GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 7-May-10 COURSE: ECE-2025

NAME: ______________________________ GT username: ____________________

LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Michaels)	L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Michaels)	L08:Thur-1:30pm (Bhatti)
L01:M-3pm (Lee)	L09:Tues-3pm (Fekri)	
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	30	
4	30	
5	30	
6	30	
7	30	

PROBLEM sp-10-F.1:

Pick the correct frequency response characteristic and enter the number in the answer box:

Time-Domain Description

(a) $y[n] = \frac{1}{2}y[n-1] + x[n]$

ANS =

(b) $h[n] = (-\frac{1}{2})^n u[n]$

ANS =

(c) `yn = filter([1,1],1,xn)`

ANS =

(d) $y[n] = x[n] + x[n-1] + x[n-2]$

ANS =

(e) $h[n] = \sum_{k=0}^{3} \delta[n-k]$

ANS =

(f) `yn = conv([1,0,2,0,1],xn)`

ANS =

Frequency Response Characteristic

1. $|H(e^{j\hat{\omega}})|^2 = 2 + 2\cos(\hat{\omega})$ *(Mag-squared)*

2. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$

3. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

4. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$

5. $H(e^{j\hat{\omega}}) = 1 - \frac{1}{2}e^{-j\hat{\omega}}$

6. $\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$

7. $H(e^{j\hat{\omega}}) = \dfrac{\sin 2\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})} e^{-j1.5\hat{\omega}}$

8. $H(e^{j\hat{\omega}}) = \dfrac{\sin \hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}$

PROBLEM sp-10-F.2:

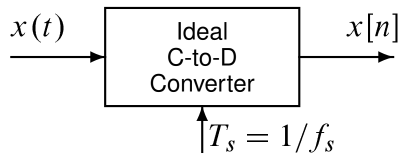
For each part, pick a correct frequency[1] ***from the list*** and enter its value in the answer box.[2]
Write a brief explanation of your answers to receive any credit.

Frequency

- 5000 Hz
- 4000 Hz
- 3000 Hz
- 2700 Hz
- 2400 Hz
- 2100 Hz
- 1800 Hz
- 1500 Hz
- 1200 Hz

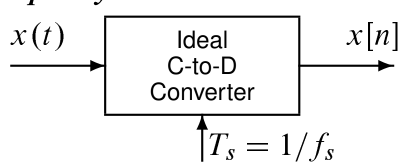
(a) If the output from an ideal C/D converter is $x[n] = A\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value of the ***input frequency*** of $x(t)$:

ANS =



(b) If the output from an ideal C/D converter is $x[n] = A\cos(0.5\pi n)$, and the input signal $x(t)$ defined by: $x(t) = A\cos(3000\pi t)$ then determine one possible value of the ***sampling frequency*** of the C-to-D converter:

ANS =

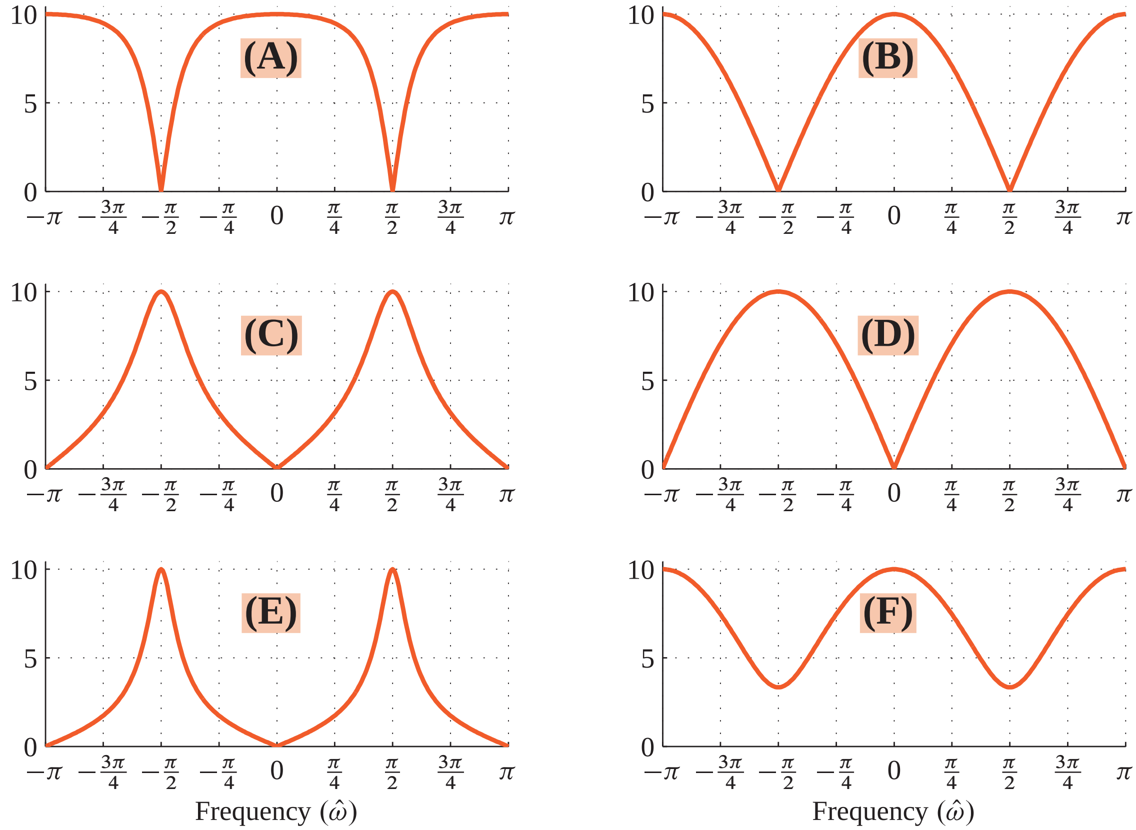


(c) Determine the ***Nyquist rate*** for sampling the signal $x(t)$ defined by: $x(t) = (A - B\cos(800\pi t))\sin(2200\pi t)$.

ANS =

[1] Some questions have more than one answer, but you ***must pick your correct answer from the specified list.***
[2] It is possible to use an answer more than once.

PROBLEM sp-10-F.3:



Each of the discrete-time systems below is described by its poles and zeros. Determine the matching frequency response (magnitude) plot for each one. Use the ***letter*** on the plot for your answer, or ***None***.
Note: The plots on the left are all IIR filters; the ones on the right are FIR.

ANS =	Poles at: $0.707e^{\pm j\pi/3}$	Zeros at: $e^{\pm j\pi/3}$
ANS =	Poles at the origin	Zeros at: $e^{\pm j\pi/2}$
ANS =	Poles at the origin	Zeros at: ± 1
ANS =	Poles at: $0.707e^{\pm j\pi/2}$	Zeros at: ± 1
ANS =	Poles at: $0.837e^{\pm j\pi/2}$	Zeros at: ± 1
ANS =	Poles at the origin	Zeros at: $0.707e^{\pm j\pi/2}$
ANS =	Poles at: $0.707e^{\pm j\pi/2}$	Zeros at: $e^{\pm j\pi/2}$

PROBLEM sp-10-F.4:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes next to the question.*** (The operator $*$ denotes convolution.)

(a) $\square$ $x(t) = e^{-(t-4)}\delta(t-4)$

(b) $\square$ $x(t) = u(t-3) - u(t-5)$

(c) $\square$ $x(t) = u(t+4) - u(t-4)$

(d) $\square$ $x(t) = \delta(t-2) * e^{-t+1}u(t-1) * \delta(t-1)$

(e) $\square$ $x(t) = \delta(t) - \delta(t-8)$

(f) $\square$ $x(t) = \cos(\pi t) * \delta(t-4)$

(g) $\square$ $x(t) = \displaystyle\int_{-\infty}^{t} e^{-t+\tau}\delta(\tau)d\tau$

(h) $\square$ $x(t) = -e^{-t}u(t) + \delta(t)$

Each of the time signals above has a Fourier transform that should be in the list below.

[0] $X(j\omega)$ not in the list below.

[1] $X(j\omega) = \dfrac{j\omega}{1+j\omega}$

[2] $X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

[3] $X(j\omega) = \dfrac{1}{1+j\omega}$

[4] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[5] $X(j\omega) = e^{-j4\omega}$

[6] $X(j\omega) = e^{-j4\omega}[\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)]$

[7] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(\omega)}{\omega}$

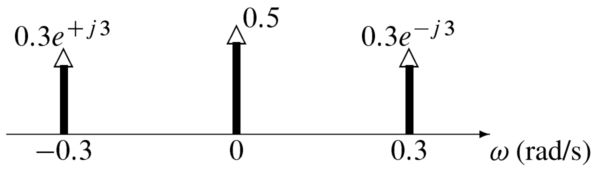
[8] $X(j\omega) = 0$

[9] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(4\omega)}{\omega}$

[10] $X(j\omega) = e^{-j4\omega}[u(\omega) - u(\omega-8)]$

PROBLEM sp-10-F.5:

(a) A real signal $x(t)$ has a Fourier Transfrom that consists of the impulses shown below.
The frequency axis has units of rad/s.
Define a new signal by squaring $x(t)$, i.e., $y(t) = x^2(t)$. The Fourier transform of $y(t)$ consists of impulses; determine the locations of all the impulses in $Y(j\omega)$. *Only the locations in* ω.



Deltas located at $\omega =$

(b) Define a continuous-time rectangular pulse via $p(t) = 2[u(t-2) - u(t-5)]$. Then convolve the pulse with itself to get $q(t) = p(t) * p(t)$ which is a triangle. Determine the location of the maximum value of $q(t)$, t_{qmax}, and also the maximum value q_{max}.

