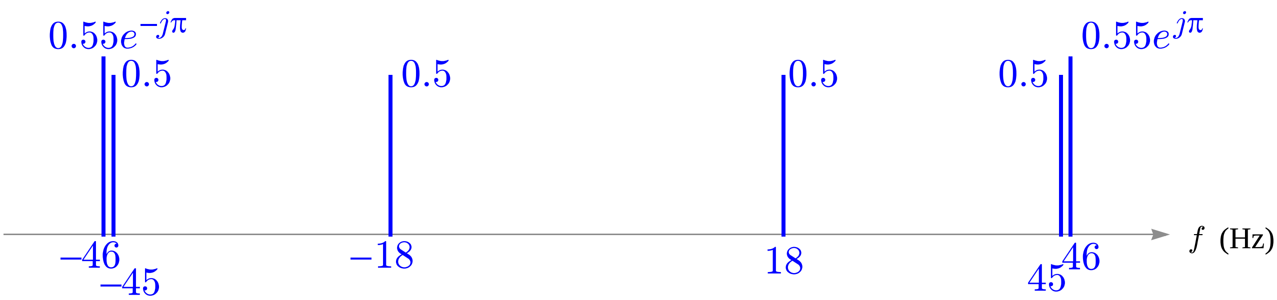
The DC gain is $H(1) = H(z)|_{z=1} = \frac{1 + 1 + 1}{1 + 0.5} = 2.$

PROB. SU14-Final-5.

Consider the signal: $x(t) = \cos(36\pi t) + \cos(90\pi t) - A_3\cos(2\pi f_3 t)$,

where the amplitude $A_3 > 0$ and frequency $f_3 > 0$ of the third sinusoid are not specified. (They may be different in each part below.)

(a) Sketch the two-sided spectrum in the space below, assuming that $A_3 = 1.1$ and $f_3 = 46$ Hz, taking care to **label** the frequency **and** complex amplitude (in polar form) of each line:



(b) If $f_3 = 3$ Hz, then the fundamental fundamental frequency of $x(t)$ is $f_0 =$ 3 Hz.

(c) If $f_3 = 36$ Hz, then the fundamental fundamental frequency of $x(t)$ is $f_0 =$ 9 Hz.

(d) If $x(t)$ is periodic with a fundamental frequency of $f_0 = 18$ Hz then $f_3 =$ 45 Hz.

(And $A_3 = 1$ so that 45-Hz is eliminated)

(e) If $x(t)$ is periodic with a fundamental frequency of $f_0 = 45$ Hz then $A_3 =$ 1.

(And $f_3 = 18$ Hz so that 18-Hz is eliminated)

(f) An example of an f_3 for which $x(t)$ is *not* periodic is $f_3 =$ $\sqrt{2}$ Hz.

or π

or any other irrational number

PROB. SU14-Final-6. Consider the six lines of MATLAB code shown below:

```
dur   = [ 2    ];
fsamp = [ 3200 ];
ALPHA = [ 300  ];

tt = 0:(1/fsamp):dur;
xx = real((1 + 2*j)*exp(j*2*pi*ALPHA*(tt + exp(tt))));
spectrogram(xx, ... );   % see footnote1
```

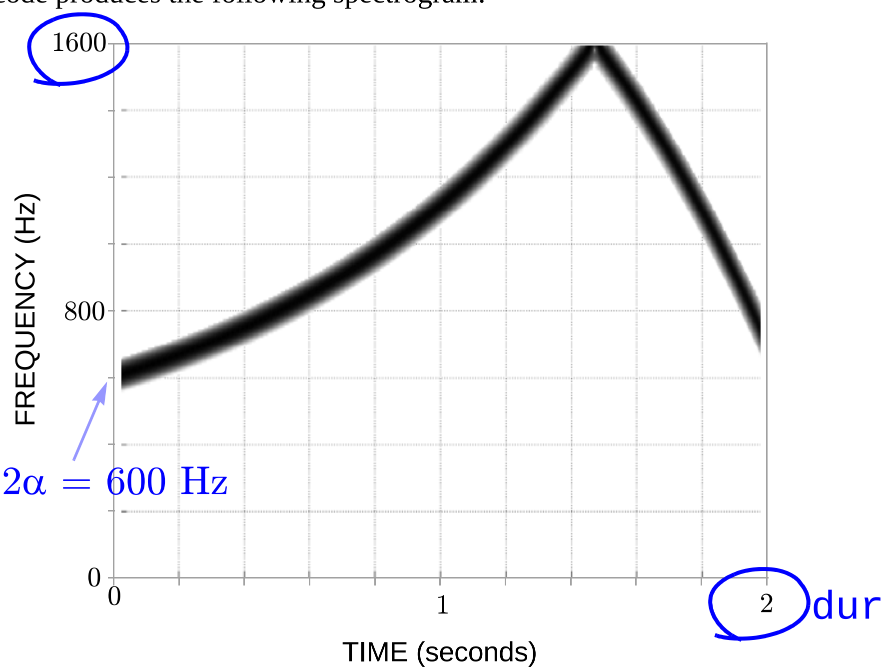
Find numerical values for the three unspecified parameters `dur`, `fsamp`, and `ALPHA` so that running the above code produces the following spectrogram:

aliasing beyond 1600 Hz

$\Rightarrow$ `fsamp = 3200`

$f_i(t) = \alpha(1 + e^t)$

$\Rightarrow f_i(0) = \alpha(1 + 1) = 2\alpha = 600$ Hz



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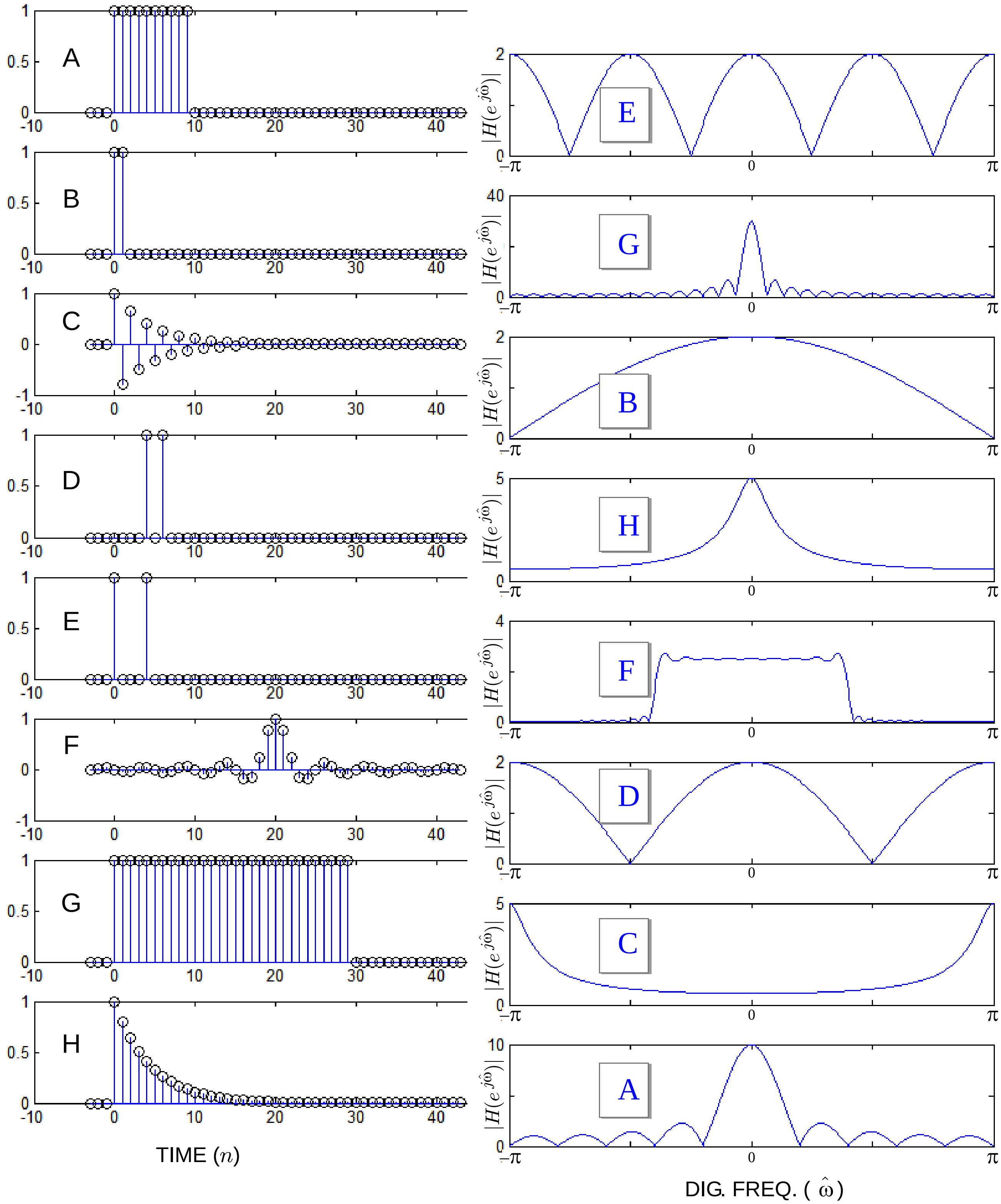
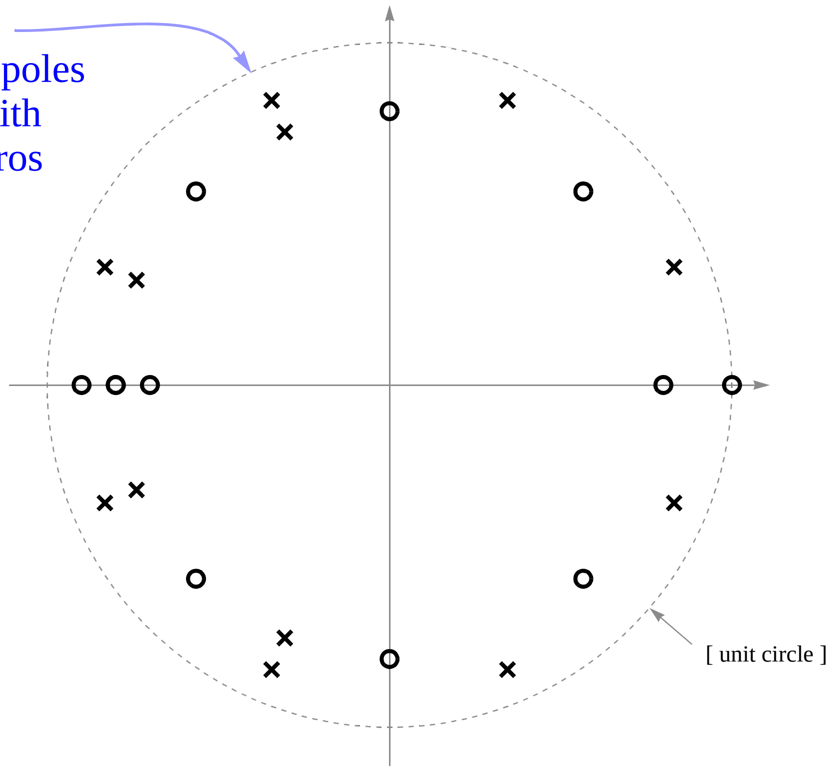
1. To avoid confusion, the remaining arguments of `spectrogram` are not shown. They are not relevant.
If you are curious, however, the complete command is `spectrogram(xx,128,120,512,fsamp,'yaxis')`.

