GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 Spring 2025
Final Exam

April 25, 2025

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section:

L01 (Daniela)	L05 (Chun-Wei)	L07 (Chun-Wei)	L09 (Daniela)
L02 (Greg)	L06 (Kennedy)	L08 (Kennedy)	L10 (Greg)

Important Notes:

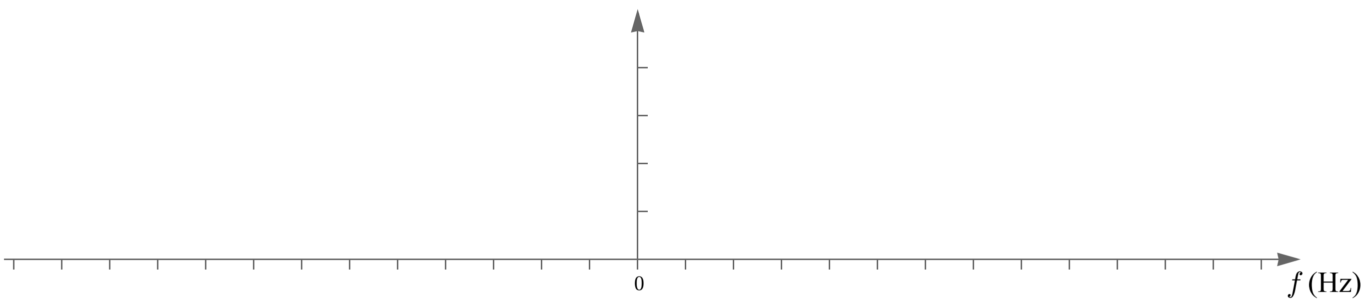
- You may tear off the tables from the back of the exam, but otherwise do not unstaple the rest.
- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no other electronics (no sphones/watches/readers/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
- Only the fronts of the each page will be scanned and graded.

Problem	Value	Score
1	15	
2	16	
3	17	
4	9	
5	16	
6	16	
7	11	
Total		

PROB. Sp25-F.1. Consider the signal $s(t) = 4\cos(16\pi t) + 2\sin(24\pi t)$.

(a) The signal $s(t)$ is periodic with fundamental frequency ☐ Hz.

(b) In the space below, carefully sketch the two-sided spectrum for $s(t)$;
full credit requires that the <u>frequency</u> and <u>complex</u> <u>coefficient</u> for *each line* be clearly *labeled*:



(c) Construct a new signal according to $x(t) = 2\cos(4\pi t)s(t)$.

The Fourier series representation for this new signal is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_0 t}$, where:

$f_0 =$ ☐ Hz, $a_0 =$ ☐ ,

$a_1 =$ ☐ ,

$a_2 =$ ☐ ,

$a_3 =$ ☐ ,

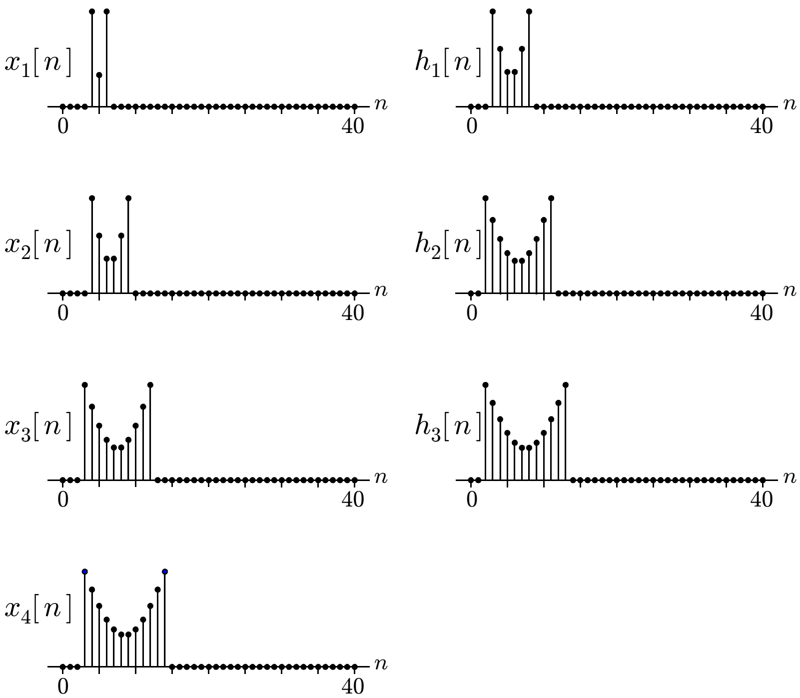
$a_4 =$ ☐ ,

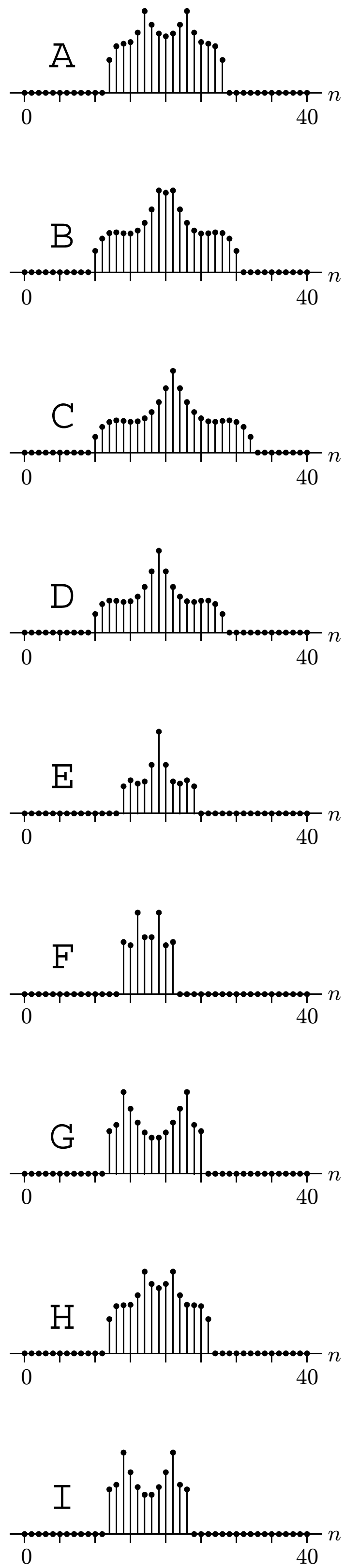
$a_5 =$ ☐ ,

$a_6 =$ ☐ ,

$a_7 =$ ☐ .