$q_{\text{max}} =$ | $t_{\text{qmax}} =$

(c) Suppose that the Fourier series coefficients of a periodic signal $s(t)$ are defined via the integral:

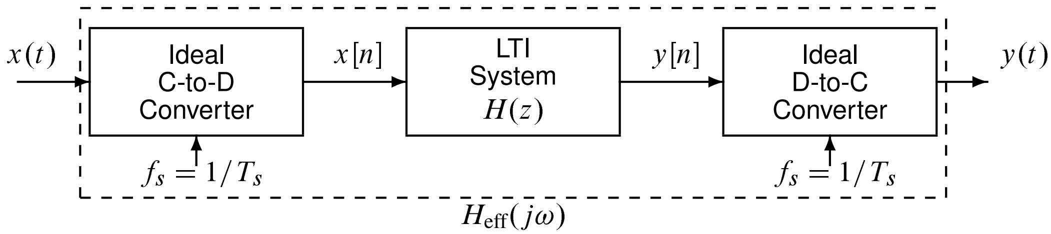
$$a_k = \frac{1}{9}\int_0^9 s(t)\, e^{-j(2\pi/9)kt}\, dt \qquad \text{where} \quad s(t) = \begin{cases} 0 & 0 \le t < 6 \\ 10 & 6 \le t < 9 \end{cases}$$

Evaluate a_k for $k = 5$; *express your answer as a complex number in polar form,* i.e., $r_k e^{j\theta_k}$.

$r_k =$ | $\theta_k =$

PROBLEM sp-10-F.6:

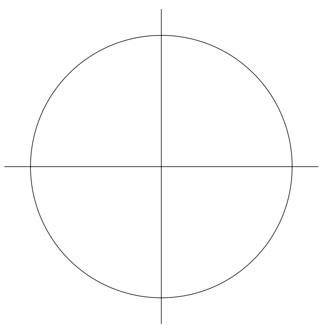
Consider the following system for discrete-time filtering of a continuous-time signal:



The system function $H(z)$ of the discrete-time IIR system can be derived from the following MATLAB code:

```
yn = filter([1, alf, 1], [1, 0.9*alf, 0.81], xn);
```

(a) For the case where **`alf = 0.7`**, determine the poles and zeros of the system, and then give your answer as a pole-zero plot. Account for ***all*** poles and zeros.



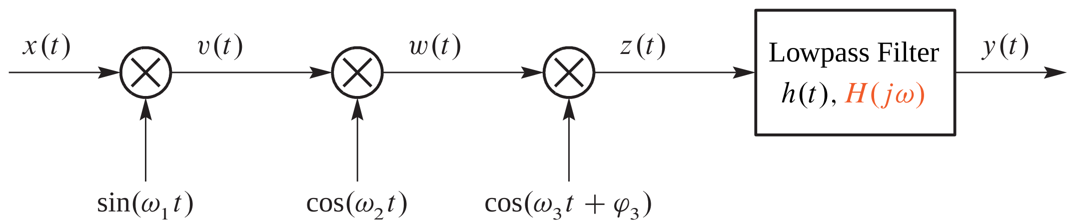
(b) This filter is a notch filter, so its frequency response has a null and passband regions that are nearly flat. For the case where **`alf = 0.7`**, determine the maximum value of the frequency response magnitude in the passbands.

(c) *Note: In this part the value of* **`alf`** *is no longer* **`0.7`**.
The effective frequency response of this system (using the $H(z)$ above) is able to null out one sinusoid. It is similar to the system used in the lab to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient parameter **`alf`** controls the (frequency) location of the null. If the sampling rate is $f_s = 10000$ Hz, determine the value of **`alf`** so that the overall effective frequency response has a null at 2000 Hz.

alf =

PROBLEM sp-10-F.7:

The system below involves the cascade of several modulators followed by a filter:



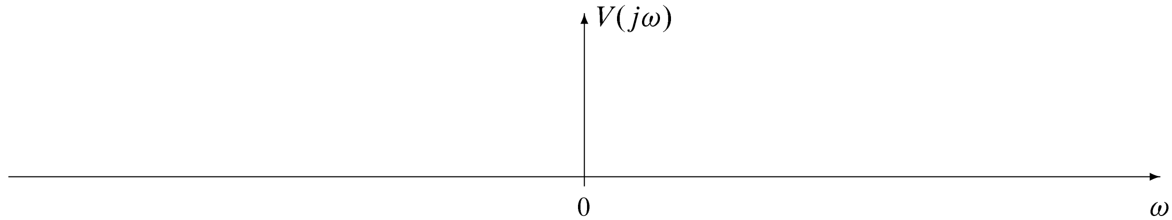
The signals are defined by

$$v(t) = x(t)\sin(55t) \qquad w(t) = v(t)\cos(15t) \qquad z(t) = w(t)\cos(\omega_3 t + \varphi_3)$$