PROB. SU14-Final-7. Shown on the left are impulse responses of eight different filters, labeled A through H. (Values not shown are zero.) Shown on the right are the magnitude responses for these filters, but the order is scrambled. Match each magntiude response to its corresponding impulse response by writing a letter (A through H) in each answer box:



PROB. SU14-Final-8. Below is shown the pole-zero plot for an IIR system:

max here: (multiple poles nearby, with fewest zeros nearby)



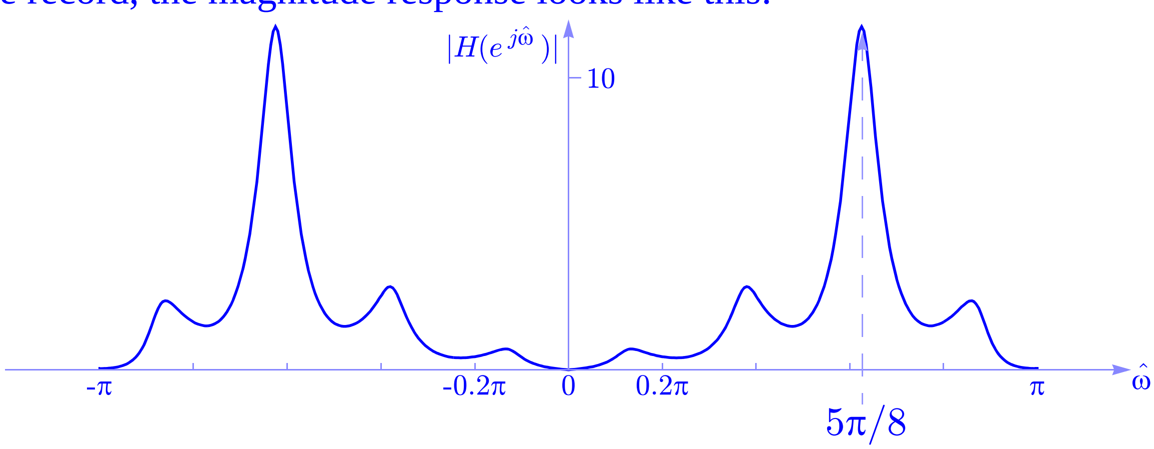
(a) The DC gain of the system is [0].

