GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL

DATE: 26-Qpril-24 COURSE: ECE-2026 **Section B**

NAME: ______________________________ ______________

LAST, FIRST (ex: 901109911)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01:Mons-3:30pm (Chen) L07:Tues-12:30pm (Aghazadeh) L08:Thurs-12:30pm (Wang)

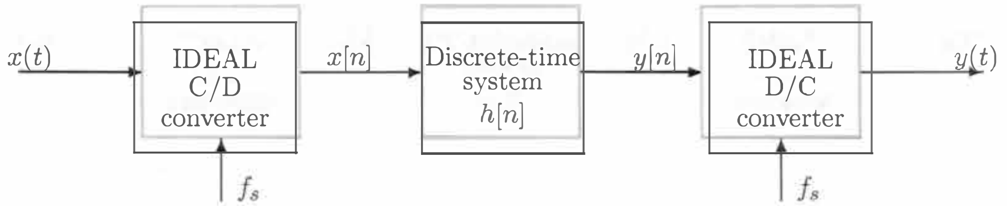
L02:Weds-3:30pm (Lee) L09:Tues-2:00pm (Wang) L10:Thurs-2:00pm (Lee)

L05:Tues-3:30pm (Chen)

- Write your name on the front page ONLY. **DO NOT** unstaple the test. You **MAY detach** the three pages with the tables at the end of the exam paper.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning CLEARLY .
  Justification is required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided boxes or on the provided plots.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	16	
2	13	
3	24	
4	18	
5	17	
6	12	
Total	100	
Missing Rec. Section	-3	

PROBLEM sp-24-FINAL.1:



This problem is related to the above block diagram of an ideal C-to-D converter, a filter, and an ideal D-to-C converter.

(a) **[2 points]** If the output from the ideal C-to-D converter is $x[n] = 3\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value of the input frequency of $x(t)$.

frequency
8000 Hz
4000 Hz
2000 Hz
1600 Hz
1200 Hz
1000 Hz
800 Hz
500 Hz
400 Hz
300 Hz

Pick a number from the list and enter its value in the answer box.

frequency =

(b) [**3 points**] If the input is $x(t) = 2\cos(100\pi t - 0.1\pi) - 7\cos(250\pi t + 0.2\pi)$, and the system is given as $h[n] = \delta[n] - \frac{\sin(0.4\pi n)}{\pi n}$, find the smallest sampling frequency, $f_s = f_{min}$, such that $y(t) = 0$ without aliasing.

Enter the value in the answer box. $f_{min} =$

(c) **[2 points]** Suppose that

$$x(t) = \cos(400\pi t + \pi/2) + \cos(200\pi t + \pi/2).$$

Determine one value of f_s such that $x[n] = 0$.

frequency
8000 Hz
4000 Hz
2000 Hz
1600 Hz
1200 Hz
1000 Hz
800 Hz
500 Hz
400 Hz
300 Hz

Pick a number from the list and enter its value in the answer box.

$f_s =$

(d) [**5 points**] Suppose that $x(t) = \cos(500\pi t)$ and $y(t) = 0.75\cos(200\pi t)$. Also, suppose that the discrete-time IIR system is given by the following difference equation: $y[n] = a_1 y[n-1] + x[n] + b_2 x[n-2]$. Determine a value for f_s that is larger than 200 Samples/s. The same f_s is used at both the Ideal C-to-D converter and the Ideal D-to-C converter.

frequency
750 Hz
700 Hz
650 Hz
600 Hz
550 Hz
500 Hz
450 Hz
400 Hz
350 Hz
300 Hz

[2 points] Pick a number from the list and enter its value in the answer box. $f_s =$

[3 points] Determine the value of a_1 when the value for b_2 is given as $b_1 = 0.25$.

$a_1 =$

(e) **[2 points]** Suppose that the discrete-time LTI system is defined by the following difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2] + a_1 y[n-1].$$

In this part, let $a_1 = -0.3$. The overall system can be used to null one continuous-time sinusoid. The frequency that is nulled is controlled by the value of b_1. If the sampling rate is $f_s = 8000$ Hz, find the value of b_1 so that the overall system nulls out a sinusoid at 60 Hz.

$b_1 =$

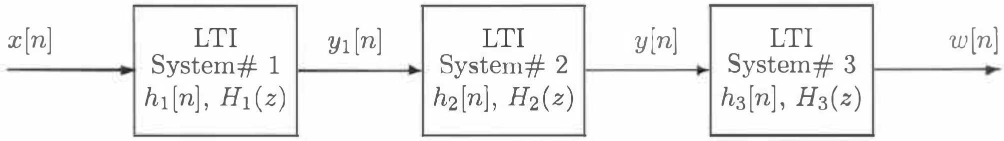
(f) **[2 points]** Suppose that the discrete-time LTI system is defined by the following difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2] + a_1 y[n-1].$$

In this part, let $a_1 = 0$ and $b_1 = 1$. If the input $x(t) = 1$ for all t, what is $y(t)$ for all t?

$y(t) =$

PROBLEM sp-24-FINAL.2:



The above block diagram depicts a cascade connection of three LTI systems. Suppose that the system function for system # 1 is given as $H_1(z) = 2 - 2z^{-5}$. Also, suppose that system # 2 is an FIR filter described by the following difference equation: $y[n] = 3y_1[n] + 3y_1[n-5]$.

(a) **[2 points]** Make a pole-zero plot for the first system, system # 1. Account for all poles and zeros.

(b) **[2 points]** If we would like to replace the first two systems, # 1 and # 2, with a single overall system that has an impulse response $h[n]$, determine $h[n]$ as a sum of scaled and shifted delta functions.

$h[n] =$

(c) [**3 points**] What is $y[n]$ when $x[n] = \cos(\pi n + \pi/5)$.

$y[n] =$

(d) [**2 points**] Suppose that $y[n] = 6\delta[n-2] - 6\delta[n-12]$ is the output of system# 2 when $x[n] = \delta[n-B]$, where B is an integer. Determine the value of B.

$B =$

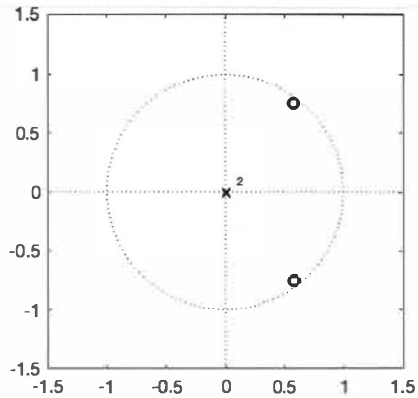
(e) [**4 points**] We would like to design system# 3 such that it inverts the cascade of the first two systems, $H_1(z)$ and $H_2(z)$. Suppose that system# 3 can be defined by the following difference equation: $w[n] = a_1 w[n-1] + a_{10} w[n-10] + b_0 y[n] + b_{10} y[n-10]$.

Determine the values of a_1, a_{10}, b_0, and b_{10}.

$a_1 =$	$a_{10} =$	$b_0 =$	$b_{10} =$

PROBLEM sp-24-FINAL.3:

(a) **[2 points Each]** Below are the pole-zero plots of the system functions, H(z), of several discrete-time systems. Also, there are plots of impulse responses, $h[n]$, on the next page. For each pole-zero plot, enter the letters of the matching impulse response. There are more impulse response plots than pole-zero plots.



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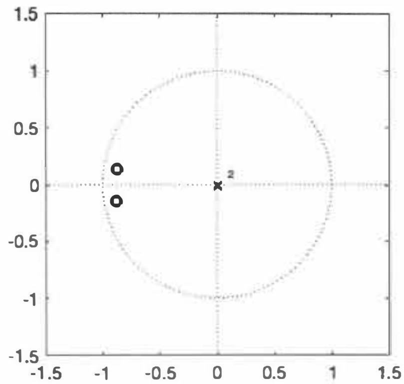

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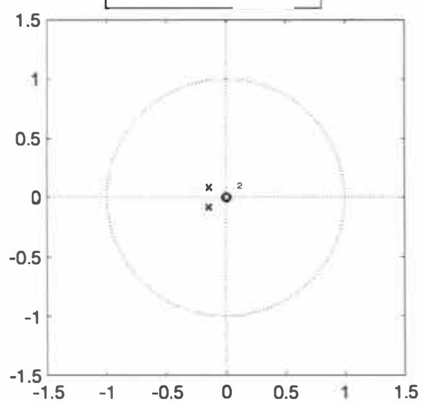

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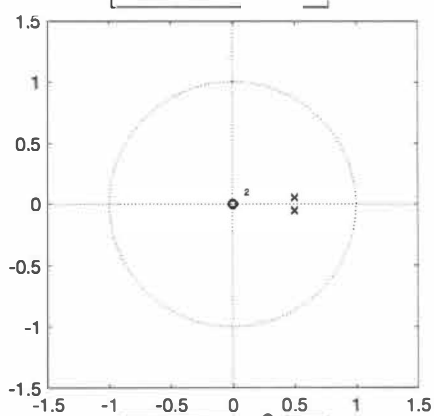

