GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 Spring 2023
Final Exam

April 28, 2023

NAME: ____________ ______________ GT username: ______________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section:

L01 (Chen)	L07 (Davenport)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Duan)	L10 (Chen)	

Important Notes:

- You may tear off the tables from the back of the exam, but otherwise do not unstaple the rest.
- One two-sided page (8.5” × 11”) of notes permitted.
- Calculators are allowed, but no other electronics (no smartphones/watches/readers/tablets/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score
1	15	
2	15	
3	18	
4	10	
5	18	
6	12	
7	12	
Total		

PROB. Sp23-F.1.

Suppose that the following equation is true, for all time t:

$$t_0 \cos(2\pi f_0(t - t_0)) \;=\; \sqrt{2}\cos(2\pi f_0 t) \;+\; \sqrt{2}\sin(2\pi f_0 t).$$

(a) As a function of the unspecified parameters f_0 and t_0, the corresponding phasor equation is:

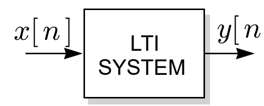
$$\boxed{} = \boxed{} + \boxed{}.$$

(b) Find the *smallest* positive values for f_0 and t_0 so that the given equation is true, for all time t:

$f_0 = \boxed{} > 0,$

$t_0 = \boxed{} > 0.$

PROB. Sp23-F.2. Let $x[n] = -2\delta[n] + 2\delta[n-2] + \delta[n-4]$ be the input to an LTI filter whose frequency response is $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(3 + 4\cos(\hat{\omega}))$, resulting in an output $y[n]$.



(a) T $\square$ F $\square$ (True or False): The system is FIR.

(b) The impulse response $h[n]$ satisfies:

$h[0] = \boxed{\quad}$,

$h[1] = \boxed{\quad}$,

$h[2] = \boxed{\quad}$.

(c) The filter output $y[n]$ satisfies:

$y[0] = \boxed{\quad}$,

$y[1] = \boxed{\quad}$,

$y[2] = \boxed{\quad}$,

$y[3] = \boxed{\quad}$,

$y[4] = \boxed{\quad}$,

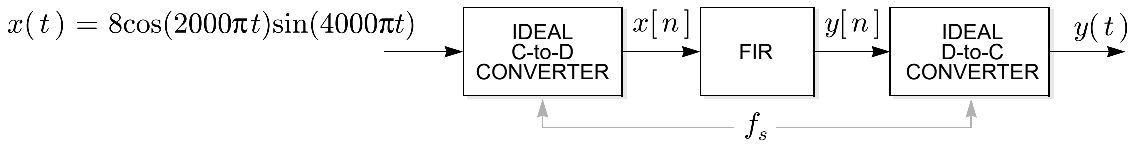
$y[5] = \boxed{\quad}$,

$y[6] = \boxed{\quad}$,

$y[7] = \boxed{\quad}$.

PROB. Sp23-F.3.

Suppose that a continuous-time signal $x(t) = 8\cos(2000\pi t)\sin(4000\pi t)$ is fed to an ideal sampling/filtering/reconstruction system, producing an output $y(t)$, as shown below:



The difference equation for the FIR filter is $y[n] = x[n] + Bx[n-1] + x[n-2]$.
The parameter B and the sampling rate f_s are unspecified and different in each part below.

(a) How big must f_s be to avoid aliasing? $f_s >$ ______ Hz.

(b) If a sampling rate of $f_s = 12000$ Hz results in a D-to-C output $y(t)$ that is *periodic* with fundamental frequency $f_0 = 3000$ Hz, then it must be that: $B =$ ______ .