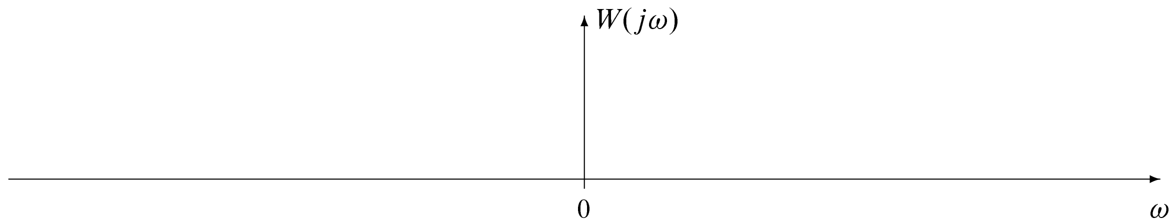
Suppose that the Fourier transform of $x(t)$ is

$$X(j\omega) = |\omega|\{u(\omega+5) - u(\omega-5)\}$$

(a) Determine the Fourier transform, $V(j\omega)$, giving your answer as a plot. Sketch the magnitude of $V(j\omega)$, but label the sketch with the complex amplitude, i.e., keep track of the magnitude and phase.



(b) Determine the Fourier transform, $W(j\omega)$, giving your answer as a plot. Sketch the magnitude of $W(j\omega)$, but label the sketch with the complex amplitude, i.e., keep track of the magnitude and phase.



(c) Determine the phase φ_3 and the frequency ω_3, so that the input signal can be recovered with an ideal LPF whose cutoff frequency is $\omega = 5$ rad/s and whose passband gain is 4. Recovery means that the output signal $y(t)$ would be equal to the input $x(t)$.
Note: There are two possible answer sets, but you only have to give one.

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3	30	
4	30	
5	30	
6	30	
7	30	

PROBLEM sp-10-F.1:

Pick the correct frequency response characteristic and enter the number in the answer box:

Time-Domain Description

(a) $y[n] = \frac{1}{2}y[n-1] + x[n]$

ANS = 4

(b) $h[n] = (-\frac{1}{2})^n u[n]$

ANS = 2

(c) `yn = filter([1,1],1,xn)`

ANS = 1

(d) $y[n] = x[n] + x[n-1] + x[n-2]$

ANS = 3

(e) $h[n] = \sum_{k=0}^{3} \delta[n-k]$

ANS = 7

(f) `yn = conv([1,0,2,0,1],xn)`

ANS = 6

Frequency Response Characteristic

1. $|H(e^{j\hat{\omega}})|^2 = 2 + 2\cos(\hat{\omega})$ *(Mag-squared)*
2. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$
3. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$
4. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$
5. $H(e^{j\hat{\omega}}) = 1 - \frac{1}{2}e^{-j\hat{\omega}}$
6. $\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$
7. $H(e^{j\hat{\omega}}) = \dfrac{\sin 2\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})} e^{-j1.5\hat{\omega}}$
8. $H(e^{j\hat{\omega}}) = \dfrac{\sin \hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}$

- The difference equation $y[n] = \frac{1}{2}y[n-1] + x[n]$ defines a filter whose system function is $H(z) = \dfrac{1}{1-0.5z^{-1}}$ which then gives the frequency response $H(e^{j\omega}) = \dfrac{1}{1-0.5e^{-j\hat{\omega}}}$.
- $h[n] = (-\frac{1}{2})^n u[n]$ has a z-transform equal to $H(z) = \dfrac{1}{1+0.5z^{-1}}$, so $H(e^{j\omega}) = \dfrac{1}{1+0.5e^{-j\hat{\omega}}}$.
- `yn = filter([1,1],1,xn)` has a frequency response $H(e^{j\omega}) = 1 + e^{-j\hat{\omega}}$. The magnitude squared is $H(e^{j\omega})H^*(e^{j\omega}) = \left(1 + e^{-j\hat{\omega}}\right)\left(1 + e^{j\hat{\omega}}\right) = e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}} = 2 + 2\cos(\hat{\omega})$.
- $y[n] = x[n] + x[n-1] + x[n-2]$ defines a 3-point summing filter so its frequency response can be written as a Dirichlet form, $\dfrac{\sin(L/\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$, times an $e^{j\,\text{phase}}$ term: $H(e^{j\omega}) = \dfrac{\sin(3\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega}}$.
- $h[n] = \sum_{k=0}^{3} \delta[n-k]$ defines a 4-point summing filter, so $H(e^{j\omega}) = \dfrac{\sin(4\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j1.5\hat{\omega}}$.
- The MATLAB code `yn = conv([1,0,2,0,1],xn)` defines the difference equation $y[n] = x[n] + 2x[n-2] + x[n-4]$ which has a frequency response $H(e^{j\omega}) = 1 + 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$ which can be rewritten as $e^{-j2\hat{\omega}}(2 + 2\cos(2\hat{\omega}))$; thus, the phase is $\angle H(e^{j\omega}) = -2\hat{\omega}$.