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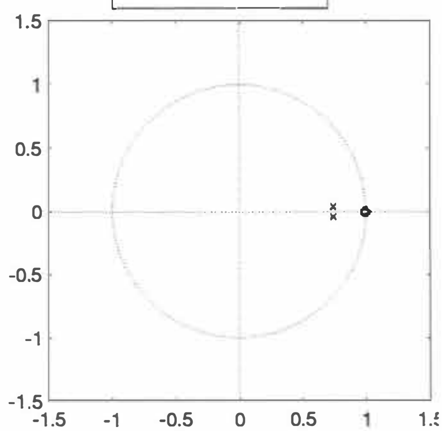

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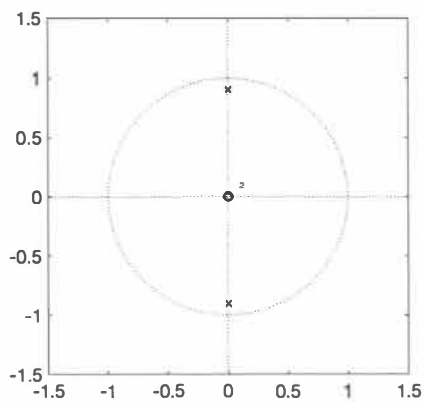

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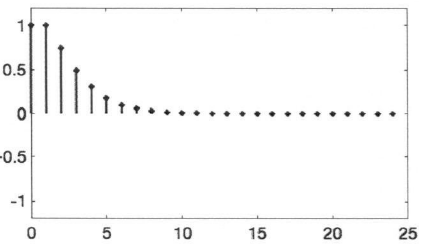
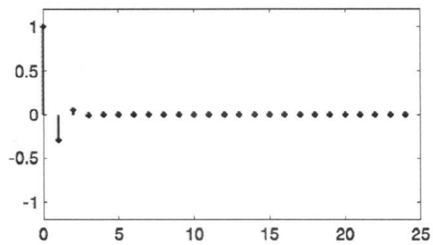
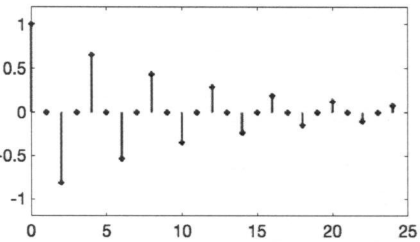

1
0.5
0
-0.5
-1
0 5 10 15 20 25

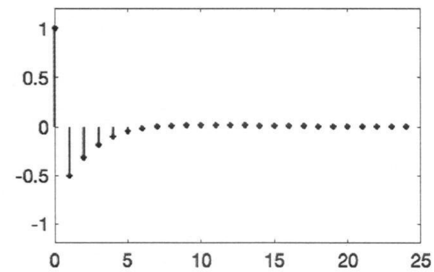
A


1
0.5
0
-0.5
-1
0 5 10 15 20 25

B


1
0.5
0
-0.5
-1
0 5 10 15 20 25

C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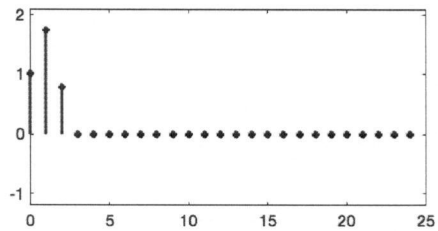
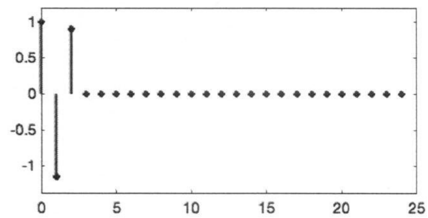
0.5
0
-0.5
-1
0 5 10 15 20 25

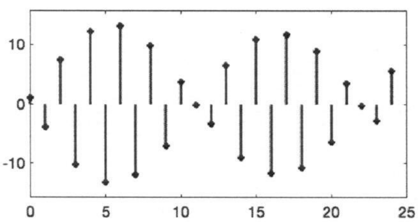
D


2
1
0
-1
0 5 10 15 20 25

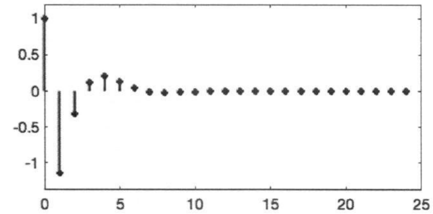
E


1
0.5
0
-0.5
-1
0 5 10 15 20 25

F


10
0
-10
0 5 10 15 20 25

G


1
0.5
0
-0.5
-1
0 5 10 15 20 25

H

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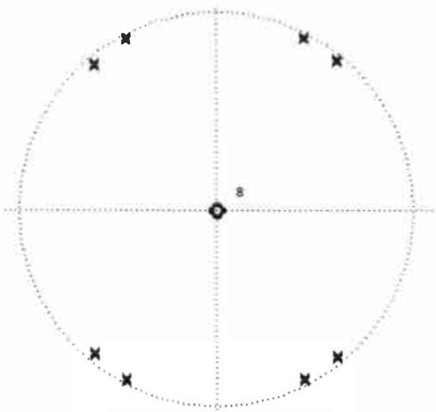
(b) [**2 points Each**] Below are the pole-zero plots of the system functions, H(z), of several discrete-time systems. Also, there are plots of frequency responses, $H(e^{j\omega})$, on the next page. For each pole-zero plot, enter the letters of the matching frequency response. There are more frequency response plots than pole-zero plots.



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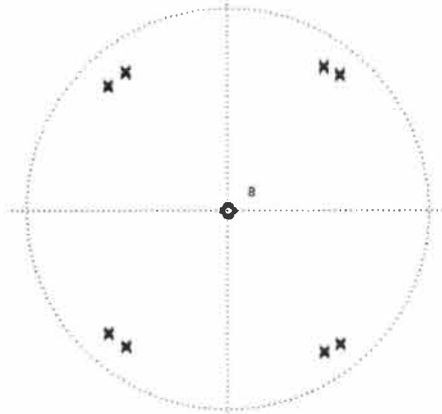

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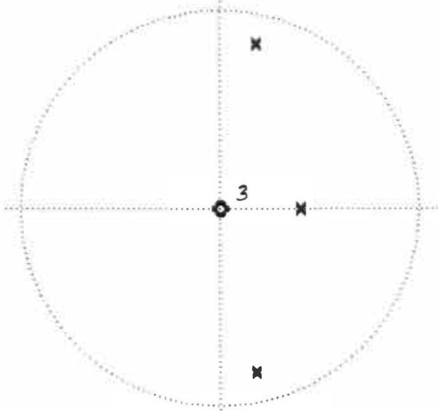
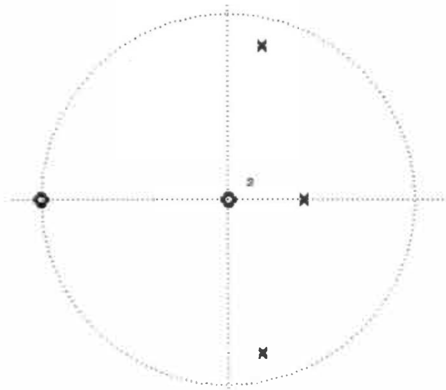
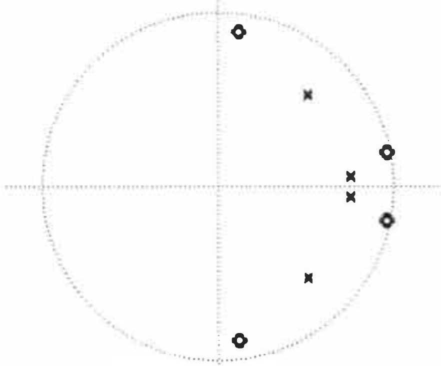


ANS =



ANS =



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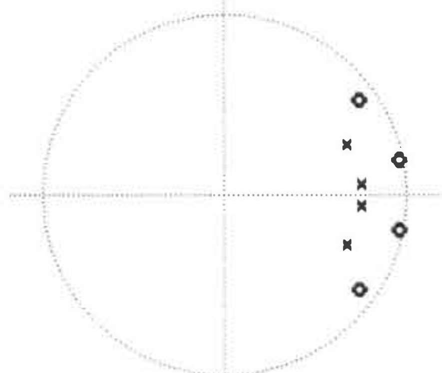
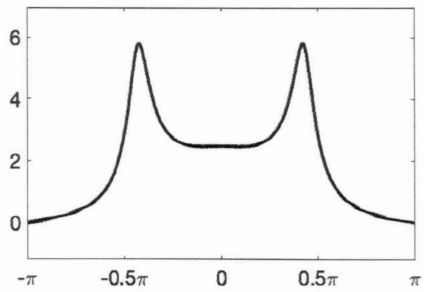
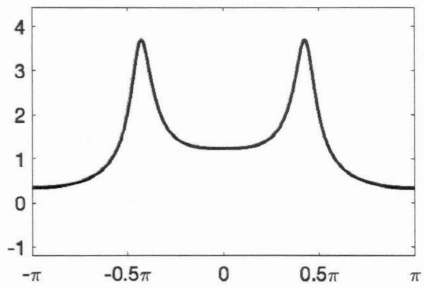