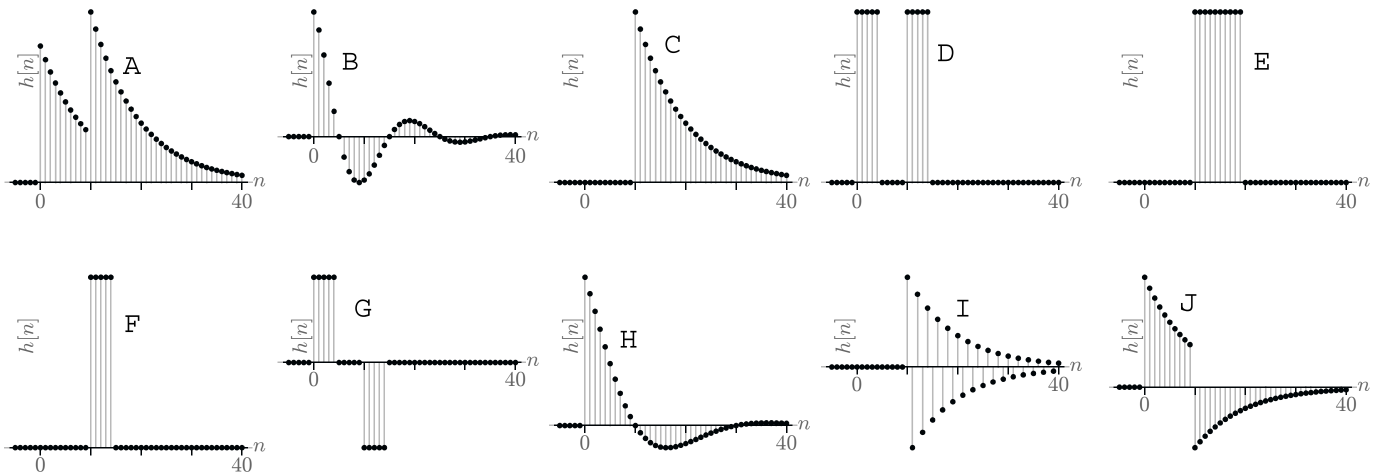
(c) When $f_s = 6000$ Hz and $B = 2$, the D-to-C output has the standard form $y(t) = A\cos(2\pi f_0 t + \varphi)$, where:

$A =$ ______ , > 0

$f_0 =$ ______ Hz, > 0

$\varphi =$ ______ . $\in [-\pi, \pi]$

PROB. Sp23-F.4. Shown below are ten different impulse responses $h[n]$, labeled A through J, where $h[n]$ is plotted for times $n \in \{-5, \ldots, 40\}$ only:



(The y-axis scales are not specified, they are not needed to solve the problem.)
Each $h[n]$ shown above is the impulse response of a filter whose frequency response $H(e^{j\hat{\omega}})$ is listed below, but the order is scrambled. Match each $H(e^{j\hat{\omega}})$ below to its corresponding stem plot for $h[n]$ above. Indicate your answers by writing a letter (from A through J) into each of the ten answer boxes.

(i) $\boxed{}$ $$H_1(e^{j\hat{\omega}}) = \frac{e^{-j10\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$$

(ii) $\boxed{}$ $$H_2(e^{j\hat{\omega}}) = \frac{e^{-j10\hat{\omega}}}{1 + 0.9e^{-j\hat{\omega}}}$$

(iii) $\boxed{}$ $$H_3(e^{j\hat{\omega}}) = \frac{1 + 0.9e^{-j10\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$$

(iv) $\boxed{}$ $$H_4(e^{j\hat{\omega}}) = \frac{1 - 0.9e^{-j10\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$$

(v) $\boxed{}$ $$H_5(e^{j\hat{\omega}}) = 2e^{-j7\hat{\omega}}\frac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}\cos(5\hat{\omega})$$

(vi) $\boxed{}$ $$H_6(e^{j\hat{\omega}}) = 2je^{-j7\hat{\omega}}\frac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}\sin(5\hat{\omega})$$

(vii) $\boxed{}$ $$H_7(e^{j\hat{\omega}}) = \frac{0.5}{1 - 0.9e^{-j(\hat{\omega} - 0.1\pi)}} + \frac{0.5}{1 - 0.9e^{-j(\hat{\omega} + 0.1\pi)}}$$

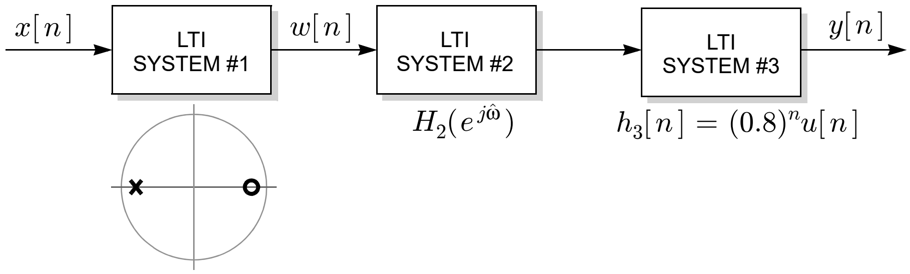
(viii) $\boxed{}$ $$H_8(e^{j\hat{\omega}}) = \frac{0.5}{1 - 0.9e^{-j(\hat{\omega} - 0.05\pi)}} + \frac{0.5}{1 - 0.9e^{-j(\hat{\omega} + 0.05\pi)}}$$

(ix) $\boxed{}$ $$H_9(e^{j\hat{\omega}}) = e^{-j12\hat{\omega}}\frac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}.$$

(x) $\boxed{}$ $$H_{10}(e^{j\hat{\omega}}) = e^{-j14.5\hat{\omega}}\frac{\sin(5\hat{\omega})}{\sin(0.5\hat{\omega})}.$$

PROB. Sp23-F.5.