PROB. Sp25-F.2. Shown below are the stem plots of four input sequences $x_1[n]$ through $x_4[n]$, along with three impulse responses $h_1[n]$ through $h_3[n]$:





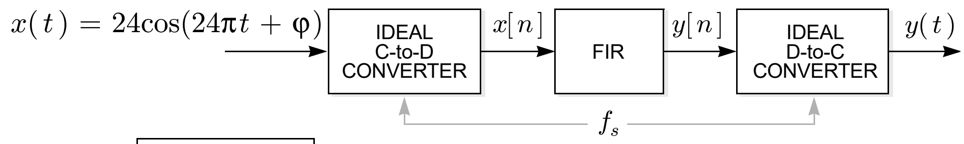
Match each convolution below to one of the nine stem plots shown to the right. Answer by writting a letter from {A, ... I} into each answer box. (Some letters used more than once.)
(Hint: Pay close attention to the durations of the signals.)

- [ ] (a) $x_1[n] * h_1[n]$
- [ ] (b) $x_2[n] * h_1[n]$
- [ ] (c) $x_3[n] * h_1[n]$
- [ ] (d) $x_4[n] * h_1[n]$
- [ ] (e) $x_1[n] * h_2[n]$
- [ ] (f) $x_2[n] * h_2[n]$
- [ ] (g) $x_3[n] * h_2[n]$
- [ ] (h) $x_4[n] * h_2[n]$
- [ ] (i) $x_1[n] * h_3[n]$
- [ ] (j) $x_2[n] * h_3[n]$
- [ ] (k) $x_3[n] * h_3[n]$
- [ ] (l) $x_4[n] * h_3[n]$

PROB. Sp25-F.3.

Consider the FIR filter defined by the difference equation: $y[n] = x[n] + x[n-1] + x[n-2]$.

Suppose the sinusoid $x(t) = 24\cos(24\pi t + \varphi)$ with unspecified phase φ is fed to an ideal sampling/filtering/reconstruction system using the above FIR filter, producing an output $y(t)$:



(a) To avoid aliasing we need $f_s >$ ______ Hz.

(b) For the case when $\varphi = 0$,
there are many sampling rates for which the D-to-C output $y(t)$ is *zero* for all time t.

Of these, the *largest* is $f_s =$ ______ Hz, and the *second-largest* is $f_s =$ ______ Hz.

(c) For the case when $\varphi = -\pi/2$,
there are again many sampling rates for which $y(t)$ is *zero* for all t,
and the largest of these is the same as in (b), but now the *second-largest* is $f_s =$ ______ Hz.

different from part (b)

PROB. Sp25-F.4. Shown below are nine plots of $\left|\sum_{n=0}^{49} \cos(\hat{\omega} n)\, e^{-jk2\pi n/50}\right|$ versus $k \in \{0, \ldots 49\}$, labeled `A` through `I`. (The y-axis scales are not labeled, only the shapes matter.) Match each plot to the corresponding value (in units of radians) of the parameter $\hat{\omega}$. Answer by writing a letter (from A through I) into each answer box.