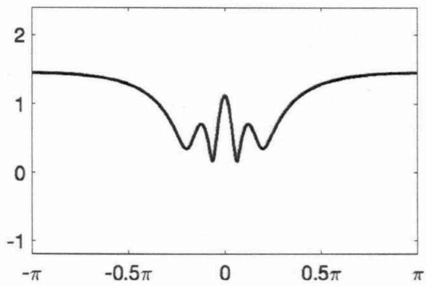
ANS =


0
2
4
6
-π
-0.5π
0
0.5π
π

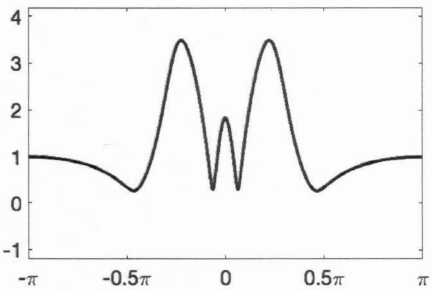
A


-1
0
1
2
3
4
-π
-0.5π
0
0.5π
π

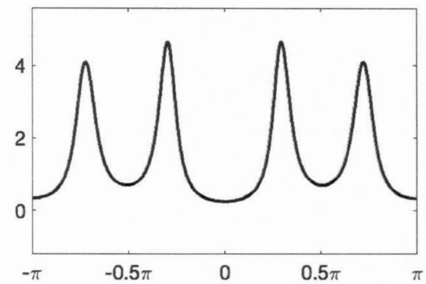
B


-1
0
1
2
-π
-0.5π
0
0.5π
π

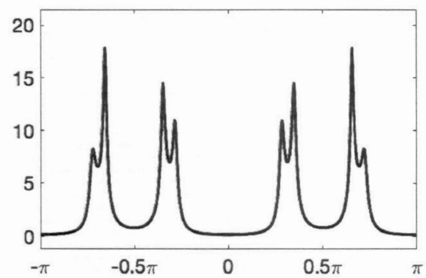
C


-1
0
1
2
3
4
-π
-0.5π
0
0.5π
π

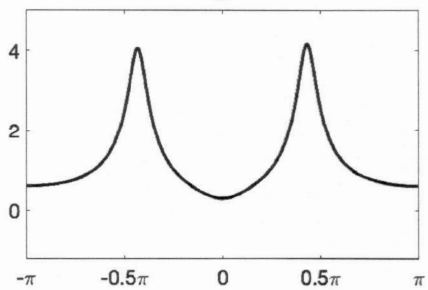
D


0
2
4
-π
-0.5π
0
0.5π
π

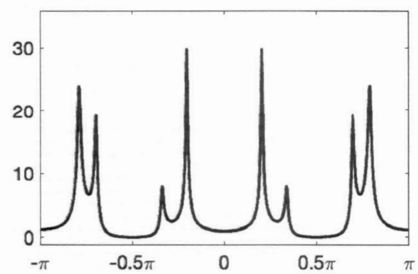
E


0
5
10
15
20
-π
-0.5π
0
0.5π
π

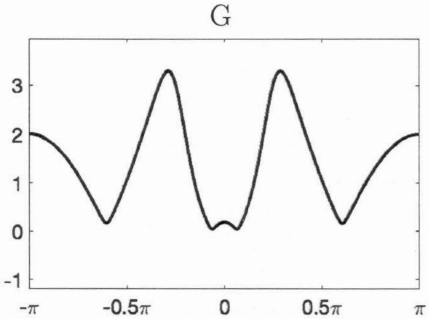
F


0
2
4
-π
-0.5π
0
0.5π
π

G


0
10
20
30
-π
-0.5π
0
0.5π
π

H


-1
0
1
2
3
-π
-0.5π
0
0.5π
π

I

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PROBLEM sp-24-FINAL.4:

[3 points Each] Pick the correct frequency response characteristic and enter the number in the answer box. Each frequency response is used only once. There are more items on the right-hand list than on the left-hand list.

(a) $h[n] = \left(-\frac{1}{2}\right)^n u[n]$

(b) $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

(c) `yn = filter([1,1],1,xn)`

(d) $h[n] = \sum_{k=0}^{3} \delta[n-k]$

(e) $y[n] = x[n-1] + 2x[n-3] + x[n-5]$

(f) $y[n] = \frac{1}{2}y[n-1] + x[n]$

1. $H(e^{j\hat{\omega}}) = \frac{1}{1-0.5e^{-j\hat{\omega}}}$
2. $H(e^{j\hat{\omega}}) = \frac{1}{1+0.5e^{-j\hat{\omega}}}$
3. $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}(0.5 + \cos(\hat{\omega}) + \cos(2\hat{\omega}))$
4. $H(e^{j\hat{\omega}}) = \frac{1}{1-e^{-j0.5\hat{\omega}}}$
5. $|H(e^{j\hat{\omega}})| = 2\cos(\hat{\omega}/2)$
6. $H(e^{j\hat{\omega}}) = \frac{\sin(2\hat{\omega})}{\sin(0.5\hat{\omega})}e^{-j1.5\hat{\omega}}$
7. $\angle H(e^{j\hat{\omega}}) = -3\hat{\omega}$
8. $|H(e^{j\hat{\omega}})| = 0.5\cos(2\hat{\omega})$

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PROBLEM sp-24-FINAL.5:

The parts in this problem are independent from each other.

(a) **[6 points]** Two periodic sequences are given as follows:

n	...	0	1	2	3	4	5	6	7	8	9	10	11	12	...
$x_1[n]$	...	1	0.5	-0.5	-1	-0.5	0.5	1	0.5	-.5	-1	-0.5	0.5	1	...
$x_2[n]$	...	0	2	0	-2	0	2	0	-2	0	2	0	-2	0	...

Note that the two signals have different periodicity. Suppose that we add the two signals to have a new periodic signal $x[n] = x_1[n] + x_2[n]$. For the sequence $x[n], n = 0, ..., 11$, determine the 12-point DFT sequence and write the values in the following table.

Hint: Write $x_1[n]$ and $x_2[n]$ as simple sinusoids.

k	0	1	2	3	4	5
$X[k]$						

k	6	7	8	9	10	11
$X[k]$						

(b) **[2 points]** Suppose the DFT, $X[k]$, of a sequence $x[n]$ shown below, is real. That is, $X^*[k] = X[k]$ for $k = 0, ..., 7$. Can the unknown values of $x[n]$ be determined? If yes, give the missing values. If no, then justify your answer.

