

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

**ECE 2026 Spring 2022
Final Exam**

April 29, 2022

NAME: _____
(FIRST) (LAST)

GT username: _____
(e.g., gtxyz123)

To earn 2 points, circle your recitation section:

L01 (Tai)	L07 (Tai)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Sadiq)	L10 (Sadiq)	L12 (Duan)

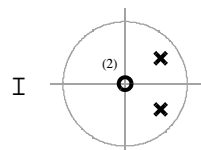
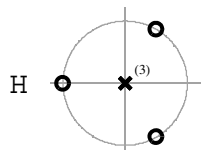
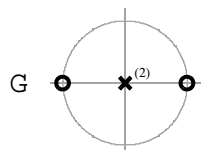
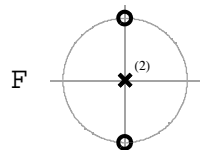
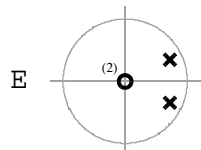
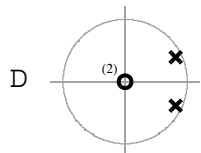
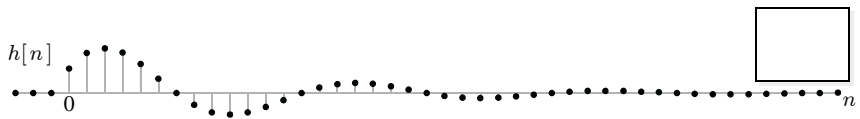
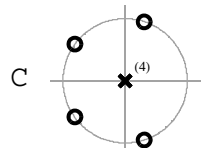
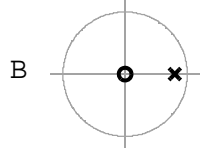
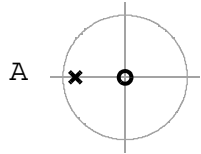
Important Notes:

- You may tear off the tables from the back of the exam, but otherwise do not unstaple the rest.
- One two-sided page (8.5" × 11") of notes permitted.
- Calculators are allowed, but no smartphones/tablets/readers/watches/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π . For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the provided answer boxes. If more space is needed for scratch work, use the backs of the previous pages.
- Problem 5: Each answer worth 4 points; Problem 6: Each answer worth 3 points. For all other problems (1, 2, 3, 4, 7, 8): each answer is worth a single point.

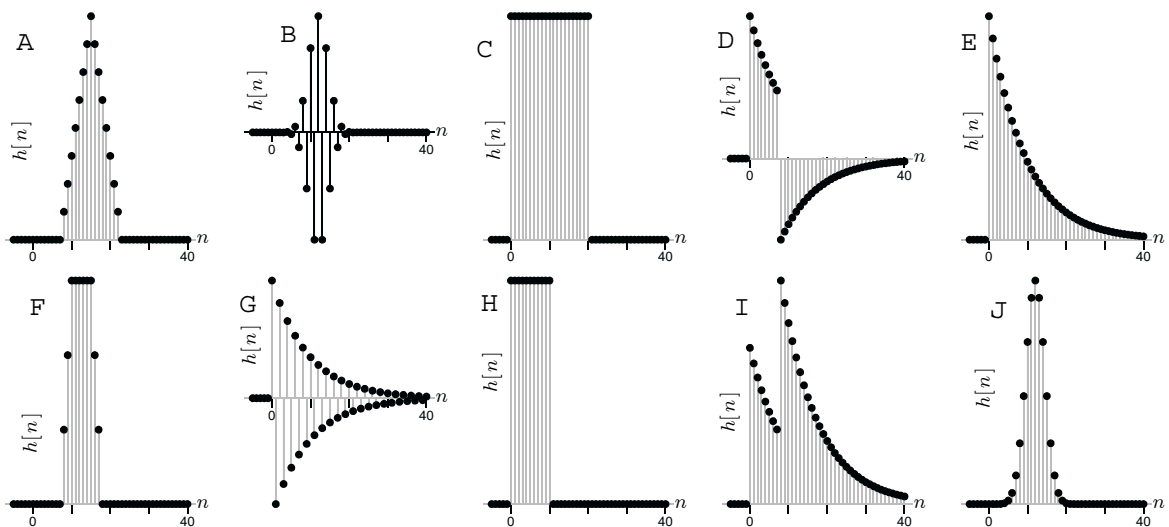
Problem	Value	Score
1	9	
2	10	
3	12	
4	12	
5	16	
6	15	
7	12	
8	12	
RECITATION	2	
Total		

PROB. Sp22F.1.

Shown below on the left are nine pole-zero plots for $H(z)$, labeled A through I. Shown on the right are stem plots of the corresponding time-domain signal $h[n]$ for $n \in \{-3, -2, \dots, 42, 43\}$, but in a scrambled order. Match each impulse response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through I) into each answer box.