(1) [ ] $\hat{\omega} = 1$

(2) [ ] $\hat{\omega} = 3$

(3) [ ] $\hat{\omega} = 6$

(4) [ ] $\hat{\omega} = 9$

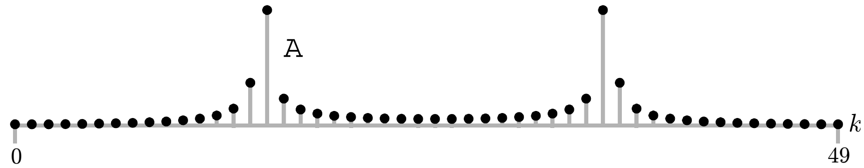
(5) [ ] $\hat{\omega} = 17$

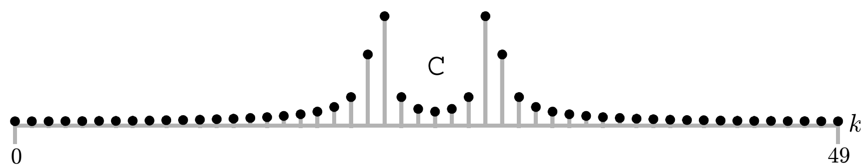
(6) [ ] $\hat{\omega} = 20$

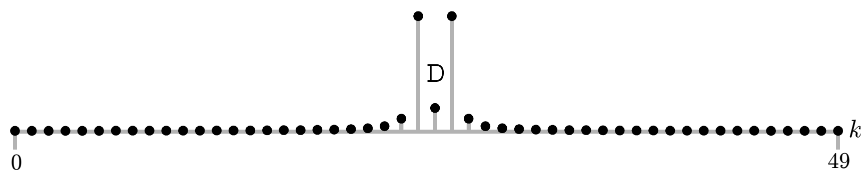
(7) [ ] $\hat{\omega} = 22$

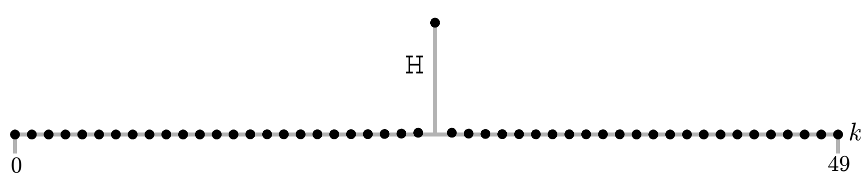
(8) [ ] $\hat{\omega} = 33$

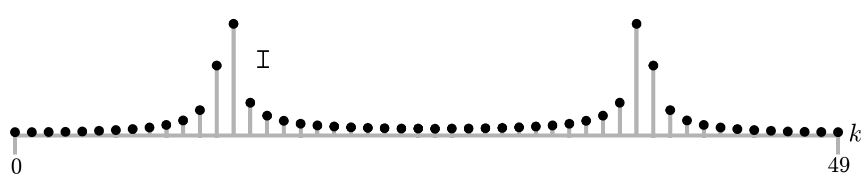
(9) [ ] $\hat{\omega} = 333$







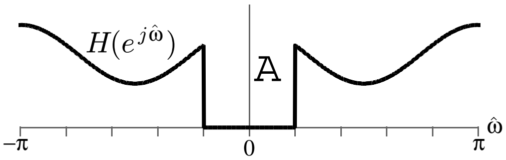


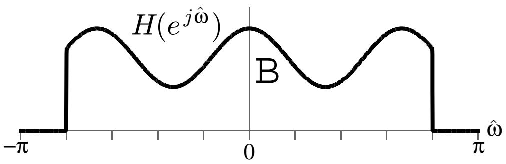


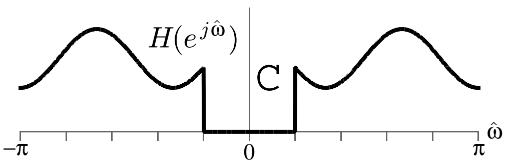
PROB. Sp25-F.5. An LTI filter's impulse response $h[n]$ is defined in terms of $g[n] = \dfrac{\cos(\hat{\omega}_0 n)\sin(\hat{\omega}_1 n)}{\pi n}$ according to:

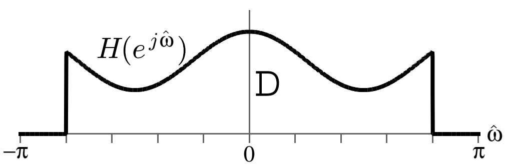
$$h[n] = 5g[n] + g[n - N] + g[n + N].$$

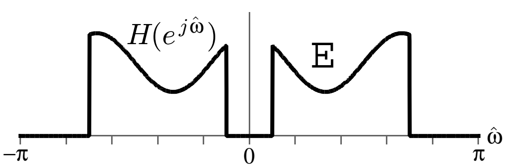
Shown on the right are eight different frequency response plots for this filter, each resulting from a different set of values for the parameters $\hat{\omega}_0$, $\hat{\omega}_1$, and N. Match each plot to one of the eight parameter sets listed below.

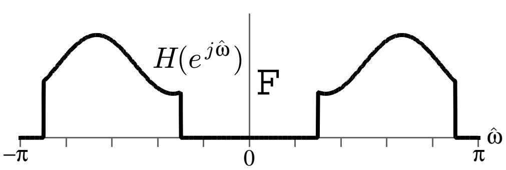


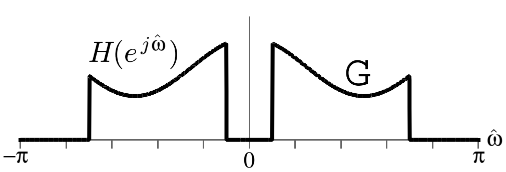


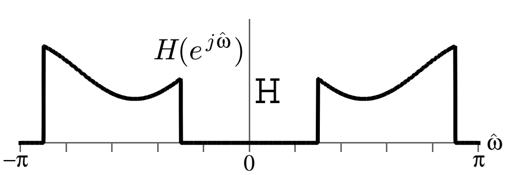












(1) ☐ $\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 2$

(2) ☐ $\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 2$

(3) ☐ $\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 2$

(4) ☐ $\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 2$

(5) ☐ $\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 3$

(6) ☐ $\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 3$

(7) ☐ $\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 3$

(8) ☐ $\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 3$

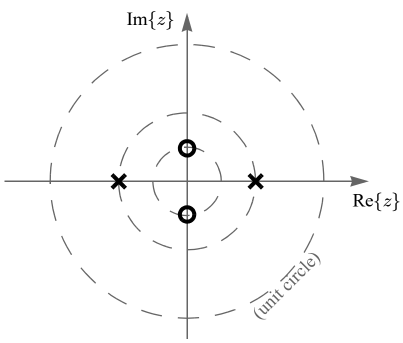
PROB. Sp25-F.6. An LTI filter is defined by the following difference equation, with parameters a and b that differ in the different parts below:

$$y[n] = x[n] + ay[n-2] + bx[n-2].$$

(a) If the output is $y[n] = 0$ (for all n) when the filter parameters are $a = 0.2$ and $b = 1$ and when the input is $x[n] = 0.3\cos(\hat{\omega}_0 n + 0.3\pi)$, the input frequency must be

$\hat{\omega}_0 =$ ☐ .

(b) Find a and b so that the pole-zero plot for the system function $H(z)$ is as shown below, with the zeros falling on the intersection of the vertical axis and a circle of radius 0.25, and the poles falling on the intersection of the horizontal axis and a circle of radius 0.5:



$a =$ ☐ ,

$b =$ ☐ .