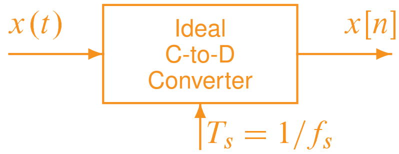
PROBLEM sp-10-F.2:

For each part, pick a correct frequency[1] ***from the list*** and enter its value in the answer box.[2]
Write a brief explanation of your answers to receive any credit.

Frequency

- 5000 Hz
- 4000 Hz
- 3000 Hz
- 2700 Hz
- 2400 Hz
- 2100 Hz
- 1800 Hz
- 1500 Hz
- 1200 Hz

(a) If the output from an ideal C/D converter is $x[n] = A\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value of the ***input frequency*** of $x(t)$:

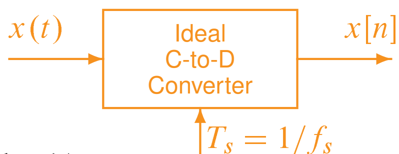


$\hat{\omega} = 0.5\pi$ rads, $f_s = 2000$ Hz

$$\hat{\omega} = \pm\frac{2\pi f}{f_s} + 2\pi\ell \quad \Rightarrow f = \pm\frac{\hat{\omega} - 2\pi\ell}{2\pi} f_s$$

For $\ell = 1$, $f = \pm\left(\dfrac{0.5\pi - 2\pi}{2\pi}\right)2000 = \pm1500$ Hz $\boxed{\text{ANS} = \mathbf{1500\ Hz}}$

(b) If the output from an ideal C/D converter is $x[n] = A\cos(0.5\pi n)$, and the input signal $x(t)$ defined by: $x(t) = A\cos(3000\pi t)$ then determine one possible value of the ***sampling frequency*** of the C-to-D converter:



$$\hat{\omega} = \frac{2\pi f}{f_s} + 2\pi\ell \qquad (f \text{ can be } \pm)$$

$$\Rightarrow f_s = \frac{\pm 2\pi f}{\hat{\omega} - 2\pi\ell} = \frac{\pm 3000\pi}{0.5\pi - 2\pi\ell} = \frac{\pm 6000}{1 - 4\ell}\text{ Hz}$$

For $\ell = 0$, $f_s = 6000$ Hz

For $\ell = -1$, numerator $= 6000$, $f_s = 1200$ Hz $\boxed{\text{ANS} = \mathbf{1200\ Hz}}$

(c) Determine the ***Nyquist rate*** for sampling the signal $x(t)$ defined by: $x(t) = (A - B\cos(800\pi t))\sin(2200\pi t)$.

$$y(t) = \left(A - \frac{B}{2}e^{j800\pi t} - \frac{B}{2}e^{-j800\pi t}\right)\left(\frac{1}{2j}e^{j2200\pi t} - \frac{1}{2j}e^{-j2200\pi t}\right)$$

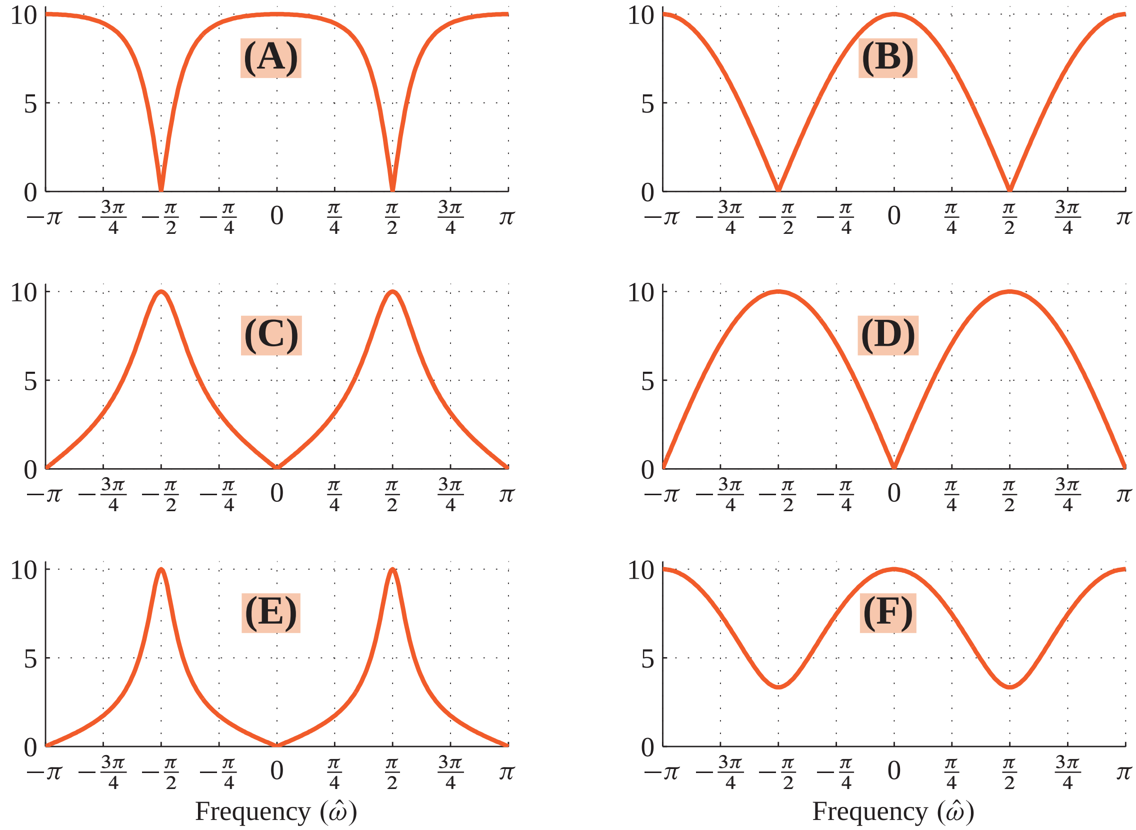
$\Rightarrow$ highest frequency in $x(t)$ is $\omega_{\max} = 800\pi + 2200\pi = 3000\pi$ rad/s.