Consider the following serial cascade of three LTI systems:



As shown above, an input sequence $x[n]$ is fed to a first LTI system, whose output $w[n]$ is fed as an input to a second system, whose output is fed to a third system, producing an overall output $y[n]$.

- The *first* system (with input $x[n]$ and output $w[n]$) has the pole-zero plot for its system function shown above, with a zero at 0.8 and a pole at -0.8.
- The *second* system has frequency response: $H_2(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} + b_2 e^{-2j\hat{\omega}}$, where $\{b_k\}$ are unspecified.
- The *third* system has impulse response: $h_3[n] = (0.8)^n u[n]$.

(a) T ☐ F ☐ (True or False): The first system is FIR.

(b) T ☐ F ☐ (True or False): The second system is FIR.

(c) T ☐ F ☐ (True or False): The third system is FIR.

(d) If the difference equation relating the overall output to the overall input is:

$$y[n] = x[n] - x[n-1],$$

then it must be that:

$b_0 =$ ☐ ,

$b_1 =$ ☐ ,

$b_2 =$ ☐ .

PROB. Sp23-F.6. Let $x[n] = \delta[n] + \beta\delta[n-1] + \delta[n-2]$.

Shown below are plots of $\left|\sum_{n=0}^{2} x[n] e^{-jk2\pi n/N}\right|$ versus $k \in \{0, \ldots N-1\}$, labeled A through L.
(The axes are not labeled, only the shapes matter.)
Match each plot to the corresponding values of the parameters β and N.
Indicate your answer by writing a letter (from A through L) in each answer box.