(b) *Estimate* (to within 20% or better) the value of $\hat{\omega}$ in the range $0 \le \hat{\omega} \le \pi$ that *maximizes* the system's magnitude response:

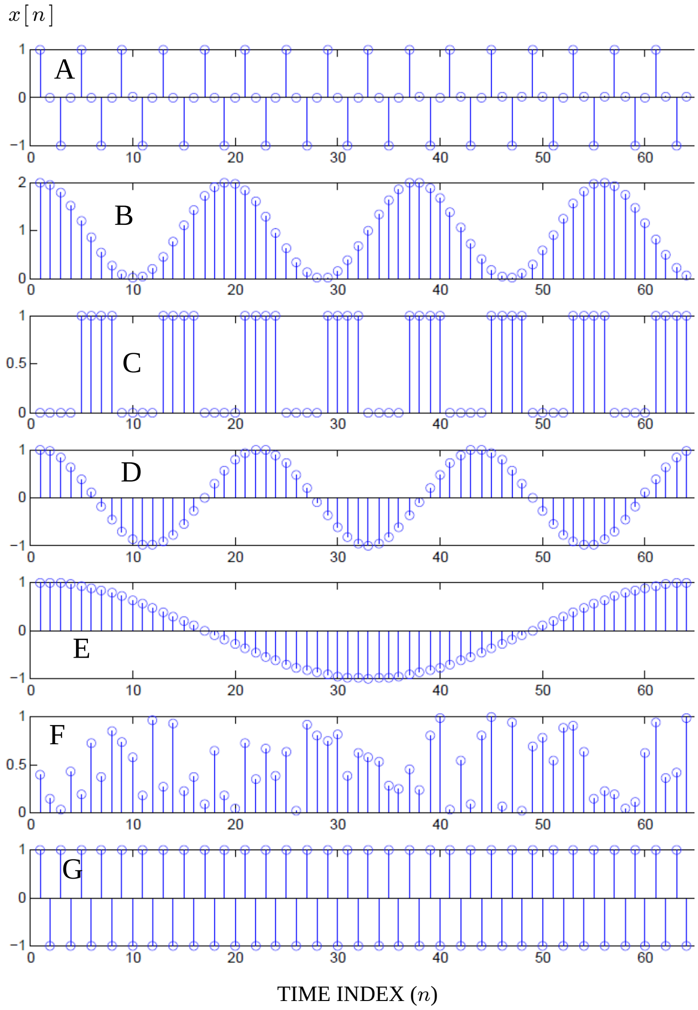
$|H(e^{j\hat{\omega}})|$ achieves a maximum at $\hat{\omega} \approx$ [$5\pi/8$] radians.

(You must *estimate* because there are too many zeros and poles to do an exact calculation, and further because you are not told the exact locations of these zeros and poles.)