$$f_{\max} = 1500\text{ Hz} \quad \Longrightarrow \quad f_s > 2f_{\max} = f_{\text{Nyquist}} = 3000\text{ Hz}$$

$\boxed{\text{ANS} = \mathbf{3000\ Hz}}$

[1] Some questions have more than one answer, but you ***must pick your correct answer from the specified list.***
[2] It is possible to use an answer more than once.

PROBLEM sp-10-F.3:



Each of the discrete-time systems below is described by its poles and zeros. Determine the matching frequency response (magnitude) plot for each one. Use the ***letter*** on the plot for your answer, or ***None***.
Note: The plots on the left are all IIR filters; the ones on the right are FIR.

ANS = NONE	Poles at: $0.707e^{\pm j\pi/3}$	Zeros at: $e^{\pm j\pi/3}$
ANS = B	Poles at the origin	Zeros at: $e^{\pm j\pi/2}$
ANS = D	Poles at the origin	Zeros at: ± 1
ANS = C	Poles at: $0.707e^{\pm j\pi/2}$	Zeros at: ± 1
ANS = E	Poles at: $0.837e^{\pm j\pi/2}$	Zeros at: ± 1
ANS = F	Poles at the origin	Zeros at: $0.707e^{\pm j\pi/2}$
ANS = A	Poles at: $0.707e^{\pm j\pi/2}$	Zeros at: $e^{\pm j\pi/2}$

PROBLEM sp-10-F.4:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes next to the question.*** (The operator $*$ denotes convolution.)

(a) $\boxed{\mathbf{5}}$ $x(t) = e^{-(t-4)}\delta(t-4) \quad \Rightarrow \quad X(j\omega) = e^{-j4\omega}$

(b) $\boxed{\mathbf{7}}$ $x(t) = u(t-3) - u(t-5) \quad \Rightarrow \quad X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(\omega)}{\omega}$

(c) $\boxed{\mathbf{0}}$ $x(t) = u(t+4) - u(t-4) \quad \Rightarrow \quad X(j\omega) = \dfrac{\sin(4\omega)}{\omega/2}$

(d) $\boxed{\mathbf{2}}$ $x(t) = \delta(t-2) * e^{-t+1}u(t-1) * \delta(t-1) \quad \Rightarrow \quad X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

(e) $\boxed{\mathbf{4}}$ $x(t) = \delta(t) - \delta(t-8) \quad \Rightarrow \quad X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

(f) $\boxed{\mathbf{6}}$ $x(t) = \cos(\pi t) * \delta(t-4) \quad \Rightarrow \quad X(j\omega) = e^{-j4\omega}[\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)]$

(g) $\boxed{\mathbf{3}}$ $x(t) = \displaystyle\int_{-\infty}^{t} e^{-t+\tau}\delta(\tau)\,d\tau \quad \Rightarrow \quad X(j\omega) = \dfrac{1}{1+j\omega}$

(h) $\boxed{\mathbf{1}}$ $x(t) = -e^{-t}u(t) + \delta(t) \quad \Rightarrow \quad X(j\omega) = \dfrac{j\omega}{1+j\omega}$

Each of the time signals above has a Fourier transform that should be in the list below.

[0] $X(j\omega)$ not in the list below.