PROB. Sp22F.2. Shown below are ten different impulse responses $h[n]$, labeled A through J, where $h[n]$ is plotted for times $n \in \{-5, \dots, 40\}$ only:



(The y-axis scales are not specified, they are not needed to solve the problem.)

Each $h[n]$ shown above is the impulse response of a filter whose frequency response $H(e^{j\hat{\omega}})$ is listed below, but the order is scrambled. Match each $H(e^{j\hat{\omega}})$ below to its corresponding stem plot for $h[n]$ above. Indicate your answers by writing a letter (from A through J) into each of the ten answer boxes.

(i)

$$H_1(e^{j\hat{\omega}}) = \frac{1}{1 - 0.9e^{-j\hat{\omega}}}$$

(ii)

$$H_2(e^{j\hat{\omega}}) = \frac{1}{1 + 0.9e^{-j\hat{\omega}}}$$

(iii)

$$H_3(e^{j\hat{\omega}}) = \frac{1 + e^{-j8\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$$

(iv)

$$H_4(e^{j\hat{\omega}}) = \frac{1 - e^{-j8\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$$

(v)

$$H_5(e^{j\hat{\omega}}) = (e^{-j7.5\hat{\omega}} \frac{\sin(4\hat{\omega})}{\sin(0.5\hat{\omega})})^2$$

(vi)

$$H_6(e^{j\hat{\omega}}) = e^{-j12.5\hat{\omega}} \left(\frac{\sin(1.5\hat{\omega})}{\sin(0.5\hat{\omega})} \right) \left(\frac{\sin(4\hat{\omega})}{\sin(0.5\hat{\omega})} \right)$$

(vii)

$$H_7(e^{j\hat{\omega}}) = (1 + e^{-j\hat{\omega}})^{24}$$

(viii)

$$H_8(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})^{24}$$

(ix)

$$H_9(e^{j\hat{\omega}}) = e^{-j5\hat{\omega}} \frac{\sin(5.5\hat{\omega})}{\sin(0.5\hat{\omega})}$$

(x)

$$H_{10}(e^{j\hat{\omega}}) = e^{-j10\hat{\omega}} \frac{\sin(10.5\hat{\omega})}{\sin(0.5\hat{\omega})}$$

PROB. Sp22F.3.

Let $x[n] = 3\delta[n] - \delta[n - 1] + \delta[n - 4]$
be the input to an LTI filter whose frequency response is
 $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega}))$, resulting in an output $y[n]$.



- (a) ^T ^F (True or False): The system is FIR.
- (b) The impulse response $h[n]$ satisfies:

$$h[0] = \boxed{},$$

$$h[1] = \boxed{},$$

$$h[2] = \boxed{}.$$

- (c) The filter output $y[n]$ satisfies:

$$y[0] = \boxed{},$$

$$y[1] = \boxed{},$$

$$y[2] = \boxed{},$$

$$y[3] = \boxed{},$$

$$y[4] = \boxed{},$$

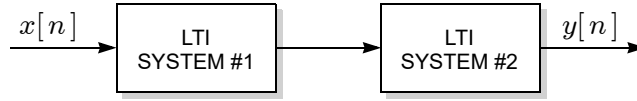
$$y[5] = \boxed{},$$

$$y[6] = \boxed{},$$

$$y[7] = \boxed{}.$$

PROB. Sp22F.4.

Consider the following serial cascade of a pair of LTI systems:



As shown in the figure, an input sequence $x[n]$ is fed to a first LTI system, whose output is fed as an input to a second LTI system, producing an overall output sequence $y[n]$.

The first system has frequency response $H_1(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} + b_2 e^{-2j\hat{\omega}}$, where the parameters $\{b_0, b_1, b_2\}$ are real but otherwise unspecified. The second system has impulse response:

$$h_2[n] = \left(\frac{-1}{2}\right)^n u[n].$$

If the difference equation relating the overall output to the overall input is:

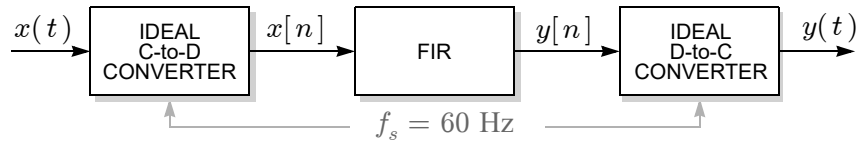
$$y[n] = 16x[n] + 8x[n - 1],$$

then it must be that:

$$b_0 = \boxed{}, \quad b_1 = \boxed{}, \quad b_2 = \boxed{}.$$

PROB. Sp22F.6.

