GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 27-Apr-09 COURSE: ECE-2025

NAME: ______________________ GT username: ______________
LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

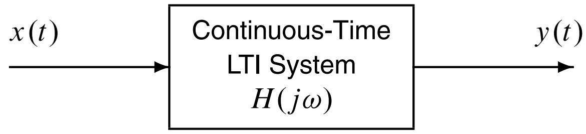
Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Bhatti)		L06:Thur-Noon (Barry)
	L07:Tues-1:30pm (Bhatti)		L08:Thur-1:30pm (Barry)
L01:M-3pm (Chang)	L09:Tues-3pm (Lee)	L02:W-3pm (Fekri)	
	L11:Tues-4:30pm (Lee)	L04:W-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	

PROBLEM sp-09-F.1:



Suppose that the frequency response $H(j\omega)$ could be written in terms of a few parameters, e.g.,

$$H(j\omega) = \frac{j\omega\, d}{a + j\omega}$$

where a and d are *real-valued parameters.*

(a) Determine the impulse response of the system above when $a = 800$ and $d = 2$. Determine the *simplest possible* formula for $h(t)$.

$h(t) =$

(b) Determine the output signal $y(t)$ when the input signal has a Fourier transform given by

$$X(j\omega) = 2\pi\delta(\omega - 800)$$

Once again assume that the parameters of the system are $a = 800$ and $d = 2$. Determine a *simple* formula for the output $y(t)$, which is *complex-valued* in this case.

$y(t) =$

PROBLEM sp-09-F.2:

The two subparts of this problem are completely independent of one another.

(a) Determine the result of the following convolution:

$$y(t) = \left\{100\,e^{-4t}u(t-5)\right\} * u(t-23)$$

$y(t) =$

(b) When two finite-duration signals are convolved, the result is a finite-duration signal. In this part,

$$h(t) = t^2\,[u(t-25) - u(t)] \qquad \text{and} \qquad x(t) = 9\,[u(t-5) - u(t-10)]$$

Determine the starting and ending times of the output signal $y(t) = x(t) * h(t)$, i.e., find T_1 and T_2 so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$. *Hint:* Visualize the flip-and-slide, but don't integrate.

$T_1 =$	sec.	$T_2 =$	sec.

PROBLEM sp-09-F.3:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \delta(t-4)$

(b) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$

(c) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \dfrac{\sin(3\pi t)}{\pi t}$

(d) $x(t) = \left(\dfrac{\sin(\pi(t+1))}{\pi(t+1)}\ \delta(t+1)\right) * \delta(t-5)$

(e) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t}\cos(4\pi t)$

Each of the time signals above has a Fourier transform that should be in the list below.

[0] $X(j\omega) = u(\omega+5\pi) - u(\omega-5\pi)$

[1] $X(j\omega) = e^{-j4\omega}[u(\omega+5\pi) - u(\omega-5\pi)]$

[2] $X(j\omega) = u(\omega+3\pi) - u(\omega-3\pi)$

[3] $X(j\omega) = j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

[4] $X(j\omega) = e^{-j4\omega}$

[5] $X(j\omega) = 0$

[6] $X(j\omega) = e^{-j4\omega}\,[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

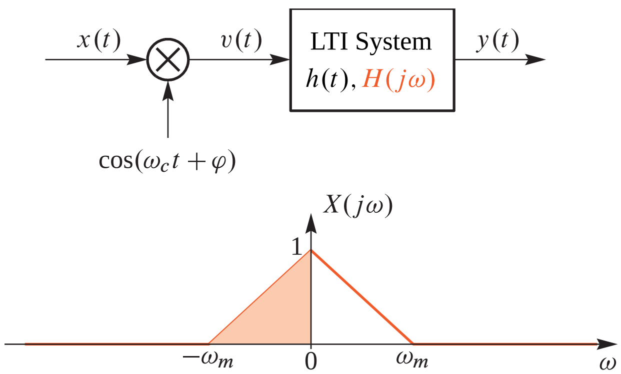
[7] $X(j\omega) = [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)] * \delta(\omega-4\pi)$

[8] $X(j\omega) = 2u(\omega+\pi) - 2u(\omega-\pi) + \pi\delta(\omega+4\pi) + \pi\delta(\omega-4\pi)$

[9] $X(j\omega) = u(\omega+5\pi) - u(\omega+3\pi) + u(\omega-3\pi) - u(\omega-5\pi)$

[None] $X(j\omega)$ not in the list above.