[1] $X(j\omega) = \dfrac{j\omega}{1+j\omega}$

[2] $X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

[3] $X(j\omega) = \dfrac{1}{1+j\omega}$

[4] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[5] $X(j\omega) = e^{-j4\omega}$

[6] $X(j\omega) = e^{-j4\omega}[\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)]$

[7] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(\omega)}{\omega}$

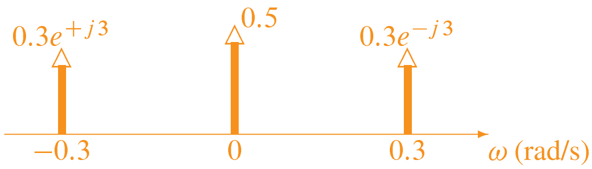
[8] $X(j\omega) = 0$

[9] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(4\omega)}{\omega}$

[10] $X(j\omega) = e^{-j4\omega}[u(\omega) - u(\omega-8)]$

PROBLEM sp-10-F.5:

(a) A real signal $x(t)$ has a Fourier Transfrom that consists of the impulses shown below.
The frequency axis has units of rad/s.
Define a new signal by squaring $x(t)$, i.e., $y(t) = x^2(t)$. The Fourier transform of $y(t)$ consists of impulses; determine the locations of all the impulses in $Y(j\omega)$. *Only the locations in ω.*



$\boxed{\text{Deltas located at } \omega = \{0, \pm 0.3, \pm 0.6\} \text{ rad/s}}$

(b) Define a continuous-time rectangular pulse via $p(t) = 2[u(t-2) - u(t-5)]$. Then convolve the pulse with itself to get $q(t) = p(t) * p(t)$ which is a triangle. Determine the location of the maximum value of $q(t)$, t_{qmax}, and also the maximum value q_{max}.

$\boxed{q_{\max} = 12}$ $\boxed{t_{\text{qmax}} = 7 \text{ secs}}$

(c) Suppose that the Fourier series coefficients of a periodic signal $s(t)$ are defined via the integral:

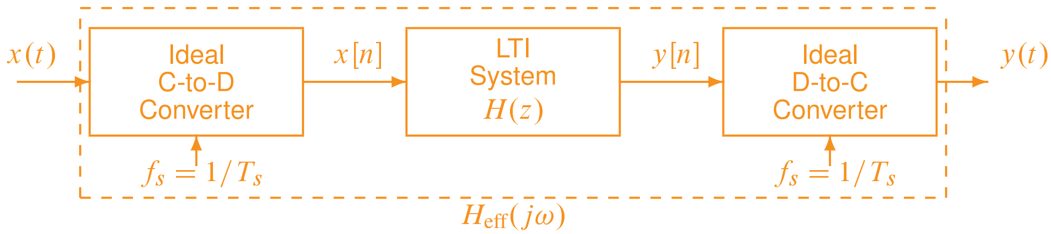
$$a_k = \frac{1}{9}\int_0^9 s(t)\, e^{-j(2\pi/9)kt}\, dt \qquad \text{where} \quad s(t) = \begin{cases} 0 & 0 \le t < 6 \\ 10 & 6 \le t < 9 \end{cases}$$

Evaluate a_k for $k = 5$; *express your answer as a complex number in polar form,* i.e., $r_k e^{j\theta_k}$.

$\boxed{r_k = 0.5513}$ $\boxed{\theta_k = 2.094 = 2\pi/3 \text{ rads}}$

PROBLEM sp-10-F.6:

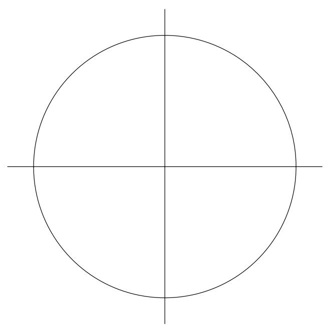
Consider the following system for discrete-time filtering of a continuous-time signal:



The system function $H(z)$ of the discrete-time IIR system can be derived from the following MATLAB code:

```
yn = filter([1, alf, 1], [1, 0.9*alf, 0.81], xn);
```

(a) For the case where **`alf = 0.7`**, determine the poles and zeros of the system, and then give your answer as a pole-zero plot. Account for ***all*** poles and zeros.



Zeros = $1\,e^{\pm j1.926} = e^{\pm j0.614\pi}$

Poles = $0.9e^{\pm j1.926} = 0.9e^{\pm j0.614\pi}$

(b) This filter is a notch filter, so its frequency response has a null and passband regions that are nearly flat. For the case where **`alf = 0.7`**, determine the maximum value of the frequency response magnitude in the passbands.

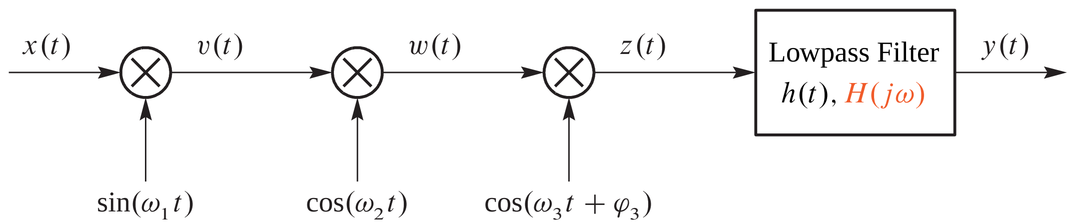
The maximum occurs at $\omega = 0$ or at $\omega = \pi$. $H(j0) = 1.107$ and $H(j\pi) = 1.102$, so the maximum is $H_{\text{max}} = 1.107$.