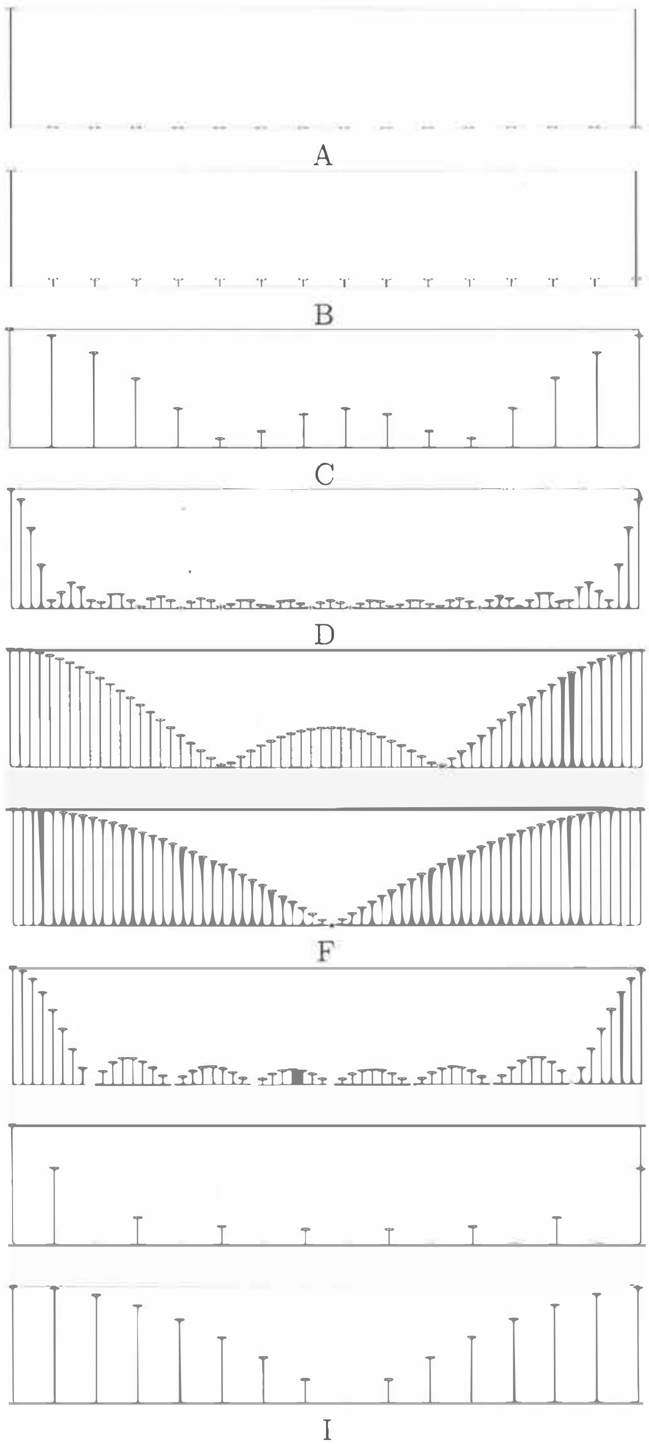
$$\{1, 7, ?, ?, 1, 6, 5, 7\}$$

circle one **YES** (give the missing values below) or **NO** (justify)

(c) [**1 point Each**] Shown below are different outcomes that result from executing the following MATLABcode:

```
stem(abs(fft(ones(1,L),N));
```

s Match each plot with the corresponding values for the variables N nd L by writing a letter (A through I) in each answer box.



L = 2, N = 16 ANS =

L = 2, N = 64 ANS =

L = 3, N = 16 ANS =

L = 3, N = 64 ANS =

L = 8, N = 16 ANS =

L = 8, N = 64 ANS =

L = 15, N = 16 ANS =

L = 15, N = 64 ANS =

L = 16, N = 16 ANS =

PROBLEM sp-24-FINAL.6:

The parts in this problem are independent from each other.

(a) **[4 points]** Simplify the following expression of $x[n]$ with a single term.

$$x[n] = \sum_{k=-\infty}^{\infty} \left(\frac{\sin(0.7\pi k)}{\pi k}\right) \cdot \left(\frac{\sin(0.85\pi(n-k))}{\pi(n-k)}\right)$$

$x[n] =$

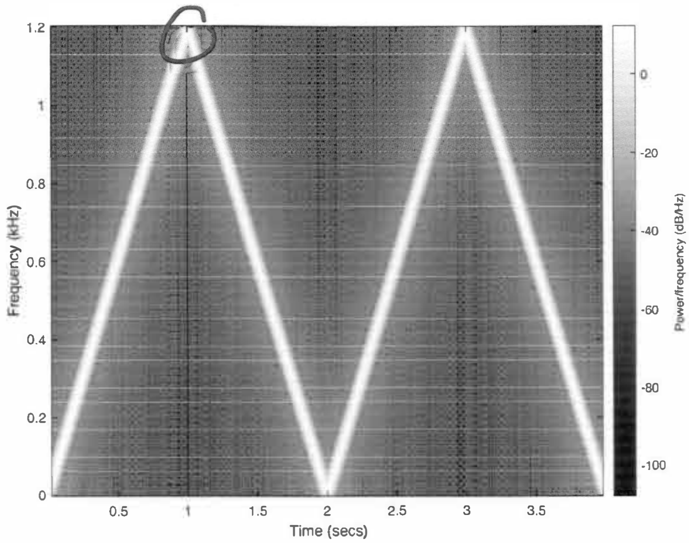
(b) **[4 points]** Determine the fundamental frequency of the signal

$$x(t) = \cos(40\pi t)\cos(24\pi t) + \cos(60\pi t)$$

$f_0 =$ Hz

(c) [**4 points**] Running the following MATLAB code produces the plot below, for a specific value of the parameter W.

```
fsamp = 2400;
tmax = 4;
tt = 0:(1/fsamp):tmax;
xx = real((20+15*j)*exp(j*2*pi*W*(tt.^2)));
spectrogram(xx,128,120,512,fsamp,'yaxis')
```



Note that the y-axis is in kHz, i.e., the highest frequency shown is 1200 Hz.

Determine the numerical value of the parameter W in the MATLAB code.

W =

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_0))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_0))} e^{-j(\hat{\omega}-\hat{\omega}_0)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Date: 24-Apr-2019

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n = 0, 1, 2, \ldots, N-1$	*Frequency-Domain:* $X[k], \quad k = 0, 1, 2, \ldots, N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
1	$N\delta[k]$
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$r_L[n] = u[n] - u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))} e^{-j(2\pi k/N)(L-1)/2}$
$r_L[n]e^{j(2\pi k_0/N)n}$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi(k-k_0)/N))}{\sin(\frac{1}{2}(2\pi(k-k_0)/N))} e^{-j(2\pi(k-k_0)/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))} e^{j(2\pi n/N)(L-1)/2}$	$N(u[k] - u[k-L])$, when $L \leq N$

Table of DFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X[k]$
Periodic	$x[n] = x[n+N]$	$X[k] = X[k+N]$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
Conjugate Symmetry	$x[n]$ is real	$X[N-k] = X^*[k]$
Conjugation	$x^*[n]$	$X^*[N-k]$
Time-Reversal	$x[N-n]$	$X[N-k]$
Delay (PERIODIC)	$x[n-n_d]$	$e^{-j(2\pi k/N)n_d}X[k]$
Frequency Shift	$x[n]e^{j(2\pi k_0/N)n}$	$X[k-k_0]$
Modulation	$x[n]\cos((2\pi k_0/N)n)$	$\frac{1}{2}X[k-k_0] + \frac{1}{2}X[k+k_0]$
Convolution (PERIODIC)	$x[n] * h[n] = \sum_{m=0}^{N-1} h[m]x[n-m]$	$X[k]H[k]$
Parseval's Theorem	$\displaystyle\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1} \lvert X[k]\rvert^2$	

Table of z-Transform Pairs

Signal Name	*Time-Domain:* $x[n]$	*z-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\frac{1}{1-az^{-1}}, \quad \lvert a \rvert < 1$
Decaying cosine	$r^n \cos(\hat{\omega}_0 n) u[n]$	$\frac{1 - r\cos(\hat{\omega}_0) z^{-1}}{1 - 2r\cos(\hat{\omega}_0) z^{-1} + r^2 z^{-2}}$
Decaying sinusoid	$Ar^n \cos(\hat{\omega}_0 n + \varphi) u[n]$	$A\frac{\cos(\varphi) - r\cos(\hat{\omega}_0 - \varphi) z^{-1}}{1 - 2r\cos(\hat{\omega}_0) z^{-1} + r^2 z^{-2}}$

Table of z-Transform Properties

Property Name	*Time-Domain* $x[n]$	*z-Domain* $X(z)$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
Delay (d=integer)	$x[n-d]$	$z^{-d} X(z)$
Convolution	$x[n] * h[n]$	$X(z)H(z)$

Date: 24-April-2019

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL

DATE: 26-Qpril-24 COURSE: ECE-2026 **Section B**

NAME: Solutions
LAST, FIRST

GT #: Ver 01
(ex: 901109911)

3 points 3 points 3 points

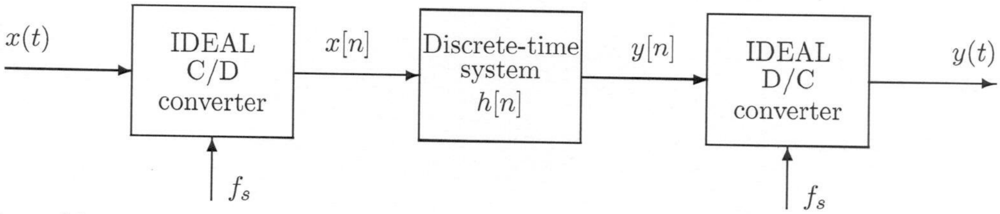
Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01:Mons-3:30pm (Chen) L07:Tues-12:30pm (Aghazadeh) L08:Thurs-12:30pm (Wang)
L02:Weds-3:30pm (Lee) L09:Tues-2:00pm (Wang) L10:Thurs-2:00pm (Lee)
L05:Tues-3:30pm (Chen)

- Write your name on the front page ONLY. **DO NOT** unstaple the test. You **MAY detach** the three pages with the tables at the end of the exam paper.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning CLEARLY .
  Justification is required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided boxes or on the provided plots.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	16	
2	13	
3	24	
4	18	
5	17	
6	12	
Total	100	
Missing Rec. Section	-3	

PROBLEM sp-24-FINAL.1:



This problem is related to the above block diagram of an ideal C-to-D converter, a filter, and an ideal D-to-C converter.