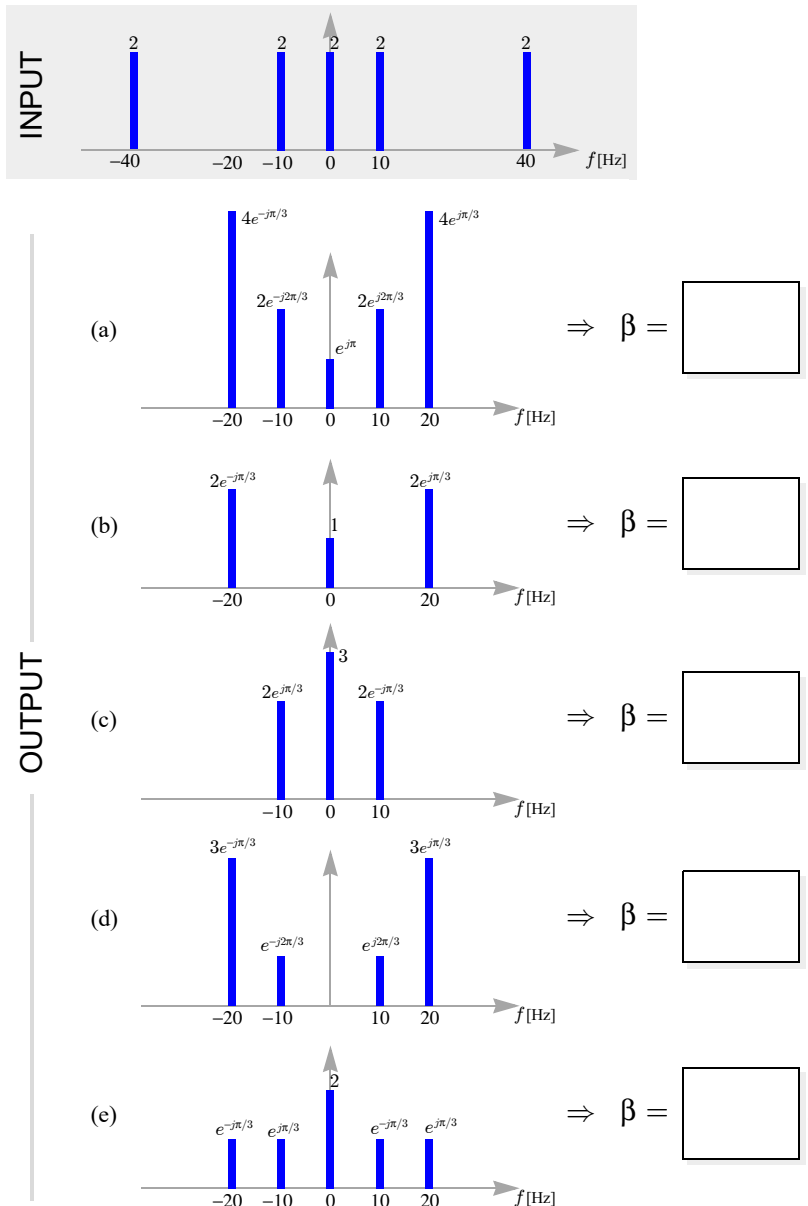
Suppose that a continuous-time signal $x(t)$ is fed to an ideal sampling/filtering/reconstruction system with sampling rate is $f_s = 60$ Hz, producing an output $y(t)$, as shown below:



The difference equation for the FIR filter is $y[n] = 0.5x[n] - \beta x[n - 1] + 0.5x[n - 2]$, where the real parameter β is unspecified.

The first spectrum (shaded) shown below is for the **input** signal $x(t)$.

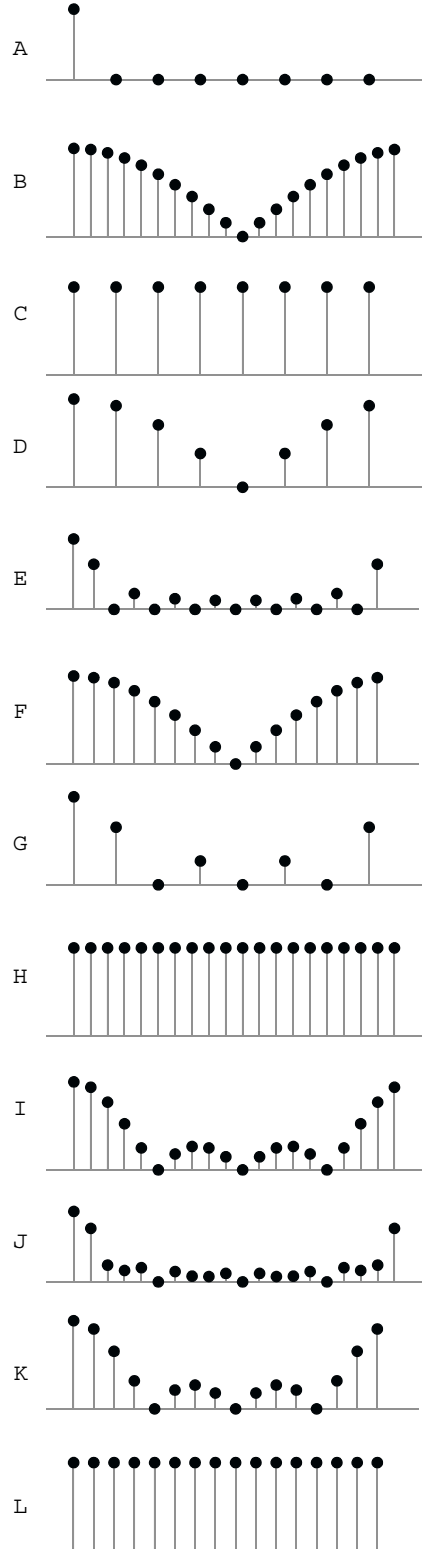
The remaining five spectra (labeled (a) through (e)) are for the **output** signal $y(t)$ for five different values of the filter parameter β . Find the numerical value of β that results in each output spectrum.