PROBLEM sp-09-F.4:
The system below involves a multiplier followed by a filter:



$\omega_c = 400\,\text{rad/s}, \varphi = 0.2\,\text{rads}$

$\omega_m = 500\,\text{rad/s}$

Suppose the frequency of the cosine in the multiplier above is $\omega_c = 400\,\text{rad/s}$, and its phase is $\varphi = 0.2\,\text{rads}$. The Fourier transform of the input, $X(j\omega)$, is also shown above.

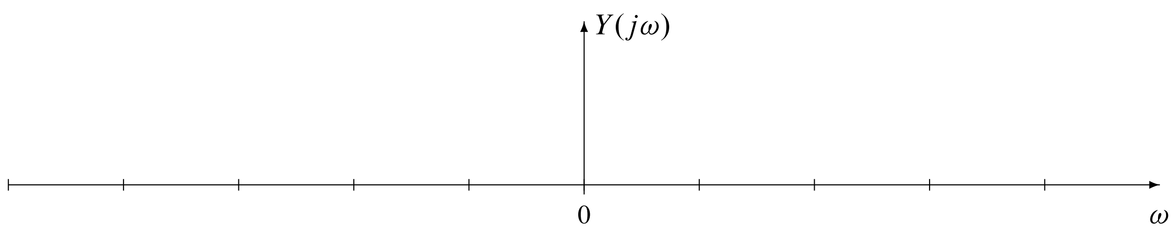
(a) Determine the DC value of $V(j\omega)$, the Fourier transform of the signal $v(t)$ after the multiplier.

$V_{\text{DC}} =$

(b) Suppose the LTI system is an *ideal* HPF defined by

$$H(j\omega) = \begin{cases} 0 & |\omega| < 400\,\text{rad/s} \\ 1 & |\omega| \geq 400\,\text{rad/s} \end{cases}$$

Make a sketch of the Fourier transform of $y(t)$, called $Y(j\omega)$, when the input is $X(j\omega)$ shown above.



PROBLEM sp-09-F.5:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

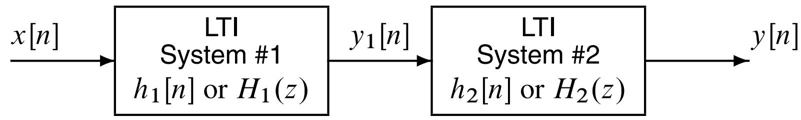


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that $H_1(z) = 3 - 3z^{-5}$ is the system function for System #1, and

System #2 is an FIR filter described by a difference equation: $y[n] = 5y_1[n-1] + 5y_1[n-6]$

(a) Determine the impulse response $h[n]$ of the overall system. ***Give your answer as a stem plot.***

(b) Determine the frequency response of System #2. Express your answer in the following form by finding values for α, β, and μ:

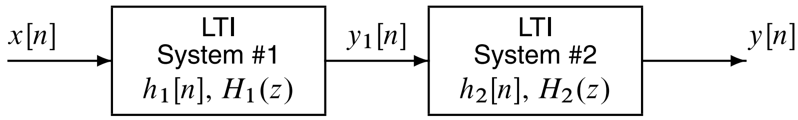
$$H_2(e^{j\hat{\omega}}) = \alpha\, e^{-j\beta\hat{\omega}} \cos(\mu\hat{\omega})$$

$\alpha =$

$\beta =$

$\mu =$

PROBLEM sp-09-F.6:
The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is a filter described by its impulse response: $h_1[n] = \frac{1}{2}\delta[n-1] - \delta[n-4]$

and System #2 is an FIR ***running sum*** filter whose system function is: $H_2(z) = \dfrac{1-z^{-7}}{1-z^{-1}}$

(a) When the input to the ***second*** system is $y_1[n] = 7\cos(2n+5.5)$, for all n, determine the output of the ***second*** system, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