(a) **[2 points]** If the output from the ideal C-to-D converter is $x[n] = 3\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value of the input frequency of $x(t)$.

frequency
8000 Hz
4000 Hz
2000 Hz
1600 Hz
1200 Hz
1000 Hz
800 Hz
500 Hz
400 Hz
300 Hz

Pick a number from the list and enter its value in the answer box.

frequency = 500Hz

$$\hat{\omega} = \pm \frac{2\pi f}{f_s} + 2\pi l$$

(b) **[3 points]** If the input is $x(t) = 2\cos(100\pi t - 0.1\pi) - 7\cos(250\pi t + 0.2\pi)$, and the system is given as $h[n] = \delta[n] - \frac{\sin(0.4\pi n)}{\pi n}$, find the smallest sampling frequency, $f_s = f_{min}$, such that $y(t) = 0$ without aliasing.

Enter the value in the answer box. $f_{min} =$ 625

The filter acts a HPF. All frequencies have to be lower than the cutoff frequency.

$$\frac{250\pi}{f_s} < 0.4\pi$$

$$\Rightarrow f_s > 625 \text{ Hz}$$

(c) **[2 points]** Suppose that

$$x(t) = \cos(400\pi t + \pi/2) + \cos(200\pi t + \pi/2).$$

Determine one value of f_s such that $x[n] = 0$.

frequency
8000 Hz
4000 Hz
2000 Hz
1600 Hz
1200 Hz
1000 Hz
800 Hz
500 Hz
400 Hz
300 Hz

Pick a number from the list and enter its value in the answer box.

$f_s =$ 300

folding

$$2\pi l + \frac{-400\pi}{f_s} = \frac{200\pi}{f_s}$$

$$\Rightarrow f_s = \frac{300}{l}$$

(d) **[5 points]** Suppose that $x(t) = \cos(500\pi t)$ and $y(t) = 0.75\cos(200\pi t)$. Also, suppose that the discrete-time IIR system is given by the following difference equation: $y[n] = a_1 y[n-1] + x[n] + b_2 x[n-2]$. Determine a value for f_s that is larger than 200 Samples/s. The same f_s is used at both the Ideal C-to-D converter and the Ideal D-to-C converter.

frequency
750 Hz
700 Hz
650 Hz
600 Hz
550 Hz
500 Hz
450 Hz
400 Hz
350 Hz
300 Hz

[2 points] Pick a number from the list and enter its value in the answer box. $f_s =$ 350

250 aliases to 100 Hz

$$f_s = \frac{100 \mp 250}{\ell}$$

[3 points] Determine the value of a_1 when the value for b_2 is given as $b_1 = 0.25$.

$a_1 =$ 0.1483

$$x[n] = \cos\left(\frac{500\pi n}{350}\right) = \cos\left(\frac{50}{35}\pi n\right) = \cos\left(-\frac{20\pi n}{35}\right)$$

$$= \cos\left(\frac{20\pi n}{35}\right)$$

$$H(z) = \frac{1 + 0.25 z^{-2}}{1 - a_1 z^{-1}}\Bigg|_{z = e^{j\frac{20}{35}\pi}}$$

$$= 0.1483$$

(e) **[2 points]** Suppose that the discrete-time LTI system is defined by the following difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2] + a_1 y[n-1].$$

In this part, let $a_1 = -0.3$. The overall system can be used to null one continuous-time sinusoid. The frequency that is nulled is controlled by the value of b_1. If the sampling rate is $f_s = 8000$ Hz, find the value of b_1 so that the overall system nulls out a sinusoid at 60 Hz.

$b_1 = -1.998$

$$H(z) = \frac{1 + b_1 z^{-1} + z^{-2}}{1 - a_1 z^{-1}} = \frac{(z - e^{j\theta})(z - e^{-j\theta})}{z(z - a_1)}$$

$$\hat{\omega} = \frac{2\pi f}{f_s} = \frac{2\pi(60)}{8000} = \frac{3\pi}{200}$$

$$\Rightarrow b_1 = -2\cos\left(\frac{3\pi}{200}\right) = -1.998$$

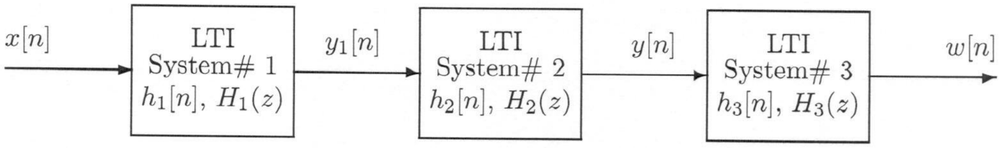
(f) **[2 points]** Suppose that the discrete-time LTI system is defined by the following difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2] + a_1 y[n-1].$$

In this part, let $a_1 = 0$ and $b_1 = 1$. If the input $x(t) = 1$ for all t, what is $y(t)$ for all t?

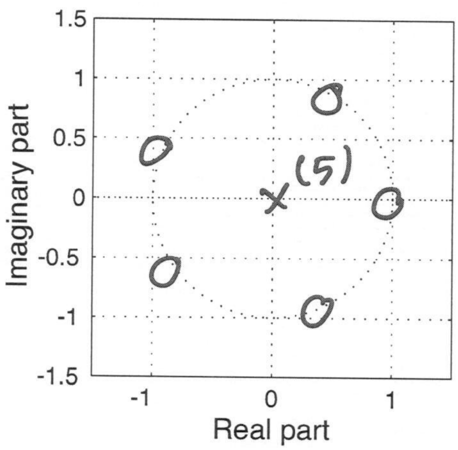
$y(t) = 3$

PROBLEM sp-24-FINAL.2:



The above block diagram depicts a cascade connection of three LTI systems. Suppose that the system function for system # 1 is given as $H_1(z) = 2 - 2z^{-5}$. Also, suppose that system # 2 is an FIR filter described by the following difference equation: $y[n] = 3y_1[n] + 3y_1[n-5]$.

(a) **[2 points]** Make a pole-zero plot for the first system, system # 1. Account for all poles and zeros.



$H_1(z) = 2 - 2z^{-5}$

(b) **[2 points]** If we would like to replace the first two systems, # 1 and # 2, with a single overall system that has an impulse response $h[n]$, determine $h[n]$ as a sum of scaled and shifted delta functions.

$h[n] =$ $6\delta[n] - 6\delta[n-10]$

(c) **[3 points]** What is $y[n]$ when $x[n] = \cos(\pi n + \pi/5)$.

$y[n] =$ 0

$\hat{\omega} = \pi \Rightarrow z = -1$

(d) **[2 points]** Suppose that $y[n] = 6\delta[n-2] - 6\delta[n-12]$ is the output of system# 2 when $x[n] = \delta[n-B]$, where B is an integer. Determine the value of B.

$B =$ 2

(e) **[4 points]** We would like to design system# 3 such that it inverts the cascade of the first two systems, $H_1(z)$ and $H_2(z)$. Suppose that system# 3 can be defined by the following difference equation: $w[n] = a_1 w[n-1] + a_{10} w[n-10] + b_0 y[n] + b_{10} y[n-10]$.

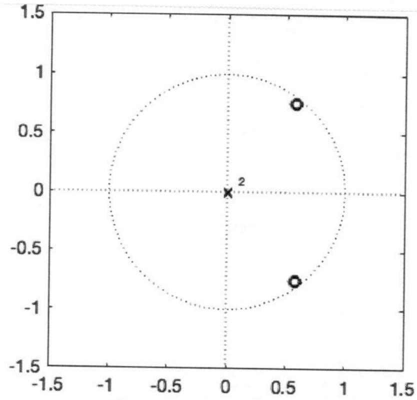
Determine the values of a_1, a_{10}, b_0, and b_{10}.

$a_1 =$ 0	$a_{10} =$ 1	$b_0 =$ 1/6	$b_{10} =$ 0

$$H_3(z) = \frac{1}{6 - 6z^{-10}}$$

PROBLEM sp-24-FINAL.3:

(a) [**2 points Each**] Below are the pole-zero plots of the system functions, H(z), of several discrete-time systems. Also, there are plots of impulse responses, $h[n]$, on the next page. For each pole-zero plot, enter the letters of the matching impulse response. There are more impulse response plots than pole-zero plots.