PROB. Sp22F.7.

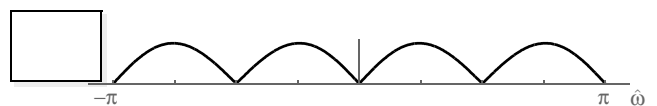
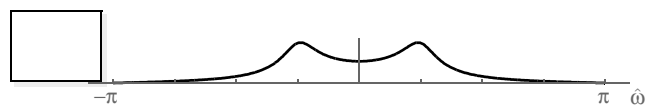
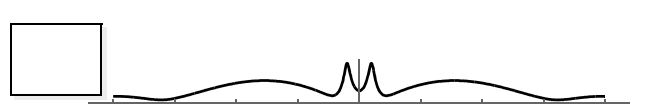
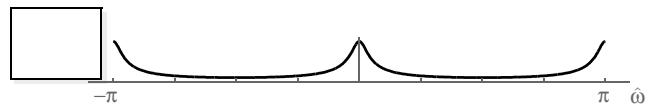
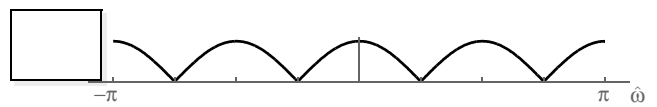
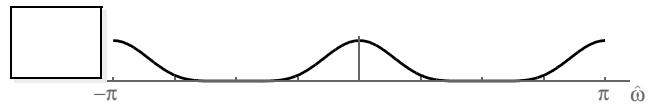
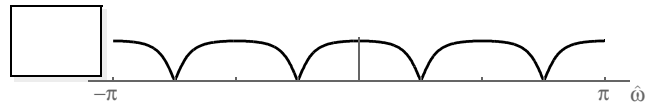
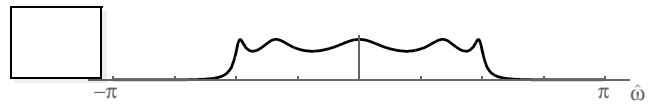
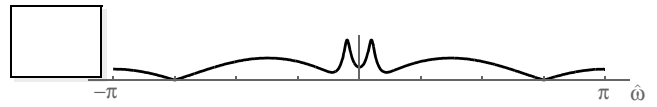
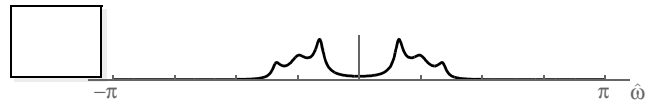
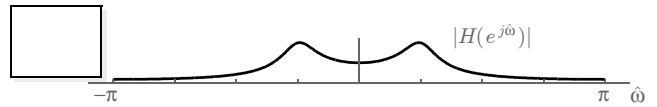
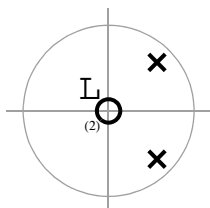
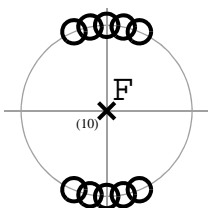
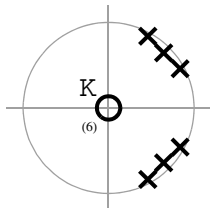
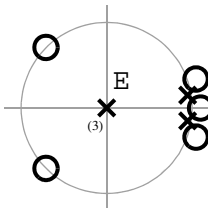
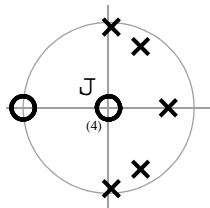
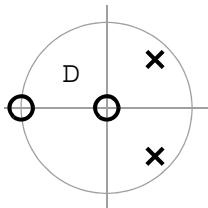
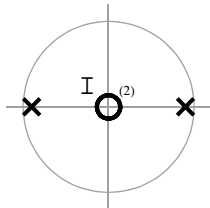
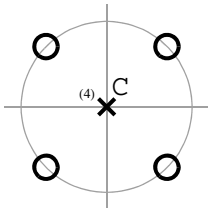
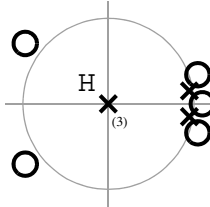
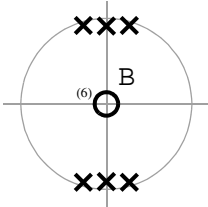
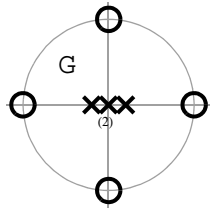
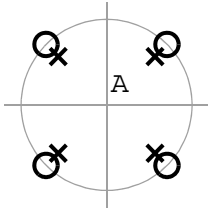
Shown below are plots of $|\sum_{n=0}^{L-1} e^{-jk2\pi n/N}|$ versus $k \in \{0, \dots, N-1\}$, labeled A through L. (The axes are not labeled, only the shapes matter.) Match each plot to the corresponding values of the parameters L and N . Indicate your answer by writing a letter (from A through L) in each answer box.