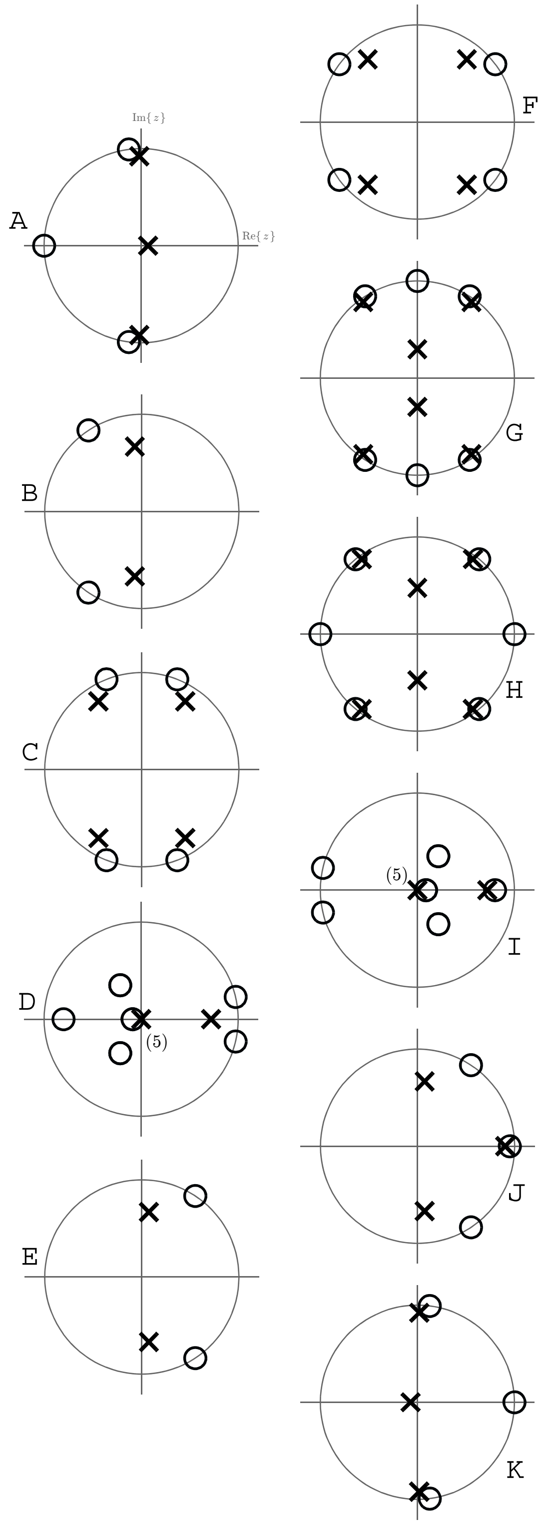
(c) Find a and b so that the output in response to $x[n] = 8 + 10\cos\big(0.5\pi(n-2)\big)$

is $y[n] = 40 + 40\cos\big(0.5\pi(n-2)\big)$:

$a =$ ☐ ,

$b =$ ☐ .

PROB. Sp25-F.7. Shown on the left are the pole-zero plots for eleven LTI systems, labeled `A` through `K`. Shown on the right are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (`A` through `K`) in each answer box.



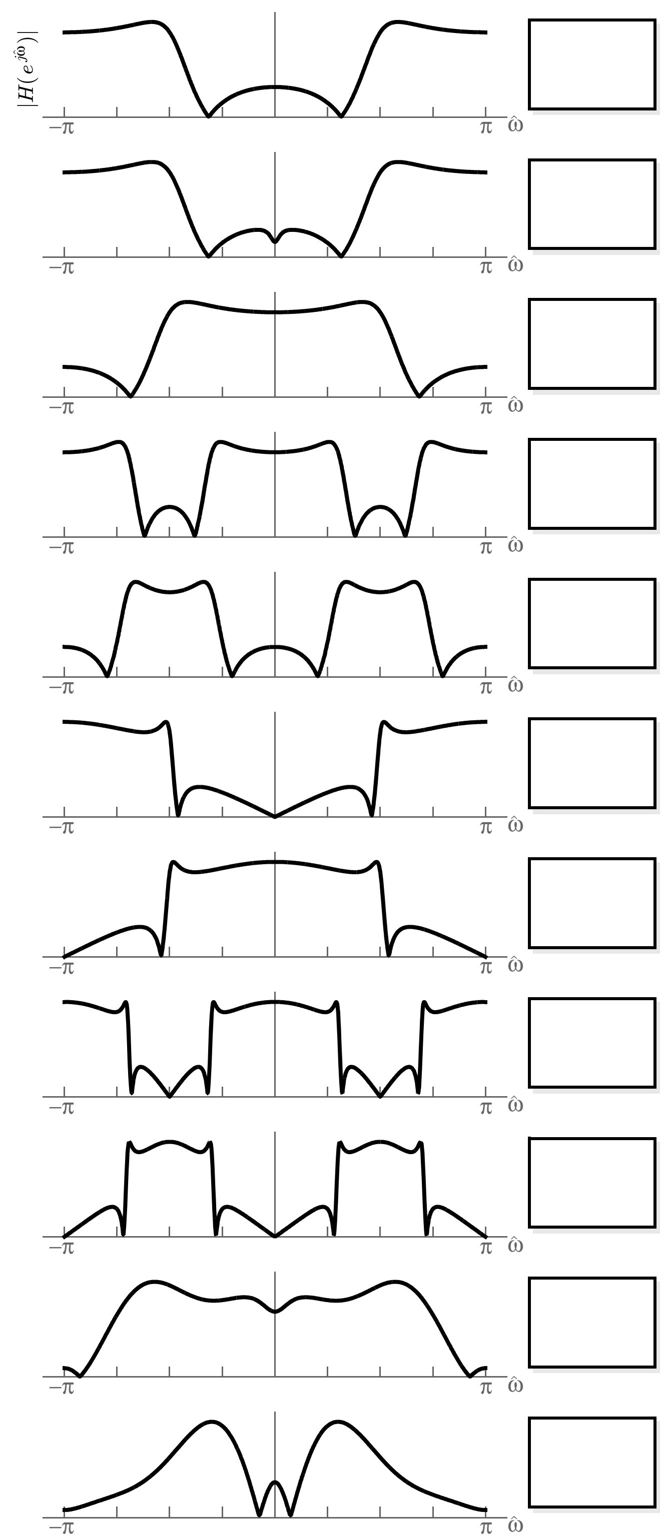


Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_0))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_0))}\, e^{-j(\hat{\omega}-\hat{\omega}_0)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n = 0, 1, 2, \ldots, N-1$	*Frequency-Domain:* $X[k], \quad k = 0, 1, 2, \ldots, N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\|_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
1	$N\delta[k]$
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$r_L[n] = u[n] - u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))}\, e^{-j(2\pi k/N)(L-1)/2}$
$r_L[n]e^{j(2\pi k_0/N)n}$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi(k-k_0)/N))}{\sin(\frac{1}{2}(2\pi(k-k_0)/N))}\, e^{-j(2\pi(k-k_0)/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))}\, e^{j(2\pi n/N)(L-1)/2}$	$N(u[k] - u[k-L])$, when $L \leq N$