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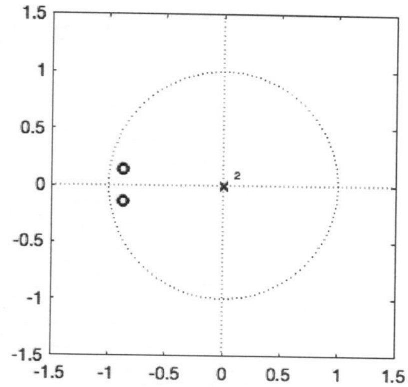

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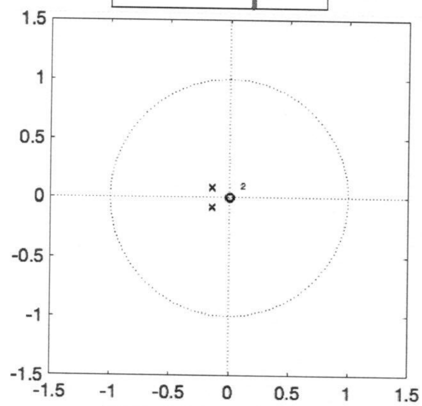
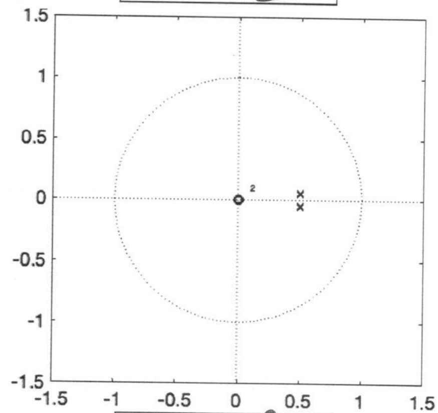


ANS = B



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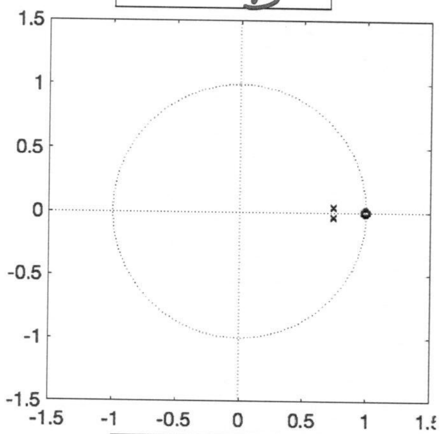

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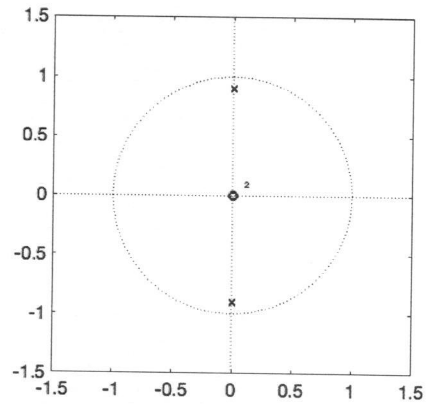

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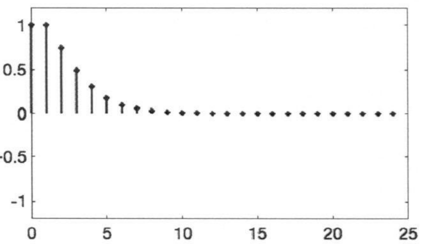
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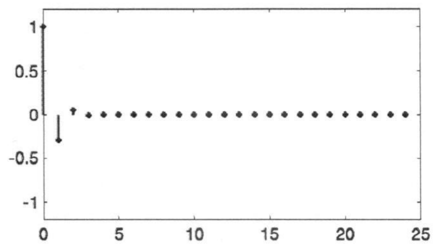
-0.5
-1
0 5 10 15 20 25

A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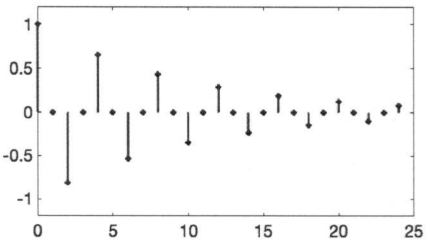
0 5 10 15 20 25

B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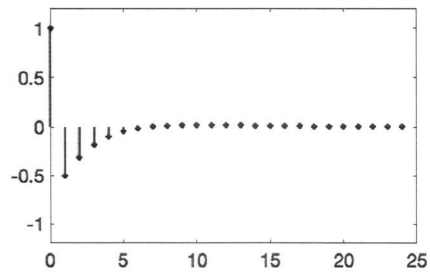
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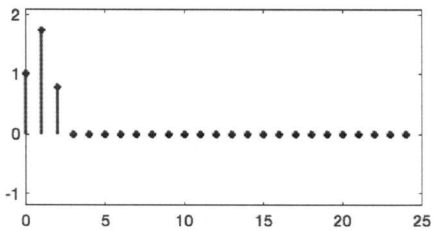
0 5 10 15 20 25

D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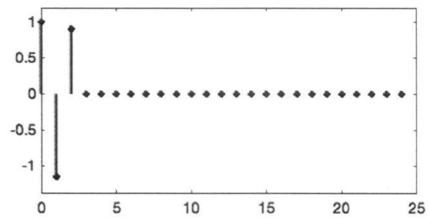
E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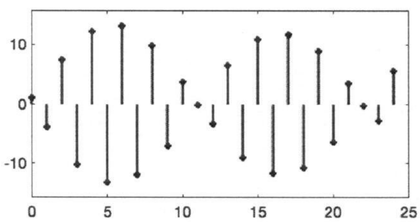
F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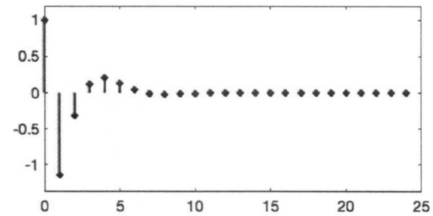
G


1
0.5
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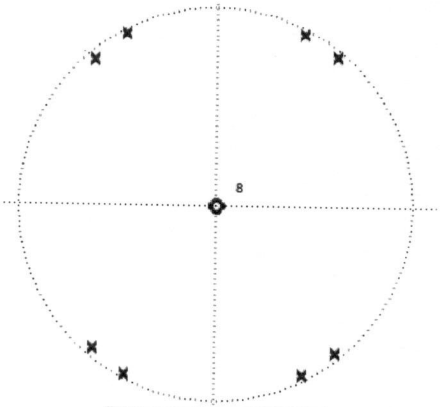
-0.5
-1
0 5 10 15 20 25

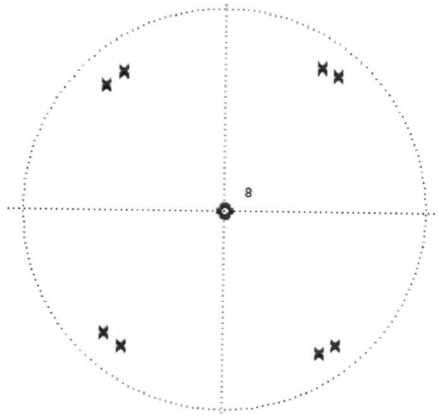
H

This page is left blank so that you can use it as work space.

(b) **[2 points Each]** Below are the pole-zero plots of the system functions, H(z), of several discrete-time systems. Also, there are plots of frequency responses, $H(e^{j\omega})$, on the next page. For each pole-zero plot, enter the letters of the matching frequency response. There are more frequency response plots than pole-zero plots.



ANS = F



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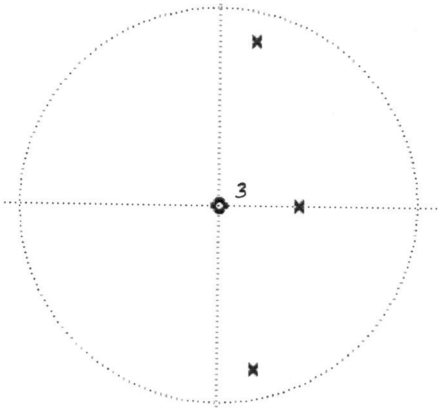
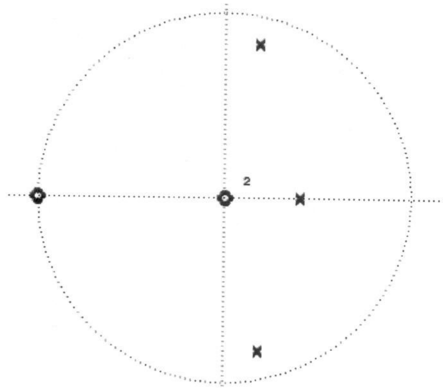


ANS = B



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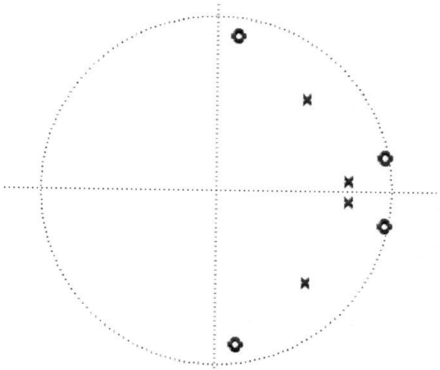