(a) $\beta = 0,\ N = 12.$

(b) $\beta = 1,\ N = 12.$

(c) $\beta = 2,\ N = 12.$

(d) $\beta = 3,\ N = 12.$

(e) $\beta = 0,\ N = 15.$

(f) $\beta = 1,\ N = 15.$

(g) $\beta = 2,\ N = 15.$

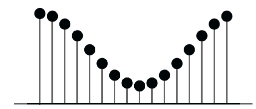
(h) $\beta = 3,\ N = 15.$

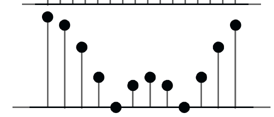
(i) $\beta = 0,\ N = 16.$

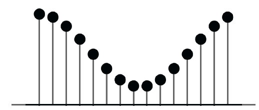
(j) $\beta = 1,\ N = 16.$

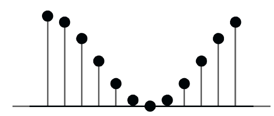
(k) $\beta = 2,\ N = 16.$

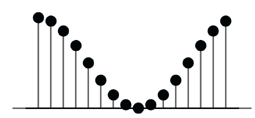
(l) $\beta = 3,\ N = 16.$

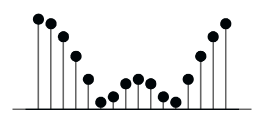
A 

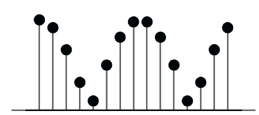
B 

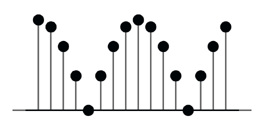
C 

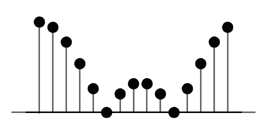
D 

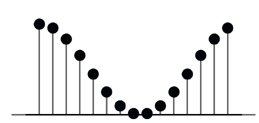
E 

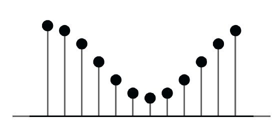
F 

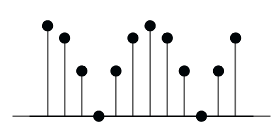
G 

H 

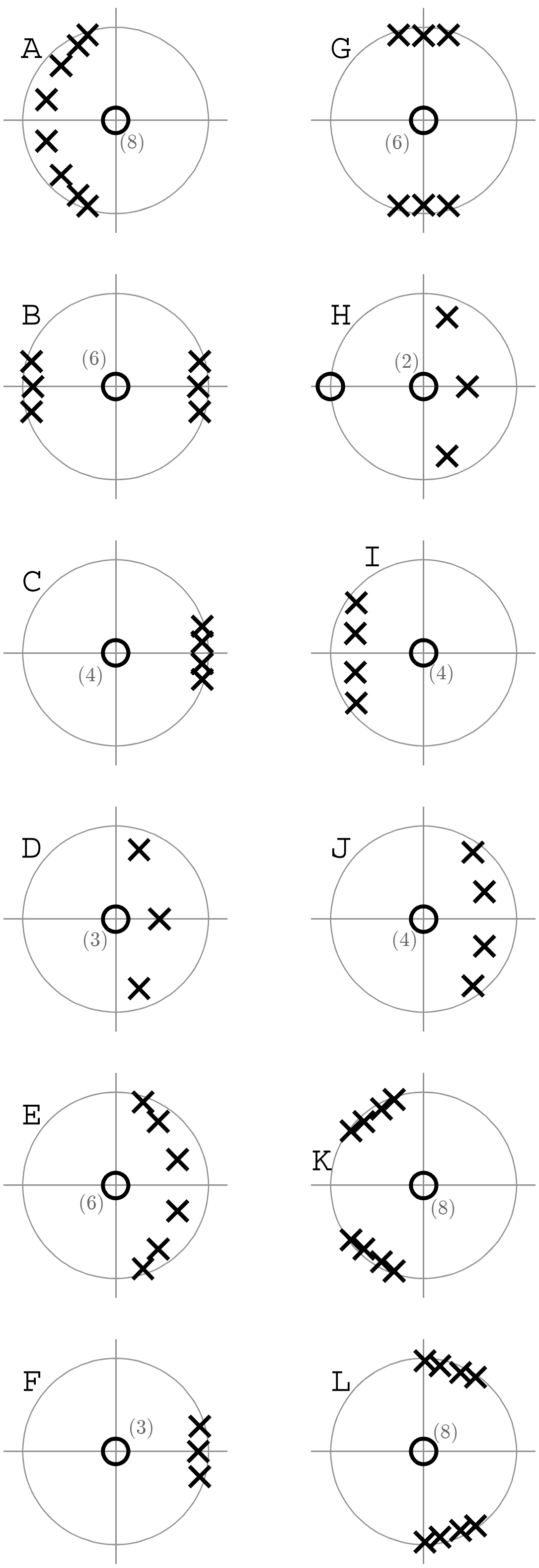
I 

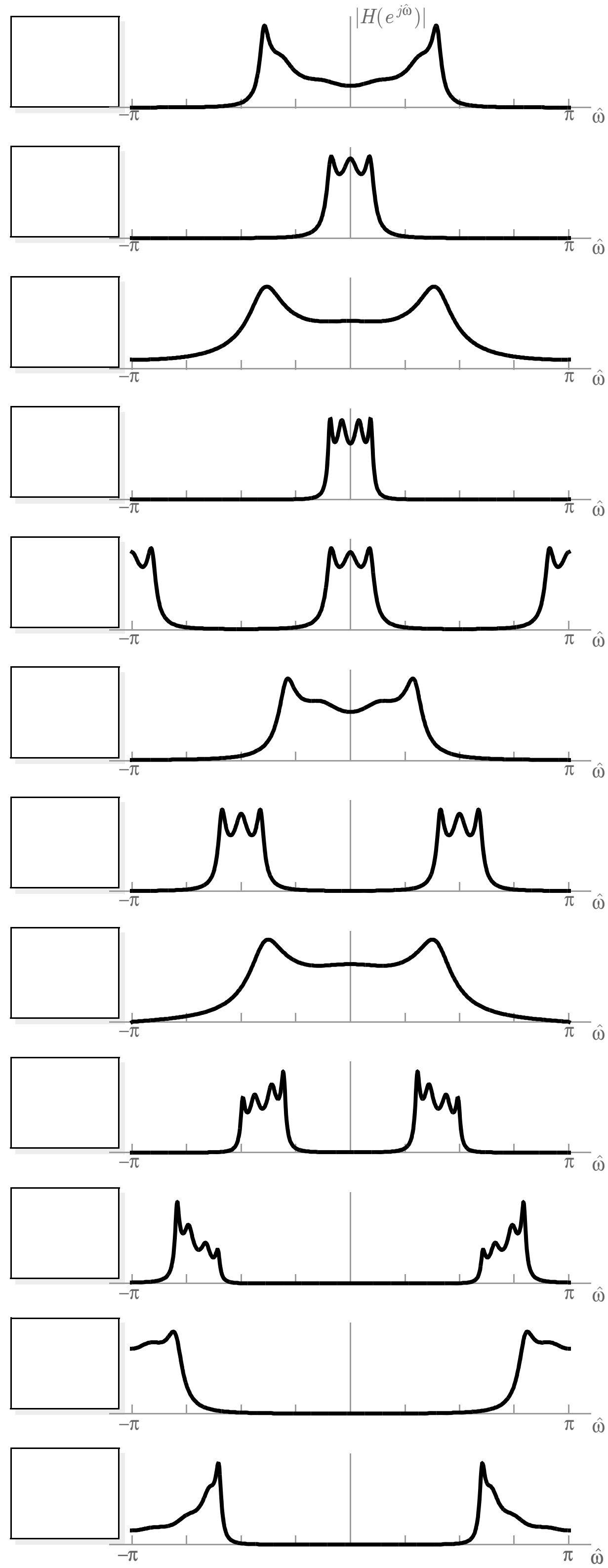
J 

K 

L 

PROB. Sp23-F.7. Shown below on the left are twelve pole-zero plots for $H(z)$, labeled A through L. Shown below on the right are the corresponding magnitude responses, $|H(e^{j\hat{\omega}})|$ plotted *vs* $\hat{\omega}$, but in a scrambled order. Match each magnitude response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through L) into each answer box.





version A

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 Spring 2023
Final Exam

April 28, 2023

NAME: ANSWER KEY ______ ______ (FIRST) (LAST) GT username: A ______ (e.g., gtxyz123)

Circle your recitation section:

L01 (Chen)	L07 (Davenport)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Duan)	L10 (Chen)	

Important Notes:

- You may tear off the tables from the back of the exam, but otherwise do not unstaple the rest.