Table of DFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X[k]$
Periodic	$x[n] = x[n+N]$	$X[k] = X[k+N]$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
Conjugate Symmetry	$x[n]$ is real	$X[N-k] = X^*[k]$
Conjugation	$x^*[n]$	$X^*[N-k]$
Time-Reversal	$x[N-n]$	$X[N-k]$
Delay (PERIODIC)	$x[n-n_d]$	$e^{-j(2\pi k/N)n_d}X[k]$
Frequency Shift	$x[n]e^{j(2\pi k_0/N)n}$	$X[k-k_0]$
Modulation	$x[n]\cos((2\pi k_0/N)n)$	$\frac{1}{2}X[k-k_0] + \frac{1}{2}X[k+k_0]$
Convolution (PERIODIC)	$x[n] * h[n] = \sum_{m=0}^{N-1} h[m]x[n-m]$	$X[k]H[k]$
Parseval's Theorem	$\displaystyle\sum_{n=0}^{N-1} \|x[n]\|^2 = \frac{1}{N}\sum_{k=0}^{N-1} \|X[k]\|^2$	

Table of z-Transform Pairs		
Signal Name	*Time-Domain: $x[n]$*	*z-Domain: $X(z)$*
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\frac{1}{1-az^{-1}}, \quad \lvert a\rvert<1$
Decaying cosine	$r^n \cos(\hat{\omega}_0 n)u[n]$	$\frac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$
Decaying sinusoid	$Ar^n \cos(\hat{\omega}_0 n+\varphi)u[n]$	$A\frac{\cos(\varphi)-r\cos(\hat{\omega}_0-\varphi)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties		
Property Name	*Time-Domain $x[n]$*	*z-Domain $X(z)$*
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay (d=integer)	$x[n-d]$	$z^{-d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

Date: 28-April-2013

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 Spring 2025
Final Exam

April 25, 2025

NAME: SOLUTIONS (FIRST) (LAST) GT username: VERSION A (e.g., gtxyz123)

Circle your recitation section:

L01 (Daniela)	L05 (Chun-Wei)	L07 (Chun-Wei)	L09 (Daniela)
L02 (Greg)	L06 (Kennedy)	L08 (Kennedy)	L10 (Greg)

Important Notes:

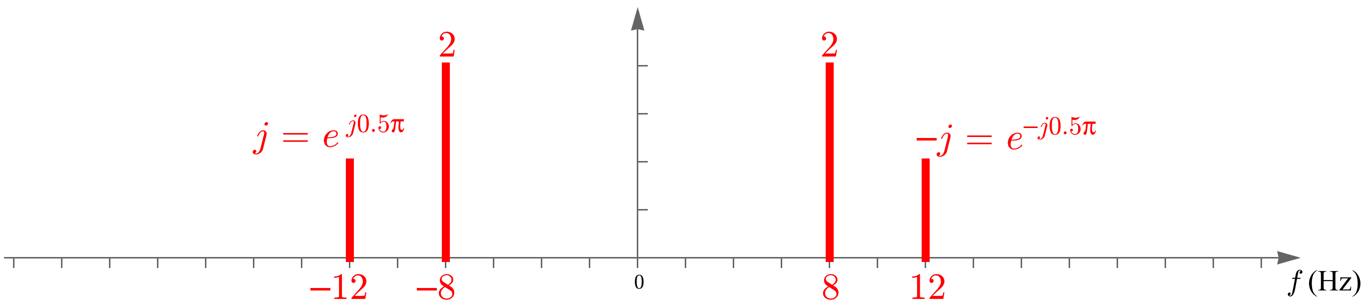
- You may tear off the tables from the back of the exam, but otherwise do not unstaple the rest.
- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” $\times$ 11”) of hand-written notes.
- Calculators are allowed, but no other electronics (no sphones/watches/readers/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to $18°$ or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
- Only the fronts of the each page will be scanned and graded.

Problem	Value	Score
1	15	
2	16	
3	17	
4	9	
5	16	
6	16	
7	11	
Total		

PROB. Sp25-F.1. Consider the signal $s(t) = 4\cos(16\pi t) + 2\sin(24\pi t)$.

(a) The signal $s(t)$ is periodic with fundamental frequency $f_0 = \gcd(8, 12) =$ $\boxed{4}$ Hz.

(b) In the space below, carefully sketch the two-sided spectrum for $s(t)$;
full credit requires that the <u>frequency</u> and <u>complex</u> <u>coefficient</u> for *each line* be clearly *labeled*:



(c) Construct a new signal according to $x(t) = 2\cos(4\pi t)s(t)$.

The Fourier series representation for this new signal is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_0 t}$, where:

$f_0 =$ $\boxed{2}$ Hz, $a_0 =$ $\boxed{0}$,

$a_1 =$ $\boxed{0}$,

$a_2 =$ $\boxed{0}$,

$a_3 =$ $\boxed{2}$,

$a_4 =$ $\boxed{0}$,

$a_5 =$ $\boxed{2 - j}$,

$a_6 =$ $\boxed{0}$,

$a_7 =$ $\boxed{-j}$.