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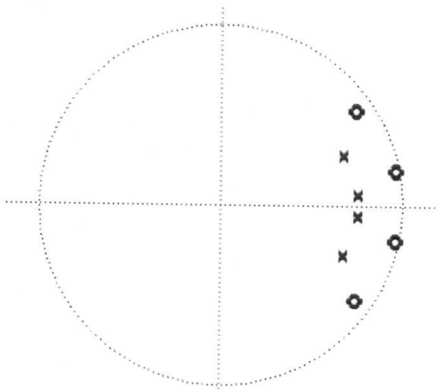

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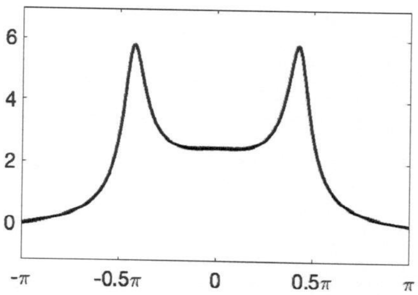
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2
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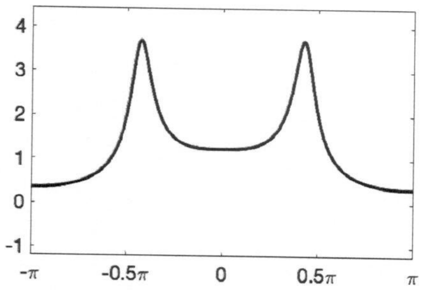
-π
-0.5π
0
0.5π
π

A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1
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-1
-π
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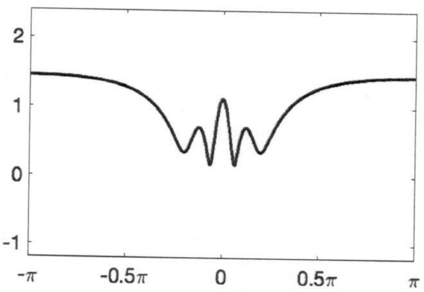
0
0.5π
π

B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-0.5π
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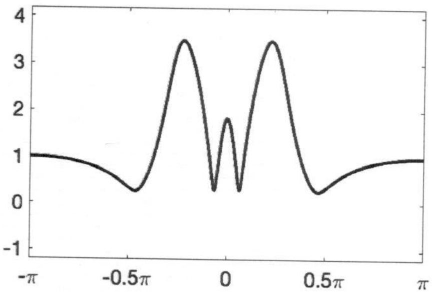
C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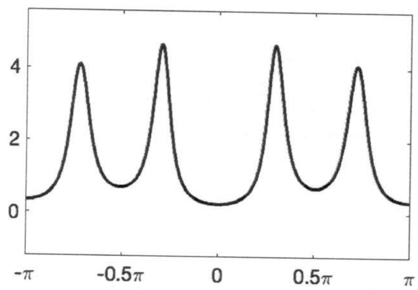
D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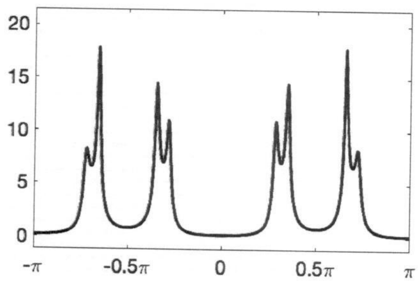
E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-π
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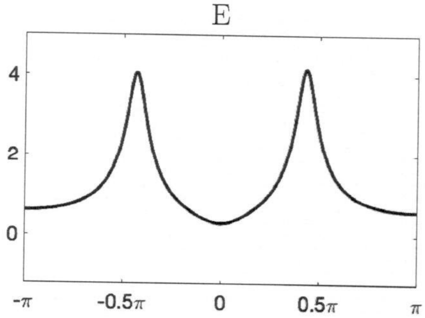
0
0.5π
π

F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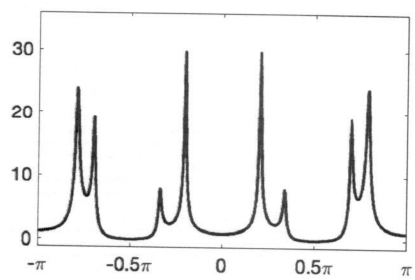
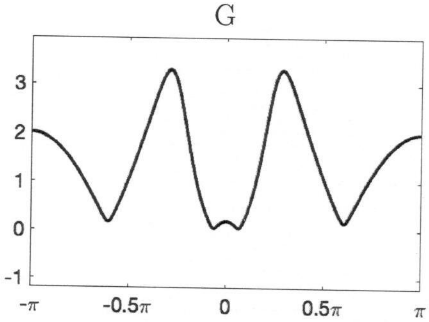
2
0
-π
-0.5π
0
0.5π
π

G


30
20
10
0
-π
-0.5π
0
0.5π
π

H


3
2
1
0
-1
-π
-0.5π
0
0.5π
π

I

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PROBLEM sp-24-FINAL.4:

[**3 points Each**] Pick the correct frequency response characteristic and enter the number in the answer box. Each frequency response is used only once. There are more items on the right-hand list than on the left-hand list.

(a) $h[n] = \left(-\frac{1}{2}\right)^n u[n]$

2

(b) $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

3

(c) `yn = filter([1,1],1,xn)`

5

(d) $h[n] = \sum_{k=0}^{3} \delta[n-k]$

6

(e) $y[n] = x[n-1] + 2x[n-3] + x[n-5]$

7

(f) $y[n] = \frac{1}{2}y[n-1] + x[n]$

1

1. $H(e^{j\hat{\omega}}) = \frac{1}{1-0.5e^{-j\hat{\omega}}}$
2. $H(e^{j\hat{\omega}}) = \frac{1}{1+0.5e^{-j\hat{\omega}}}$
3. $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}(0.5 + \cos(\hat{\omega}) + \cos(2\hat{\omega}))$
4. $H(e^{j\hat{\omega}}) = \frac{1}{1-e^{-j0.5\hat{\omega}}}$
5. $|H(e^{j\hat{\omega}})| = 2\cos(\hat{\omega}/2)$
6. $H(e^{j\hat{\omega}}) = \frac{\sin(2\hat{\omega})}{\sin(0.5\hat{\omega})}e^{-j1.5\hat{\omega}}$
7. $\angle H(e^{j\hat{\omega}}) = -3\hat{\omega}$
8. $|H(e^{j\hat{\omega}})| = 0.5\cos(2\hat{\omega})$

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PROBLEM sp-24-FINAL.5:

The parts in this problem are independent from each other.

(a) **[6 points]** Two periodic sequences are given as follows:

n	...	0	1	2	3	4	5	6	7	8	9	10	11	12	...
$x_1[n]$	...	1	0.5	-0.5	-1	-0.5	0.5	1	0.5	-.5	-1	-0.5	0.5	1	...
$x_2[n]$	...	0	2	0	-2	0	2	0	-2	0	2	0	-2	0	...

Note that the two signals have different periodicity. Suppose that we add the two signals to have a new periodic signal $x[n] = x_1[n] + x_2[n]$. For the sequence $x[n], n = 0, ..., 11$, determine the 12-point DFT sequence and write the values in the following table.

Hint: Write $x_1[n]$ and $x_2[n]$ as simple sinusoids.

k	0	1	2	3	4	5
$X[k]$	0	0	6	$12e^{-j\pi/2}$	0	0

k	6	7	8	9	10	11
$X[k]$	0	0	0	$12e^{j\pi/2}$	6	0

$x_1[n] = \cos(\frac{\pi}{3}n)$; $x_2[n] = 2\cos(\frac{\pi}{2}(n-1))$

Common period is 12 samples

$\Rightarrow$ 12-point DFT can be computed as follows using Fourier Series

$$x[n] = \frac{1}{2}\left[e^{-j\frac{\pi}{3}n} + e^{j\frac{\pi}{3}n}\right] + \left[e^{j\frac{\pi}{2}} e^{-j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}n}\right]$$

$$= \frac{1}{12}\left\{6\left[\quad\right] + 12\left[\quad\right]\right\}$$

(b) **[2 points]** Suppose the DFT, $X[k]$, of a sequence $x[n]$ shown below, is real. That is, $X^*[k] = X[k]$ for $k = 0, ..., 7$. Can the unknown values of $x[n]$ be determined? If yes, give the missing values. If no, then justify your answer.

$$\{1, 7, ?, ?, 1, 6, 5, 7\}$$

circle one **YES** (give the missing values below) or **NO** (justify)

5 and 6

$X^*[k] = X[k] \Rightarrow \text{IDFT}\{X[k]\}$ is conjugate symmetric

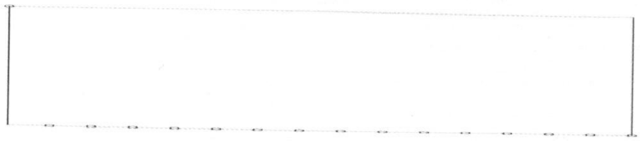
$\Rightarrow x[2] = x[8-2] = 5$

$x[3] = x[8-3] = 6$

(c) **[1 point Each]** Shown below are different outcomes that result from executing the following MATLABcode:

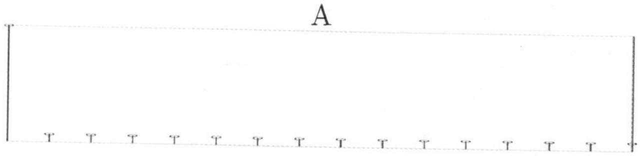
```
stem(abs(fft(ones(1,L),N));
```

s Match each plot with the corresponding values for the variables N nd L by writing a letter (A through I) in each answer box.