- One two-sided page (8.5” × 11”) of notes permitted.
- Calculators are allowed, but no other electronics (no smartphones/watches/readers/tablets/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to $18°$ or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score
1	15	
2	15	
3	18	
4	10	
5	18	
6	12	
7	12	
Total		

version A

PROB. Sp23-F.1.

Suppose that the following equation is true, for all time t:

$$t_0 \cos(2\pi f_0(t - t_0)) \;=\; \sqrt{2}\cos(2\pi f_0 t) \;+\; \sqrt{2}\sin(2\pi f_0 t).$$

(a) As a function of the unspecified parameters f_0 and t_0, the corresponding phasor equation is:

$$\boxed{t_0 e^{-j2\pi f_0 t_0}} = \boxed{\sqrt{2}} + \boxed{-j\sqrt{2}}.$$

$$\left(\text{or } \sqrt{2}e^{-j0.5\pi}\right)$$

(b) Find the *smallest* positive values for f_0 and t_0 so that the given equation is true, for all time t:

$$f_0 = \boxed{\frac{1}{16}} > 0,$$

$$t_0 = \boxed{2} > 0.$$

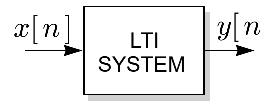
In polar form, the phasor equation simplifies to:

$$t_0 e^{-j2\pi f_0 t_0} \;=\; 2e^{-j0.25\pi}$$

- Equate magnitudes $\Rightarrow$ $|t_0 e^{-j2\pi f_0 t_0}| = |2e^{-j0.25\pi}|$ $\Rightarrow$ $t_0 = 2$
- Equate phases $\Rightarrow$ $2\pi f_0 t_0 = 0.25\pi$ $\Rightarrow$ $f_0 = \dfrac{0.25\pi}{2\pi t_0} = \dfrac{1}{8t_0} = \dfrac{1}{16}$

version A

PROB. Sp23-F.2. Let $x[n] = -2\delta[n] + 2\delta[n-2] + \delta[n-4]$ be the input to an LTI filter whose frequency response is $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(3 + 4\cos(\hat{\omega}))$, resulting in an output $y[n]$.



(a) T [X] F [ ] (True or False): The system is FIR.

(b) The impulse response $h[n]$ satisfies:

$h[0] = \boxed{2}$,

$h[1] = \boxed{3}$,

$h[2] = \boxed{2}$.