- (a) $L=1, N=8.$
- (b) $L=2, N=8.$
- (c) $L=4, N=8.$
- (d) $L=8, N=8.$
- (e) $L=1, N=16.$
- (f) $L=2, N=16.$
- (g) $L=4, N=16.$
- (h) $L=8, N=16.$
- (i) $L=1, N=20.$
- (j) $L=2, N=20.$
- (k) $L=4, N=20.$
- (l) $L=8, N=20.$



PROB. Sp22F.8.

Shown below on the left are twelve pole-zero plots for $H(z)$, labeled A through L. Shown below on the right are the corresponding magnitude responses, $|H(e^{j\hat{\omega}})|$ plotted vs $\hat{\omega}$, but in a scrambled order. Match each magnitude response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through L) into each answer box.



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NAME: _____ ANSWER KEY

(FIRST) (LAST)

GT username: _____ A

(e.g., gtxyz123)

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L01 (Tai)	L07 (Tai)	L09 (Hessler)	L11 (Hessler)
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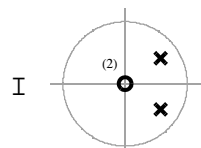
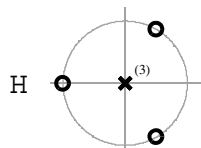
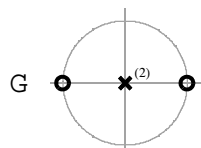
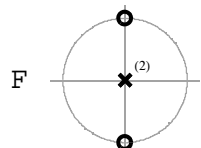
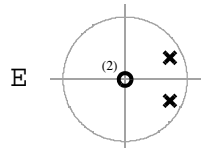
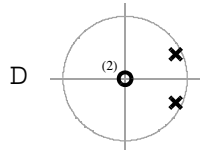
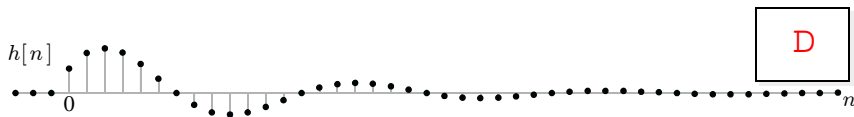
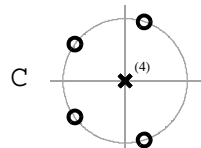
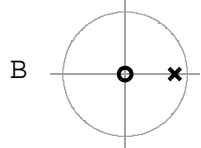
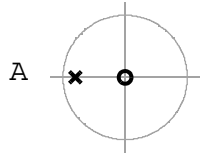
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- Problems 4 and 5: Each answer worth 4 points; Problem 6: Each answer worth 3 points. For all other problems (1, 2, 3, 7, 8): each answer is worth a single point.

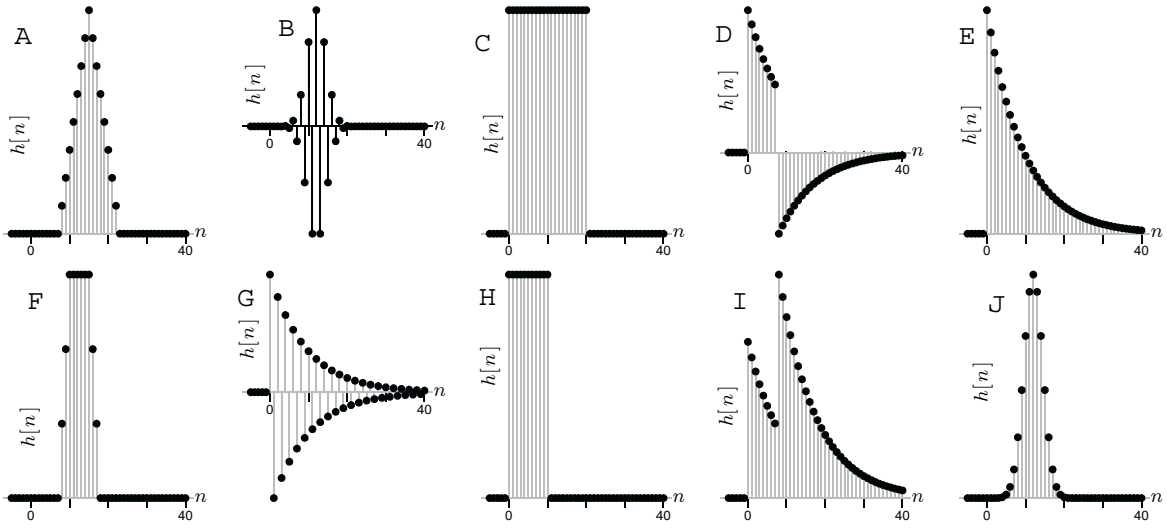
Problem	Value	Score
1	9	
2	10	
3	12	
4	12	
5	16	
6	15	
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8	12	
RECITATION	2	
Total		