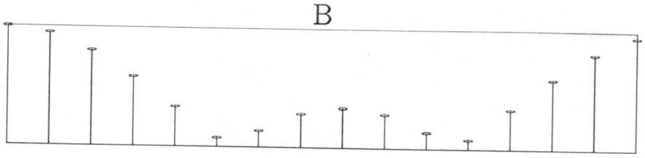
A

L = 2, N = 16 ANS = I



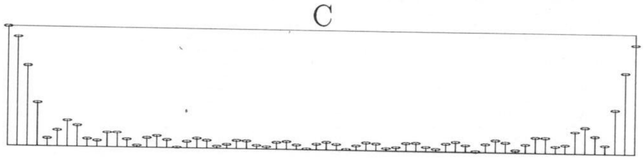
B

L = 2, N = 64 ANS = E



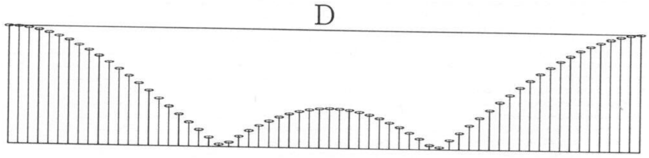
C

L = 3, N = 16 ANS = C



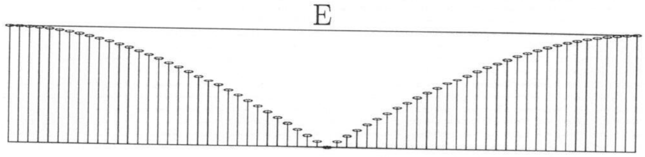
D

L = 3, N = 64 ANS = E



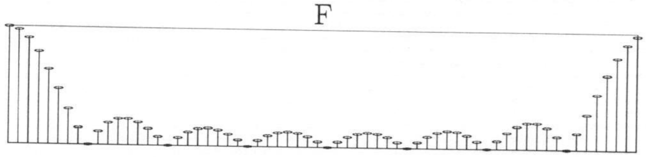
E

L = 8, N = 16 ANS = H



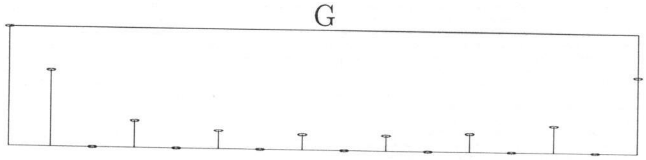
F

L = 8, N = 64 ANS = G



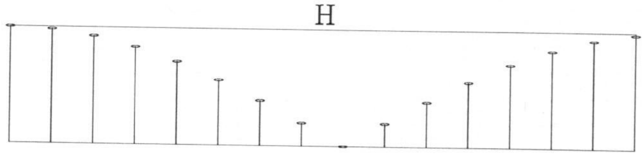
G

L = 15, N = 16 ANS = B



H

L = 15, N = 64 ANS = D



I

L = 16, N = 16 ANS = A

PROBLEM sp-24-FINAL.6:

The parts in this problem are independent from each other.

(a) [**4 points**] Simplify the following expression of $x[n]$ with a single term.

$$x[n] = \sum_{k=-\infty}^{\infty} \left(\frac{\sin(0.7\pi k)}{\pi k}\right) \cdot \left(\frac{\sin(0.85\pi(n-k))}{\pi(n-k)}\right)$$

$x[n] = \dfrac{\sin(0.7\pi n)}{\pi n}$

(b) [**4 points**] Determine the fundamental frequency of the signal

$$x(t) = \cos(40\pi t)\cos(24\pi t) + \cos(60\pi t)$$

2 sinusoids

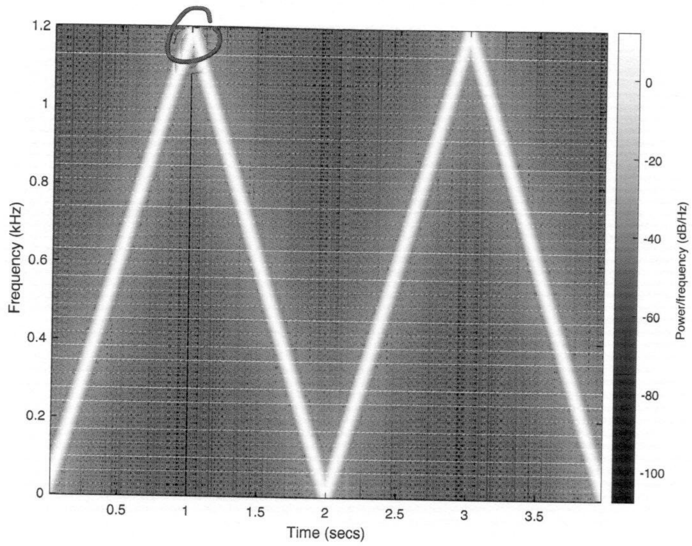
with $\omega_1 = 64\pi$

and $\omega_2 = 16\pi$

$f_0 =$ 2 Hz

(c) [**4 points**] Running the following MATLAB code produces the plot below, for a specific value of the parameter W.

```
fsamp = 2400;
tmax = 4;
tt = 0:(1/fsamp):tmax;
xx = real((20+15*j)*exp(j*2*pi*W*(tt.^2)));
spectrogram(xx,128,120,512,fsamp,'yaxis')
```



Note that the y-axis is in kHz, i.e., the highest frequency shown is 1200 Hz.

Determine the numerical value of the parameter W in the MATLAB code.

W = ~~1200~~

600

$\varphi(t) = 2\pi W t^2$

$f_i(t) = 2Wt$

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_0))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_0))} e^{-j(\hat{\omega}-\hat{\omega}_0)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1-ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Date: 24-Apr-2019

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n = 0, 1, 2, \ldots, N-1$	*Frequency-Domain:* $X[k], \quad k = 0, 1, 2, \ldots, N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
1	$N\delta[k]$
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$r_L[n] = u[n] - u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))} e^{-j(2\pi k/N)(L-1)/2}$
$r_L[n]e^{j(2\pi k_0/N)n}$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi(k-k_0)/N))}{\sin(\frac{1}{2}(2\pi(k-k_0)/N))} e^{-j(2\pi(k-k_0)/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))} e^{j(2\pi n/N)(L-1)/2}$	$N(u[k] - u[k-L])$, when $L \leq N$

Table of DFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X[k]$
Periodic	$x[n] = x[n+N]$	$X[k] = X[k+N]$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
Conjugate Symmetry	$x[n]$ is real	$X[N-k] = X^*[k]$
Conjugation	$x^*[n]$	$X^*[N-k]$
Time-Reversal	$x[N-n]$	$X[N-k]$
Delay (PERIODIC)	$x[n-n_d]$	$e^{-j(2\pi k/N)n_d}X[k]$
Frequency Shift	$x[n]e^{j(2\pi k_0/N)n}$	$X[k-k_0]$
Modulation	$x[n]\cos((2\pi k_0/N)n)$	$\frac{1}{2}X[k-k_0] + \frac{1}{2}X[k+k_0]$
Convolution (PERIODIC)	$x[n] * h[n] = \sum_{m=0}^{N-1} h[m]x[n-m]$	$X[k]H[k]$
Parseval's Theorem	$\displaystyle\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1} \lvert X[k]\rvert^2$	

Table of z-Transform Pairs

Signal Name	*Time-Domain: $x[n]$*	*z-Domain: $X(z)$*
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\frac{1}{1-az^{-1}}, \quad \lvert a\rvert < 1$
Decaying cosine	$r^n \cos(\hat{\omega}_0 n) u[n]$	$\frac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$
Decaying sinusoid	$Ar^n \cos(\hat{\omega}_0 n+\varphi) u[n]$	$A\frac{\cos(\varphi)-r\cos(\hat{\omega}_0-\varphi)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties

Property Name	*Time-Domain $x[n]$*	*z-Domain $X(z)$*
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay (d=integer)	$x[n-d]$	$z^{-d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

Date: 24-April-2019