(c) The filter output $y[n]$ satisfies:

$y[0] = \boxed{-4}$,

$y[1] = \boxed{-6}$,

$y[2] = \boxed{0}$,

$y[3] = \boxed{6}$,

$y[4] = \boxed{6}$,

$y[5] = \boxed{3}$,

$y[6] = \boxed{2}$,

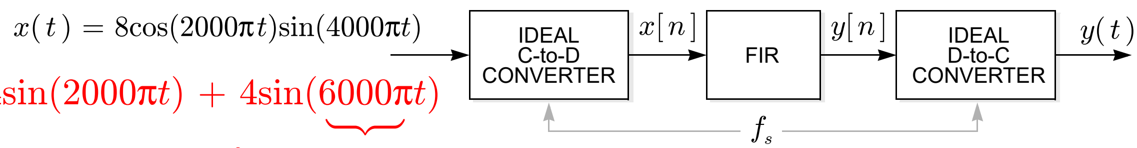
$y[7] = \boxed{0}$.

$$
\begin{array}{cccccccc}
 & -4 & 0 & 4 & 0 & 2 & & \\
 & & -6 & 0 & 6 & 0 & 3 & \\
+ & & & -4 & 0 & 4 & 0 & 2 \\
\hline
\end{array}
$$

version A

PROB. Sp23-F.3.

Suppose that a continuous-time signal $x(t) = 8\cos(2000\pi t)\sin(4000\pi t)$ is fed to an ideal sampling/filtering/reconstruction system, producing an output $y(t)$, as shown below:



$= 4\sin(2000\pi t) + 4\sin(\underbrace{6000\pi t})$
(Euler)

$f_{max} = 3000$ Hz

The difference equation for the FIR filter is $y[n] = x[n] + Bx[n-1] + x[n-2]$.
The parameter B and the sampling rate f_s are unspecified and different in each part below.

(a) How big must f_s be to avoid aliasing? $f_s >$ $\boxed{6000}$ Hz.

(b) If a sampling rate of $f_s = 12000$ Hz results in a D-to-C output $y(t)$ that is *periodic* with fundamental frequency $f_0 = 3000$ Hz, then it must be that: $B = \boxed{-\sqrt{3}}$.

To have $f_0 = 3$ kHz, the 1 kHz input must be *nulled*

With $f_s = 12$ kHz, the frequency to null is $\hat{\omega} = \pi/6$

$\Rightarrow B = -2\cos(\pi/6) = -\sqrt{3}$

(c) When $f_s = 6000$ Hz and $B = 2$, the D-to-C output has the standard form $y(t) = A\cos(2\pi f_0 t + \varphi)$, where:

$A = \boxed{12}$, > 0

$f_0 = \boxed{1000}$ Hz, > 0

$\varphi = \boxed{-\frac{5\pi}{6}}$. $\in[-\pi,\pi]$

With $f_s = 6$ kHz, the 3 kHz input $4\sin(6000\pi t)$ aliases to *zero*!

$\Rightarrow x[n] = 4\sin(\frac{\pi}{3}n)$.

But $H(e^{j\pi/3}) = 1 + 2e^{-j\pi/3} + e^{-j2\pi/3}$
$= 3e^{-j\pi/3}$

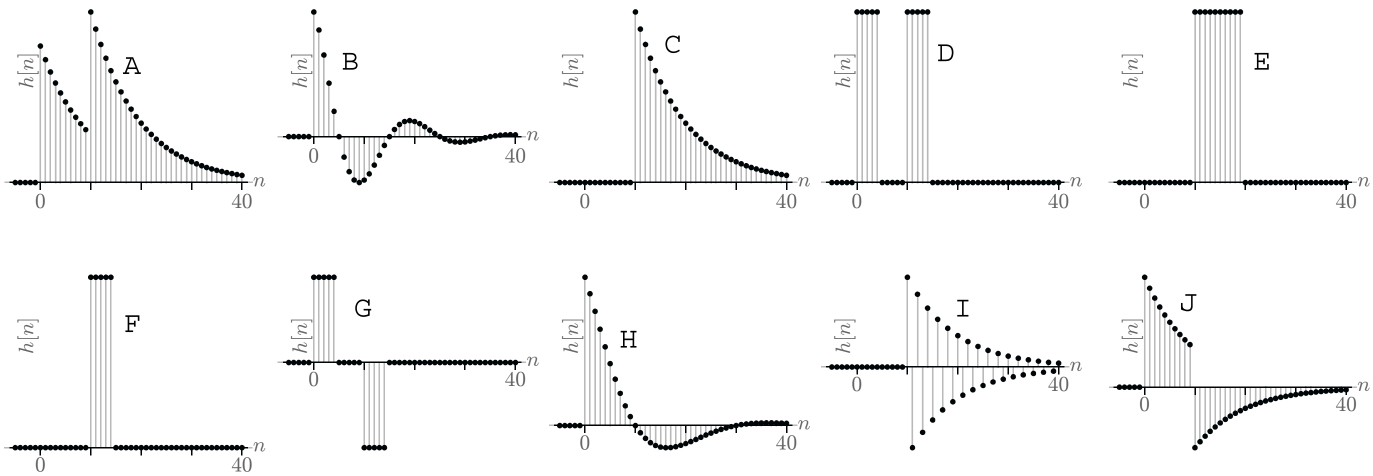
$\Rightarrow y[n] = 12\sin(\frac{\pi}{3}n - \frac{\pi}{3})$.