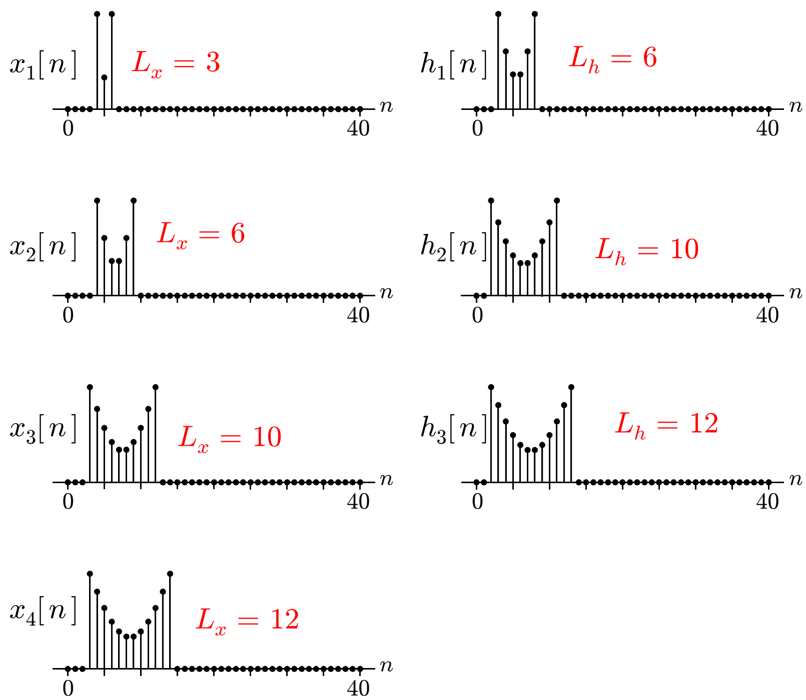
$$\begin{aligned} x(t) &= 8\cos(4\pi t)\cos(16\pi t) + 4\cos(4\pi t)\sin(24\pi t) \\ &= 4\cos(20\pi t) + 4\cos(12\pi t) + 2\sin(28\pi t) + 2\sin(20\pi t) \\ &= A\cos(20\pi t + \varphi) + 4\cos(12\pi t) + 2\sin(28\pi t) \end{aligned}$$

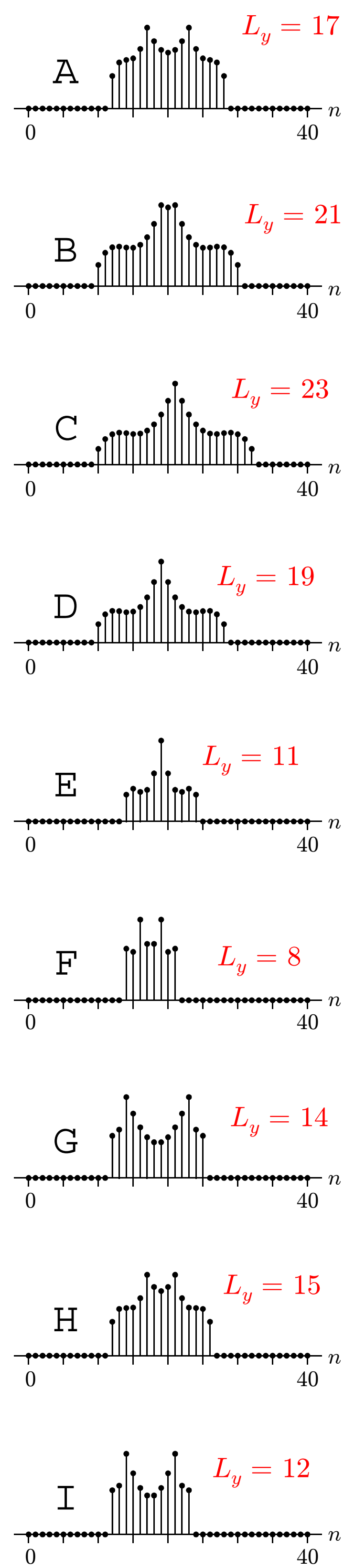
where phasor addition $\Rightarrow Ae^{j\varphi} = 4 - 2j$

$f_0 = \gcd(6, 10, 14) = 2$

Length after convolution is $L_y = L_x + L_h - 1$

PROB. Sp25-F.2. Shown below are the stem plots of four input sequences $x_1[n]$ through $x_4[n]$, along with three impulse responses $h_1[n]$ through $h_3[n]$:





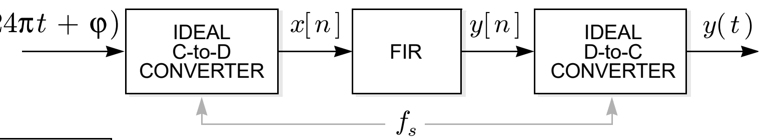
Match each convolution below to one of the nine stem plots shown to the right. Answer by writting a letter from {A, ... I} into each answer box. (Some letters used more than once.)
(Hint: Pay close attention to durations of the signals.)

Answer	Convolution	Answer	Convolution
F	(a) $x_1[n] * h_1[n]$	D	(g) $x_3[n] * h_2[n]$
E	(b) $x_2[n] * h_1[n]$	B	(h) $x_4[n] * h_2[n]$
H	(c) $x_3[n] * h_1[n]$	G	(i) $x_1[n] * h_3[n]$
A	(d) $x_4[n] * h_1[n]$	A	(j) $x_2[n] * h_3[n]$
I	(e) $x_1[n] * h_2[n]$	B	(k) $x_3[n] * h_3[n]$
H	(f) $x_2[n] * h_2[n]$	C	(l) $x_4[n] * h_3[n]$

PROB. Sp25-F.3.

Consider the FIR filter defined by the difference equation: $y[n] = x[n] + x[n-1] + x[n-2]$.

Suppose the sinusoid $x(t) = 24\cos(24\pi t + \varphi)$ with unspecified phase φ is fed to an ideal sampling/filtering/reconstruction system using the above FIR filter, producing an output $y(t)$:



(a) To avoid aliasing we need $f_s >$ **24** Hz.

(b) For the case when $\varphi = 0$,
there are many sampling rates for which the D-to-C output $y(t)$ is *zero* for all time t.

Of these, the *largest* is $f_s =$ **36** Hz, and the *second-largest* is $f_s =$ **18** Hz.