(c) *Note: In this part the value of* **`alf`** *is no longer* **`0.7`**.
The effective frequency response of this system (using the $H(z)$ above) is able to null out one sinusoid. It is similar to the system used in the lab to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient parameter **`alf`** controls the (frequency) location of the null. If the sampling rate is $f_s = 10000$ Hz, determine the value of **`alf`** so that the overall effective frequency response has a null at 2000 Hz.

The angle of the poles and zeros has to be $\theta = 2\pi(2000/10000) = 0.4\pi$.
The value of the **`alf`** coefficient is $-2\cos(\theta) = -2\cos(0.4\pi) = -0.618$.

PROBLEM sp-10-F.7:

The system below involves the cascade of several modulators followed by a filter:



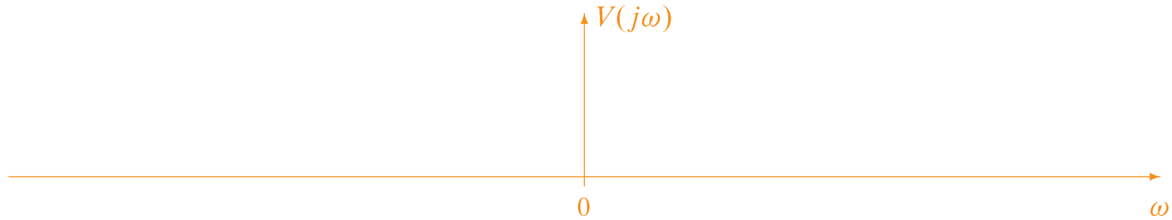
The signals are defined by

$$v(t) = x(t)\sin(55t) \qquad w(t) = v(t)\cos(15t) \qquad z(t) = w(t)\cos(\omega_3 t + \varphi_3)$$

Suppose that the Fourier transform of $x(t)$ is

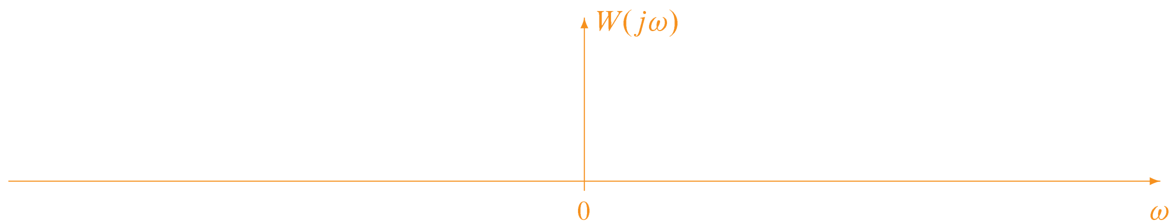
$$X(j\omega) = |\omega|\{u(\omega+5) - u(\omega-5)\}$$

(a) Determine the Fourier transform, $V(j\omega)$, giving your answer as a plot. Sketch the magnitude of $V(j\omega)$, but label the sketch with the complex amplitude, i.e., keep track of the magnitude and phase.



Draw two copies of the spectrum: one centered at $\omega = 55$ rad/s extending from $\omega = 50$ to $\omega = 60$ with its maximum complex amplitude of $2.5e^{-j\pi/2}$ at the ends; another *conjugate* copy centered at $\omega = -55$ rad/s with a maximum complex amplitude of $2.5e^{j\pi/2}$.

(b) Determine the Fourier transform, $W(j\omega)$, giving your answer as a plot. Sketch the magnitude of $W(j\omega)$, but label the sketch with the complex amplitude, i.e., keep track of the magnitude and phase.



Draw four copies of the spectrum: one centered at $\omega = 70$ rad/s extending from $\omega = 65$ and $\omega = 75$ with a maximum complex amplitude of $1.25e^{j(-\pi/2)}$ at the ends; another copy centered at $\omega = -40$ rad/s extending from $\omega = 45$ to $\omega = 35$ with a maximum complex amplitude of $1.25e^{j(-\pi/2)}$ at the ends. The other two copies are conjugates located in negative frequency.

(c) Determine the phase φ_3 and the frequency ω_3, so that the input signal can be recovered with an ideal LPF whose cutoff frequency is $\omega = 5$ rad/s and whose passband gain is 4. Recovery means that the output signal $y(t)$ would be equal to the input $x(t)$.
Note: There are two possible answer sets, but you only have to give one.

LPF cutoff = 5, phase = $\pm\pi/2$, Shift by 70, or 40. If you shift by 70, then the phase should be $-\pi/2$. When shifting by 40, use a phase of $-\pi/2$.