$\Rightarrow y(t) = y[f_s t] = 12\sin(2000\pi t - \frac{\pi}{3})$

$= 12\cos(2000\pi t - \frac{5\pi}{6})$

version A

PROB. Sp23-F.4. Shown below are ten different impulse responses $h[n]$, labeled A through J, where $h[n]$ is plotted for times $n \in \{-5, \ldots, 40\}$ only:



(The y-axis scales are not specified, they are not needed to solve the problem.)
Each $h[n]$ shown above is the impulse response of a filter whose frequency response $H(e^{j\hat{\omega}})$ is listed below, but the order is scrambled. Match each $H(e^{j\hat{\omega}})$ below to its corresponding stem plot for $h[n]$ above. Indicate your answers by writing a letter (from A through J) into each of the ten answer boxes.

(i) C $\quad H_1(e^{j\hat{\omega}}) = \dfrac{e^{-j10\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$

(ii) I $\quad H_2(e^{j\hat{\omega}}) = \dfrac{e^{-j10\hat{\omega}}}{1 + 0.9e^{-j\hat{\omega}}}$

(iii) A $\quad H_3(e^{j\hat{\omega}}) = \dfrac{1 + 0.9e^{-j10\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$

(iv) J $\quad H_4(e^{j\hat{\omega}}) = \dfrac{1 - 0.9e^{-j10\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$

(v) D $\quad H_5(e^{j\hat{\omega}}) = 2e^{-j7\hat{\omega}} \dfrac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})} \cos(5\hat{\omega})$

(vi) G $\quad H_6(e^{j\hat{\omega}}) = 2je^{-j7\hat{\omega}} \dfrac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})} \sin(5\hat{\omega})$

(vii) B $\quad H_7(e^{j\hat{\omega}}) = \dfrac{0.5}{1 - 0.9e^{-j(\hat{\omega} - 0.1\pi)}} + \dfrac{0.5}{1 - 0.9e^{-j(\hat{\omega} + 0.1\pi)}}$

(viii) H $\quad H_8(e^{j\hat{\omega}}) = \dfrac{0.5}{1 - 0.9e^{-j(\hat{\omega} - 0.05\pi)}} + \dfrac{0.5}{1 - 0.9e^{-j(\hat{\omega} + 0.05\pi)}}$

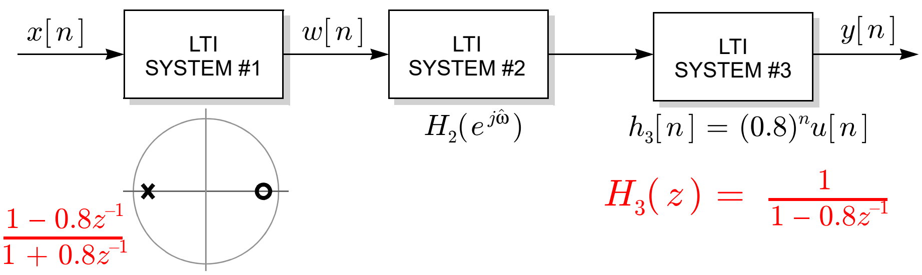
(ix) F $\quad H_9(e^{j\hat{\omega}}) = e^{-j12\hat{\omega}} \dfrac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}.$

(x) E $\quad H_{10}(e^{j\hat{\omega}}) = e^{-j14.5\hat{\omega}} \dfrac{\sin(5\hat{\omega})}{\sin(0.5\hat{\omega})}.$

version A

PROB. Sp23-F.5.

Consider the following serial cascade of three LTI systems:



$H_1(z) = \dfrac{1 - 0.8z^{-1}}{1 + 0.8z^{-1}}$ $\qquad$ $H_3(z) = \dfrac{1}{1 - 0.8z^{-1}}$

As shown above, an input sequence $x[n]$ is fed to a first LTI system, whose output $w[n]$ is fed as an input to a second system, whose output is fed to a third system, producing an overall output $y[n]$.

- The *first* system (with input $x[n]$ and output $w[n]$) has the pole-zero plot for its system function shown above, with a zero at 0.8 and a pole at –0.8.
- The *second* system has frequency response: $H_2(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} + b_2 e^{-2j\hat{\omega}}$, where $\{b_k\}$ are unspecified.
- The *third* system has impulse response: $h_3[n] = (0.8)^n u[n]$.