The $\{b_k\} = \{1\ 1\ 1\}$ filter nulls $\hat{\omega}_0 = \frac{2\pi}{3}$

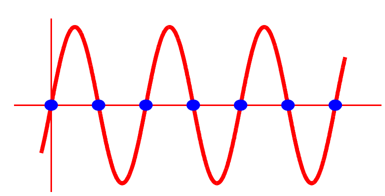
$\Rightarrow$ Smallest digital frequency nulled is $\frac{24\pi}{f_s} = \frac{2\pi}{3} \Rightarrow f_s = 36$ Hz

$\Rightarrow$ second-smallest is $\frac{24\pi}{f_s} = \left|\frac{2\pi}{3} - 2\pi\right| = \frac{4\pi}{3} \Rightarrow f_s = 18$ Hz

(c) For the case when $\varphi = -\pi/2$,
there are again many sampling rates for which $y(t)$ is *zero* for all t,
and the largest of these is the same as in (b), but now the *second-largest* is $f_s =$ **24** Hz.
different from part (b)

In this case the input is $24\sin(24\pi t)$, a pure sine,
which can be sampled at its zero crossings
to get a zero signal even before the FIR filter:

when $f_s = 24$ Hz:



Reduce digital frequencies to $\in(-\pi, \pi)$ and express in terms of π:

PROB. Sp25-F.4. Shown below are nine plots of $\left|\sum_{n=0}^{49} \cos(\hat{\omega} n)\, e^{-jk2\pi n/50}\right|$ versus $k \in \{0, \ldots 49\}$, labeled A through I. (The y-axis scales are not labeled, only the shapes matter.) Match each plot to the corresponding value (in units of radians) of the parameter $\hat{\omega}$. Answer by writing a letter (from A through I) into each answer box.