PROB. Sp22F.1.

Shown below on the left are nine pole-zero plots for $H(z)$, labeled A through I. Shown on the right are stem plots of the corresponding time-domain signal $h[n]$ for $n \in \{-3, -2, \dots, 42, 43\}$, but in a scrambled order. Match each impulse response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through I) into each answer box.



PROB. Sp22F.2. Shown below are ten different impulse responses $h[n]$, labeled A through J, where $h[n]$ is plotted for times $n \in \{-5, \dots, 40\}$ only:

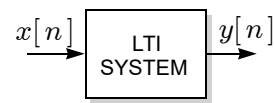


(The y-axis scales are not specified, they are not needed to solve the problem.)

Each $h[n]$ shown above is the impulse response of a filter whose frequency response $H(e^{j\hat{\omega}})$ is listed below, but the order is scrambled. Match each $H(e^{j\hat{\omega}})$ below to its corresponding stem plot for $h[n]$ above. Indicate your answers by writing a letter (from A through J) into each of the ten answer boxes.

- | | | |
|--------|---|---|
| (i) | E | $H_1(e^{j\hat{\omega}}) = \frac{1}{1 - 0.9e^{-j\hat{\omega}}}$ |
| (ii) | G | $H_2(e^{j\hat{\omega}}) = \frac{1}{1 + 0.9e^{-j\hat{\omega}}}$ |
| (iii) | I | $H_3(e^{j\hat{\omega}}) = \frac{1 + e^{-j8\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$ |
| (iv) | D | $H_4(e^{j\hat{\omega}}) = \frac{1 - e^{-j8\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$ |
| (v) | A | $H_5(e^{j\hat{\omega}}) = \left(e^{-j7.5\hat{\omega}} \frac{\sin(4\hat{\omega})}{\sin(0.5\hat{\omega})} \right)^2$ |
| (vi) | F | $H_6(e^{j\hat{\omega}}) = e^{-j12.5\hat{\omega}} \left(\frac{\sin(1.5\hat{\omega})}{\sin(0.5\hat{\omega})} \right) \left(\frac{\sin(4\hat{\omega})}{\sin(0.5\hat{\omega})} \right)$ |
| (vii) | J | $H_7(e^{j\hat{\omega}}) = (1 + e^{-j\hat{\omega}})^{24}$ |
| (viii) | B | $H_8(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})^{24}$ |
| (ix) | H | $H_9(e^{j\hat{\omega}}) = e^{-j5\hat{\omega}} \frac{\sin(5.5\hat{\omega})}{\sin(0.5\hat{\omega})}$ |
| (x) | C | $H_{10}(e^{j\hat{\omega}}) = e^{-j10\hat{\omega}} \frac{\sin(10.5\hat{\omega})}{\sin(0.5\hat{\omega})}$ |

PROB. Sp22F.3. Let $x[n] = 3\delta[n] - \delta[n - 1] + \delta[n - 4]$ be the input to an LTI filter whose frequency response is $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega}))$, resulting in an output $y[n]$.



- (a) ^T ^F (True or False): The system is FIR.
- (b) The impulse response $h[n]$ satisfies:

$$h[0] = \boxed{1},$$

$$h[1] = \boxed{2},$$

$$h[2] = \boxed{1}.$$

- (c) The filter output $y[n]$ satisfies:

$$y[0] = \boxed{3},$$

$$y[1] = \boxed{5},$$

$$y[2] = \boxed{1},$$

$$y[3] = \boxed{-1},$$

$$y[4] = \boxed{1},$$

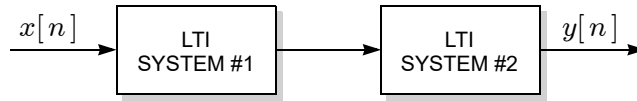
$$y[5] = \boxed{2},$$

$$y[6] = \boxed{1},$$

$$y[7] = \boxed{0}.$$

PROB. Sp22F.4.

Consider the following serial cascade of a pair of LTI systems:



As shown in the figure, an input sequence $x[n]$ is fed to a first LTI system, whose output is fed as an input to a second LTI system, producing an overall output sequence $y[n]$.