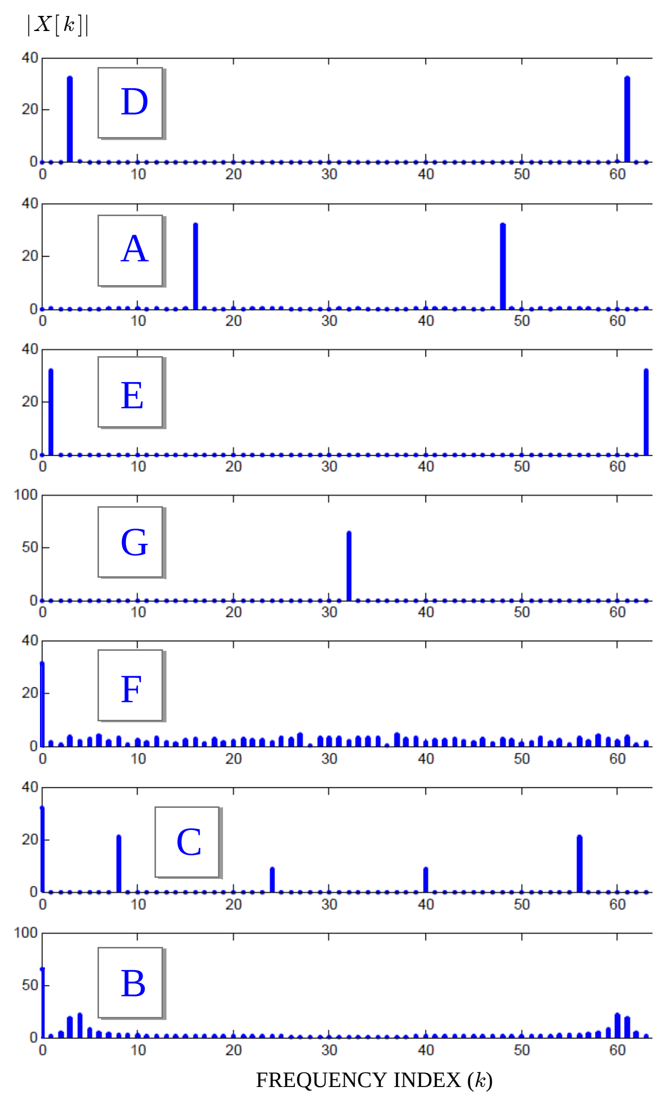
For the record, the magnitude response looks like this:



PROB. SU14-Final-9. On the left below are stem plots of seven different length-64 signal vectors, labelled A through G. On the right below are sketches of the corresponding 64-point DFT magnitude spectrum. (The spectrum is represented by a stem plot of the coefficient magnitude $|X[k]|$ versus the frequency index $k \in \{0, 1, \ldots 63\}$.)

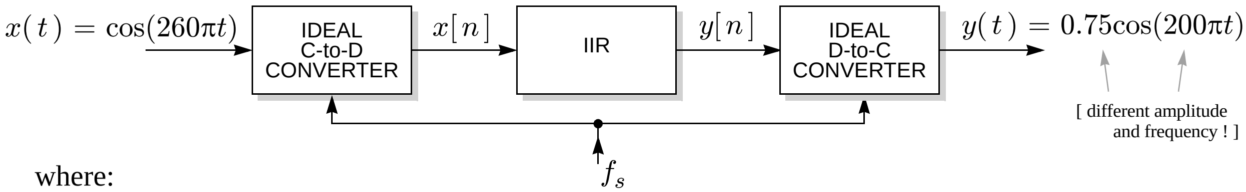


Match one of the seven signals $\{x[n]\}$ on the left to its corresponding DFT spectrum below by writing a letter (A through G) in each answer box:



PROB. SU14-Final-10.

Consider the following system for digitally filtering a continuous-time signal:



where:

- The input to the ideal C-to-D converter is $x(t) = \cos(260\pi t)$.
- The discrete-time IIR system is defined by the recursive difference equation:

$$y[n] = a_1 y[n-1] + x[n] + 0.25x[n-2].$$

- The sampling rate (common to both the C-to-D and D-to-C converters) satisfies $f_s > 200$ samples/s.

If the output of the ideal D-to-C converter is $y(t) = 0.75\cos(200\pi t)$,
then the unspecified parameters f_s and a_1 must be:

$f_s =$ **230** samples/s

$a_1 =$ **0.6115** .

(5 BONUS POINTS IF YOU GET THIS RIGHT!)

A $f_0 = 130$ Hz sinusoid aliases to a 100 Hz sinusoid
$\Rightarrow$ sampled 30 Hz too slow: $f_s = 2f_0 - 30 = 230$ Hz.

Resulting samples are:

$$x[n] = \cos(260\pi n/230) = \cos(26\pi n/23)$$
$$= \cos((26-46)\pi n/23) = \cos(-20\pi n/23) = \cos(20\pi n/23)$$
$$\Rightarrow \hat{\omega} = 20\pi/23.$$

At $z = e^{j20\pi/23}$ we want $H(z) = \dfrac{1 + 0.25z^{-2}}{1 - a_1 z^{-1}} = \dfrac{z^2 + 1/4}{z(z - a_1)} = 0.75.$

Solving last equation for a_1

$$\Rightarrow a_1 = z - \frac{z^2 + 1/4}{0.75z} = 0.6115.$$