(a) T ☐ F ☒ (True or False): The first system is FIR.

(b) T ☒ F ☐ (True or False): The second system is FIR.

(c) T ☐ F ☒ (True or False): The third system is FIR.

(d) If the difference equation relating the overall output to the overall input is:

$$y[n] = x[n] - x[n-1], \qquad \Rightarrow H(z) = 1 - z^{-1}$$

then it must be that:

Solve $H(z) = H_1(z)H_2(z)H_3(z)$ for $H_2(z)$:

$$\Rightarrow \quad H_2(z) = \frac{H(z)}{H_1(z)H_3(z)} = \frac{1 - z^{-1}}{\left(\frac{\cancel{1 - 0.8z^{-1}}}{1 + 0.8z^{-1}}\right)\left(\frac{1}{\cancel{1 - 0.8z^{-1}}}\right)}$$

$$= (1 + 0.8z^{-1})(1 - z^{-1})$$

$$= 1 - 0.2z^{-1} - 0.8z^{-2}$$

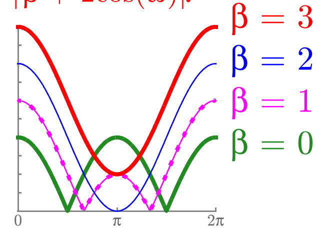
$b_0 = \boxed{1}$,

$b_1 = \boxed{-0.2}$,

$b_2 = \boxed{-0.8}$.

version A

$\left|X(e^{j\hat{\omega}})\right| = |\beta + 2\cos(\hat{\omega})|$:



PROB. Sp23-F.6. Let $x[n] = \delta[n] + \beta\delta[n-1] + \delta[n-2]$.

Shown below are plots of $\left|\sum_{n=0}^{2} x[n]e^{-jk2\pi n/N}\right|$ versus $k \in \{0, \dots N-1\}$, labeled A through L. (The axes are not labeled, only the shapes matter.)
Match each plot to the corresponding values of the parameters β and N.
Indicate your answer by writing a letter (from A through L) in each answer box.

(a) L $\beta = 0,\ N = 12.$

(b) B $\beta = 1,\ N = 12.$

(c) D $\beta = 2,\ N = 12.$

(d) K $\beta = 3,\ N = 12.$

(e) G $\beta = 0,\ N = 15.$

(f) I $\beta = 1,\ N = 15.$

(g) J $\beta = 2,\ N = 15.$

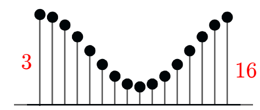
(h) C $\beta = 3,\ N = 15.$

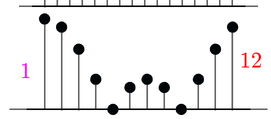
(i) H $\beta = 0,\ N = 16.$

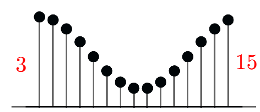
(j) F $\beta = 1,\ N = 16.$

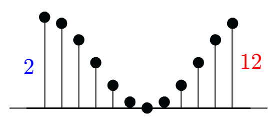
(k) E $\beta = 2,\ N = 16.$

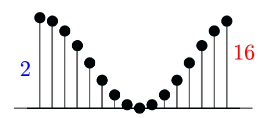
(l) A $\beta = 3,\ N = 16.$

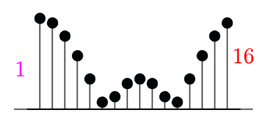
A


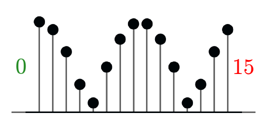
B


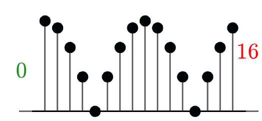
C


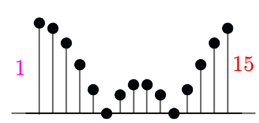
D


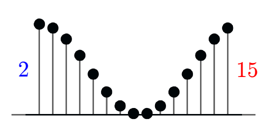
E


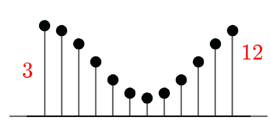
F


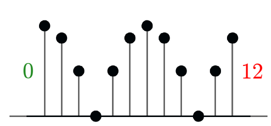
G


H


I


J


K


L


version A

PROB. Sp23-F.7. Shown below on the left are twelve pole-zero plots for $H(z)$, labeled A through L. Shown below on the right are the corresponding magnitude responses, $|H(e^{j\hat{\omega}})|$ plotted *vs* $\hat{\omega}$, but in a scrambled order. Match each magnitude response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through L) into each answer box.