The first system has frequency response $H_1(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} + b_2 e^{-2j\hat{\omega}}$, where the parameters $\{b_0, b_1, b_2\}$ are real but otherwise unspecified. The second system has impulse response:

$$h_2[n] = \left(\frac{-1}{2}\right)^n u[n].$$

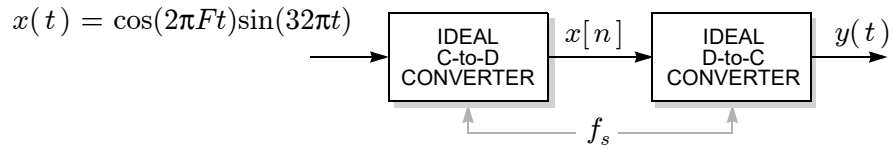
If the difference equation relating the overall output to the overall input is:

$$y[n] = 16x[n] + 8x[n - 1],$$

then it must be that:

$$b_0 = \boxed{16}, \quad b_1 = \boxed{16}, \quad b_2 = \boxed{4}.$$

PROB. Sp22F.5. Suppose the input to an ideal sampling/reconstruction system with sampling rate f_s is the product of two sinusoids, as shown below, producing an output $y(t)$:



The parameters F and f_s are positive and unspecified, they may be different in each part below.

(a) If $F = 8$ Hz, how big must f_s be to ensure that $y(t) = x(t)$? $f_s > \boxed{48}$ Hz.

(b) Find a value for $F > 0$ so that the input $x(t)$ is periodic with fundamental frequency 4 Hz:
(Not unique, there are multiple options, specify only one.)

$F = 4 + 8\ell \in \{4, 12, 20, 28, 36, \dots\}$, e.g. $F = \boxed{4}$ Hz.

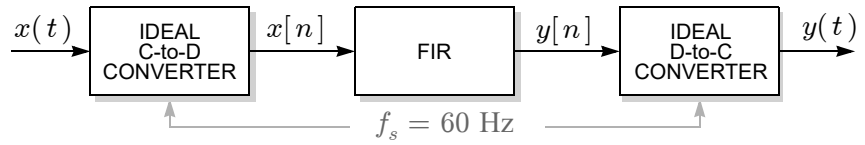
(c) If $F = 7$ Hz and $f_s = 32$ Hz, then the output is $y(t) = \boxed{0}$.
(simplify as much as possible)

(d) If $F = 11$ Hz, find the *largest* sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency 5 Hz:

$f_s = \boxed{52}$ Hz.

PROB. Sp22F.6.

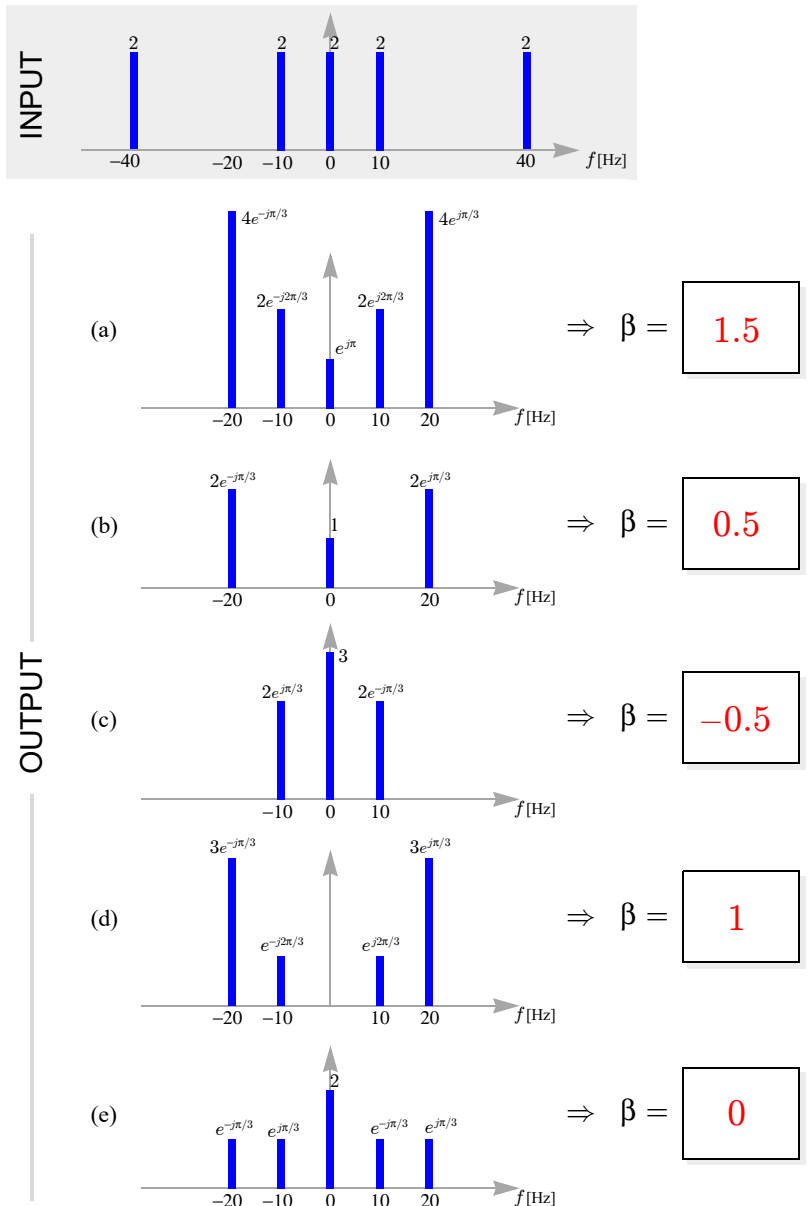
Suppose that a continuous-time signal $x(t)$ is fed to an ideal sampling/filtering/reconstruction system with sampling rate is $f_s = 60$ Hz, producing an output $y(t)$, as shown below:



The difference equation for the FIR filter is $y[n] = 0.5x[n] - \beta x[n - 1] + 0.5x[n - 2]$, where the real parameter β is unspecified.

The first spectrum (shaded) shown below is for the **input** signal $x(t)$.

The remaining five spectra (labeled (a) through (e)) are for the **output** signal $y(t)$ for five different values of the filter parameter β . Find the numerical value of β that results in each output spectrum.



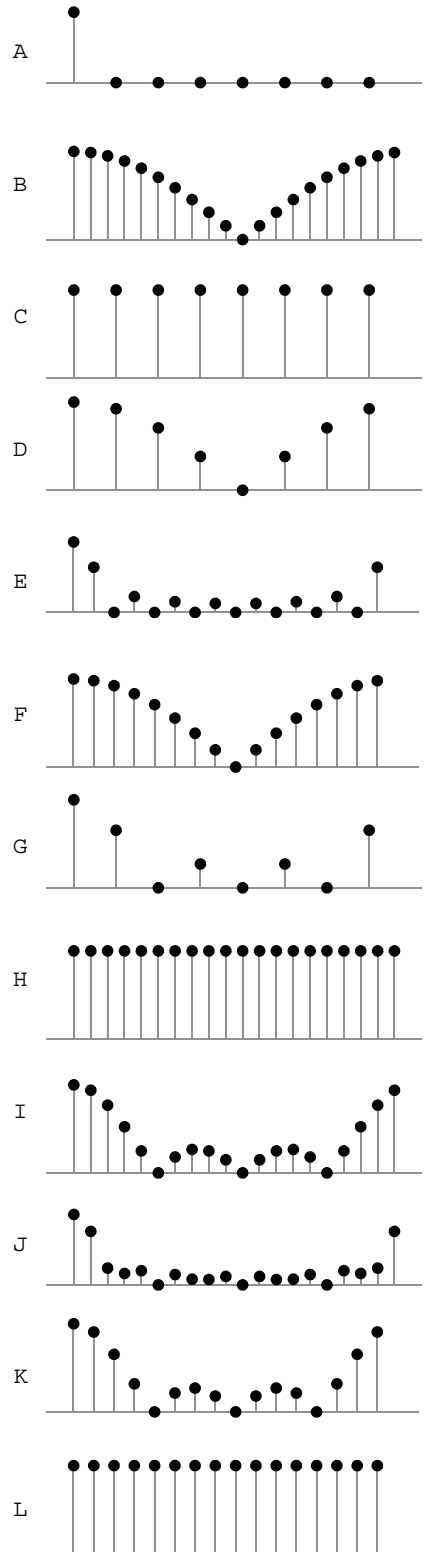
PROB. Sp22F.7.

Shown below are plots of $|\sum_{n=0}^{L-1} e^{-jk2\pi n/N}|$ versus $k \in \{0, \dots, N-1\}$, labeled A through L. (The axes are not labeled, only the shapes matter.)

Match each plot to the corresponding values of the parameters L and N .

Indicate your answer by writing a letter (from A through L) in each answer box.

- (a) C $L=1, N=8.$
- (b) D $L=2, N=8.$
- (c) G $L=4, N=8.$
- (d) A $L=8, N=8.$
- (e) L $L=1, N=16.$
- (f) F $L=2, N=16.$
- (g) K $L=4, N=16.$
- (h) E $L=8, N=16.$
- (i) H $L=1, N=20.$
- (j) B $L=2, N=20.$
- (k) I $L=4, N=20.$
- (l) J $L=8, N=20.$



PROB. Sp22F.8.

Shown below on the left are twelve pole-zero plots for $H(z)$, labeled A through L. Shown below on the right are the corresponding magnitude responses, $|H(e^{j\hat{\omega}})|$ plotted vs $\hat{\omega}$, but in a scrambled order. Match each magnitude response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through L) into each answer box.