(1) E $\hat{\omega} = 1$ $= 0.32\pi$

(2) D $\hat{\omega} = 3$ $= 0.96\pi$

(3) F $\hat{\omega} = 6$ $= -0.1\pi$

(4) C $\hat{\omega} = 9$ $= 0.86\pi$

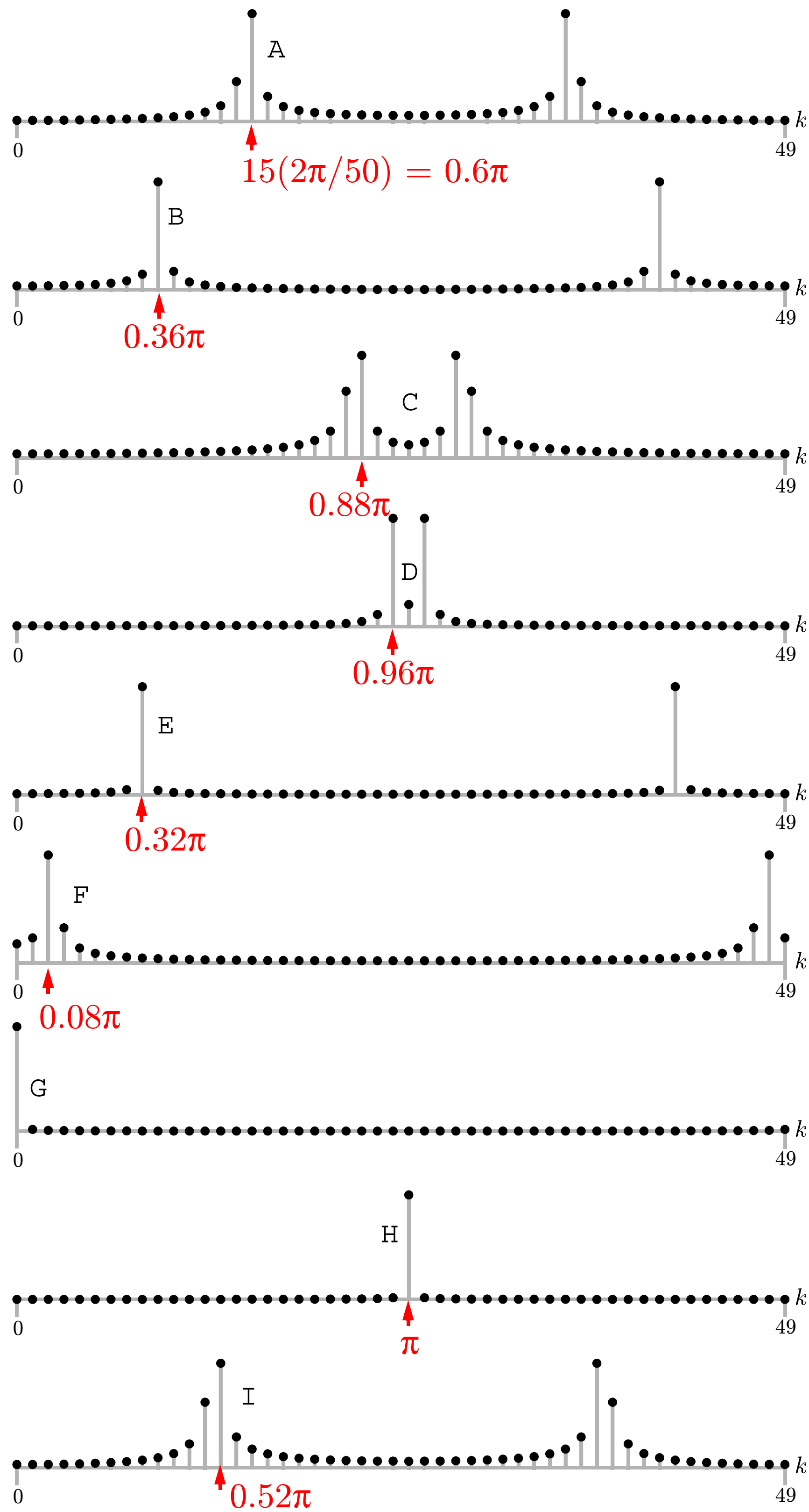
(5) A $\hat{\omega} = 17$ $= -0.59\pi$

(6) B $\hat{\omega} = 20$ $= 0.37\pi$

(7) H $\hat{\omega} = 22$ $= -0.997\pi$

(8) I $\hat{\omega} = 33$ $= 0.5\pi$

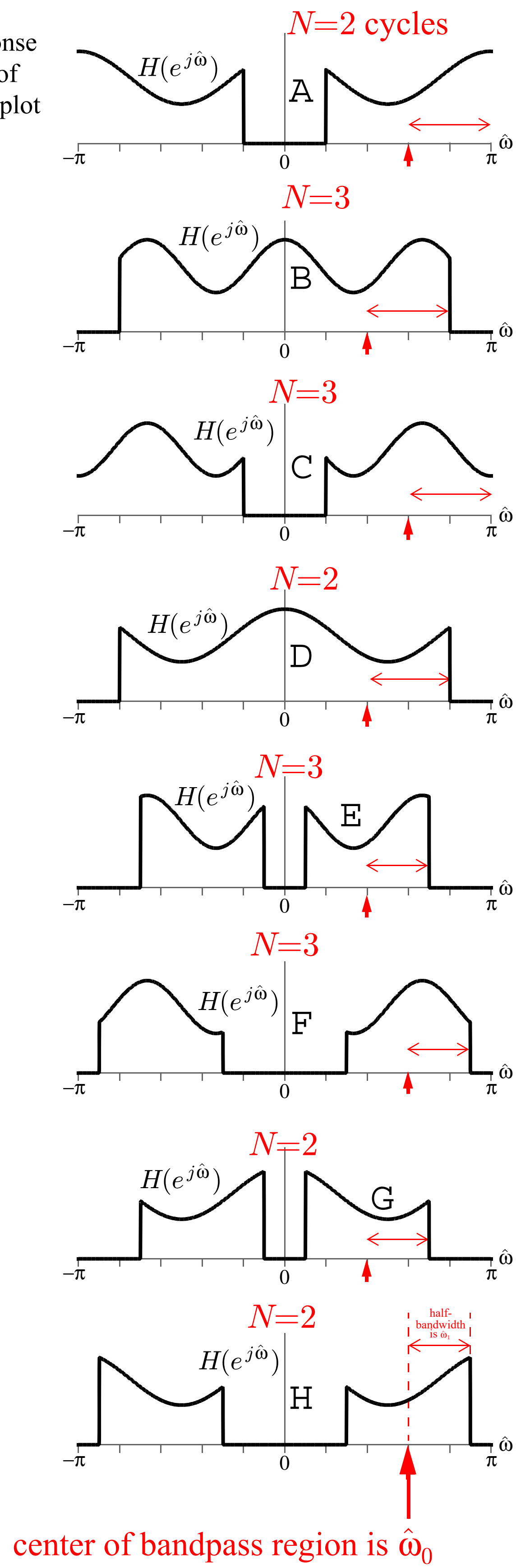
(9) G $\hat{\omega} = 333$ $= -0.003\pi$



PROB. Sp25-F.5. An LTI filter's impulse response $h[n]$ is defined in terms of $g[n] = \dfrac{\cos(\hat{\omega}_0 n)\sin(\hat{\omega}_1 n)}{\pi n}$ according to:

$$h[n] = 5g[n] + g[n-N] + g[n+N].$$

Shown on the right are eight different frequency response plots for this filter, each resulting from a different set of values for the parameters $\hat{\omega}_0$, $\hat{\omega}_1$, and N. Match each plot to one of the eight parameter sets listed below.



	Answer	Parameters
(1)	G	$\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 2$
(2)	D	$\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 2$
(3)	H	$\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 2$
(4)	A	$\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 2$
(5)	E	$\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 3$
(6)	B	$\hat{\omega}_0 = 0.4\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 3$
(7)	F	$\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.3\pi$, $N = 3$
(8)	C	$\hat{\omega}_0 = 0.6\pi$, $\hat{\omega}_1 = 0.4\pi$, $N = 3$

PROB. Sp25-F.6. An LTI filter is defined by the following difference equation, with parameters a and b that differ in the different parts below:

$$y[n] = x[n] + ay[n-2] + bx[n-2].$$

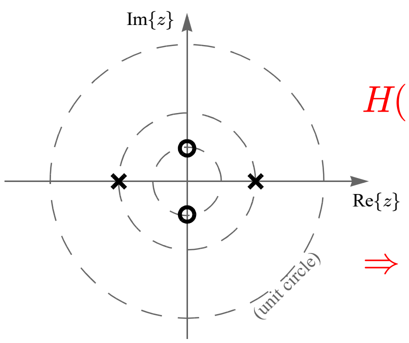
(a) If the output is $y[n] = 0$ (for all n) when the filter parameters are $a = 0.2$ and $b = 1$ and when the input is $x[n] = 0.3\cos(\hat{\omega}_0 n + 0.3\pi)$, the input frequency must be

The system function is $H(z) = \frac{1 + z^{-2}}{1 - az^{-2}} = \frac{z^2 + 1}{z^2 - a}$

$\hat{\omega}_0 =$ 0.5π .

$\Rightarrow$ zeros at $\pm j = e^{\pm j0.5\pi}$, on the unit circle $\Rightarrow$ nulls corresponding sinusoid

(b) Find a and b so that the pole-zero plot for the system function $H(z)$ is as shown below, with the zeros falling on the intersection of the vertical axis and a circle of radius 0.25, and the poles falling on the intersection of the horizontal axis and a circle of radius 0.5:



$H(z) = \frac{1 + bz^{-2}}{1 - az^{-2}} = \frac{z^2 + b}{z^2 - a}$

$a =$ $\frac{1}{4}$,

$b =$ $\frac{1}{16}$.

$\Rightarrow$ zeros at $\pm j\sqrt{b}$ $\Rightarrow b = \frac{1}{16}$

poles at $\pm\sqrt{a}$ $\Rightarrow a = \frac{1}{4}$

(c) Find a and b so that the output in response to $x[n] = 8 + 10\cos\big(0.5\pi(n-2)\big)$

is $y[n] = 40 + 40\cos\big(0.5\pi(n-2)\big)$:

$a =$ 7 ,

$b =$ -31 .

gain at $\hat{\omega} = 0$ is $H(0) = \frac{1+b}{1-a}$

gain at $\hat{\omega} = 0.5\pi$ is $H(j) = \frac{1-b}{1+a}$

Set first to 5, second to 4, solve system of equations

$$\left.\begin{aligned} \frac{1+b}{1-a} &= 5 \\ \frac{1-b}{1+a} &= 4 \end{aligned}\right\} \Rightarrow \begin{aligned} a &= 7 \\ b &= -31 \end{aligned}$$

PROB. Sp25-F.7. Shown on the left are the pole-zero plots for eleven LTI systems, labeled A through K. Shown on the right are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, but in a scrambled order. Match the pole-zero plot to its corresponding magnitude response by writing a letter (A through K) in each answer box.