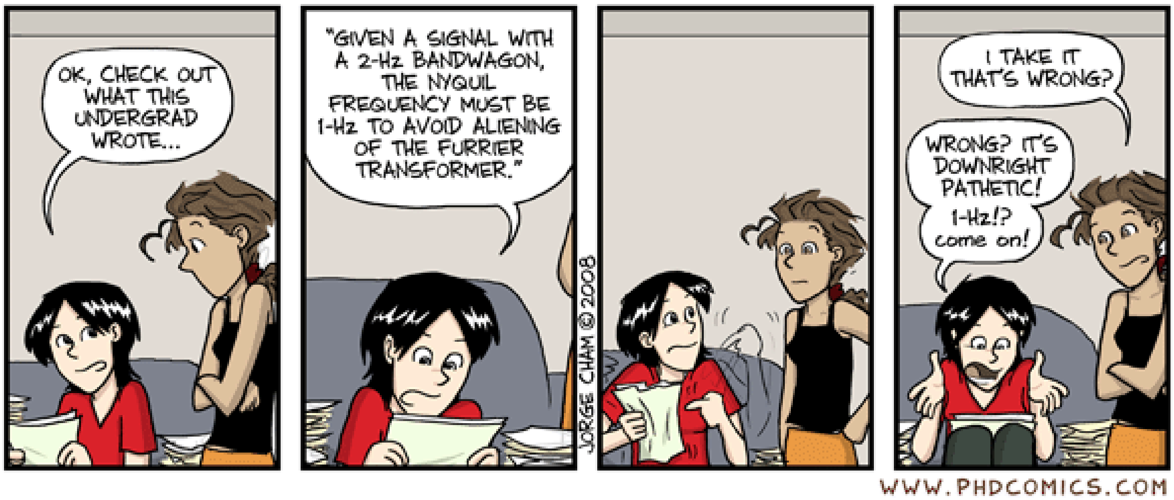
$y[n] =$

(b) When the input to the ***first*** system is $x[n] = 20$, for $-\infty < n < \infty$, determine the ***overall*** output, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

$y[n] =$

PROBLEM sp-09-F.7:

In ECE-2025 you've learned new vocabulary. Thus, the following comic strip (produced by a 1997 ME grad from Georgia Tech) now qualifies as humor:



(a) What is the correct answer that the GTA is looking for? In addition, ***explain why 1-Hz is wrong.***

(b) ***Four*** technical terms from DSP are morphed into other words in the comic strip. Identify these ***four***, and give the correct DSP technical word for each.

PROBLEM sp-09-F.8:

Circle the correct answer in both parts.

(a) Determine the amplitude (A) and phase (ϕ) of the sinusoid that is the sum of the following three sinusoids, expressed in MATLAB:

```
10*cos(6*tt + pi/2) + 7*cos(6*tt - pi/6) + 7*cos(6*tt + 7*pi/6)
```

(a) $A = 10$ and $\phi = \pi/2$.
(b) $A = 7$ and $\phi = \pi/2$.
(c) $A = 0$ and $\phi = 0$.
(d) $A = 3$ and $\phi = \pi/2$.
(e) $A = 24$ and $\phi = \pi/2$.
(f) None of the above

(b) A sinusoidal signal $x(t)$ is defined by the vector xx in the following MATLAB code:

```
tt = 0:0.001:10; xx = real( (1+j)*exp(j*pi*tt) ); plot(tt,xx)
```

When xx is plotted versus time (tt), its maximum value will be:

(a) $A = 1$
(b) $A = 1 + j$
(c) $A = \sqrt{2}$
(d) $A = 0$
(e) None of the above

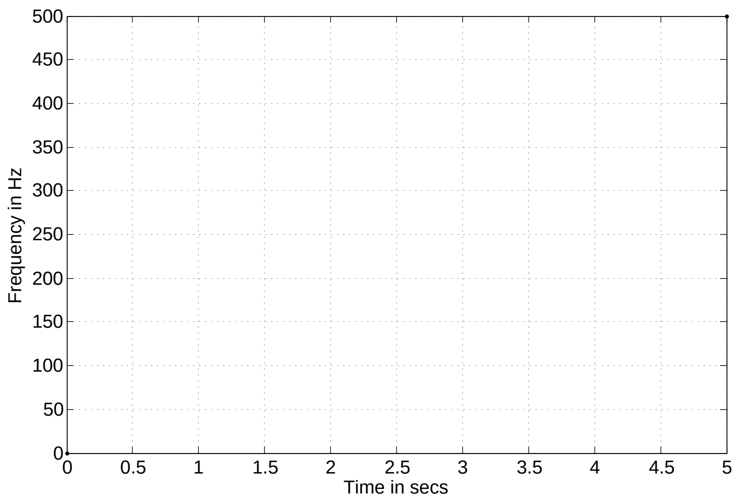
PROBLEM sp-09-F.9:

Consider the following snippet of code:

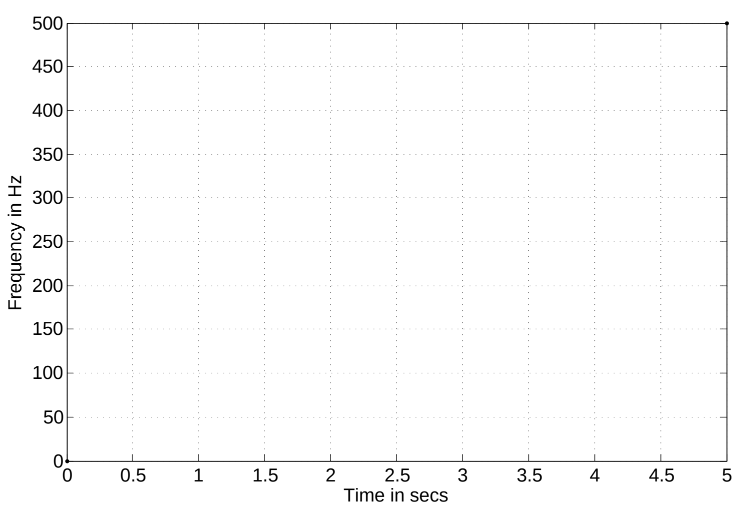
```
nn = 0:5000; fs = 1000; tt = nn/fs;
xx = ... %<-- this line is given below for each part
plotspec(xx,fs,128)   %-- same as:  specgram(xx,128,fs);
```

For two variations of the MATLAB code on the second line, sketch the resulting spectrogram.

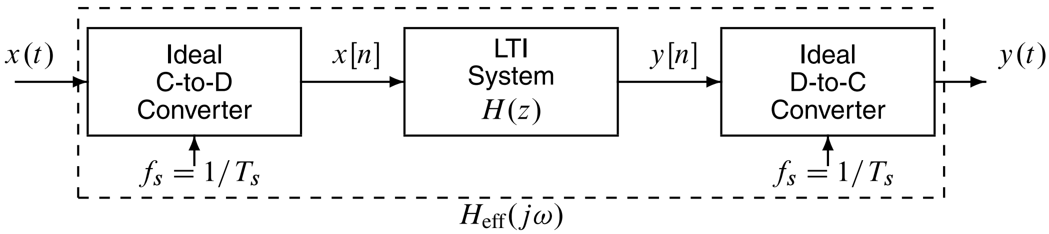
(a) `xx = cos(2*pi*100*tt) .* cos(2*pi*300*tt);`



(b) `xx = cos(2*pi*950*tt);`

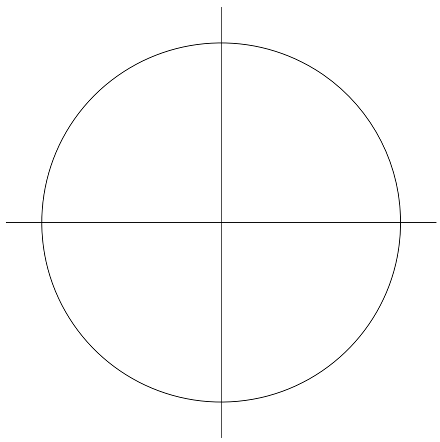


PROBLEM sp-09-F.10:
Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time FIR system has a system function $H(z)$ defined as: $H(z) = 1 + b_1 z^{-1} + z^{-2}$

(a) For the case where $b_1 = 0.7$, determine the poles and zeros of the system, and then give your answer as a pole-zero plot. Account for ***all*** poles and zeros.



(b) The effective frequency response of this system (using the $H(z)$ above) is able to null out one sinusoid. It is similar to the system used in the lab[1] to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient b_1 controls the (frequency) location of the null. If the sampling rate is $f_s = 8000$ Hz, determine the value of b_1 so that the overall effective frequency response has a null at 2000 Hz.

$b_1 =$

[1]This is *not* an IIR notch filter; it is an FIR *nulling* filter.

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NAME: Answer Key
LAST, FIRST

GT username: Ver-1
(ex: gpburdell3)

3 points 3 points 3 points

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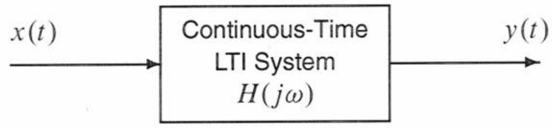
8	20	
9	20	
10	20	

PROBLEM sp-09-F.1:



Suppose that the frequency response $H(j\omega)$ could be written in terms of a few parameters, e.g.,

$$H(j\omega) = \frac{j\omega d}{a + j\omega}$$

where a and d are *real-valued parameters.*

(a) Determine the impulse response of the system above when $a = 800$ and $d = 2$. Determine the *simplest possible* formula for $h(t)$.

$h(t) = -1600e^{-800t}u(t) + 2\delta(t)$

$$H(j\omega) = j\omega\left(\frac{2}{800 + j\omega}\right)$$

$$\frac{d}{dt}\left\{2e^{-800t}u(t)\right\}$$

$$= -1600e^{-800t}u(t) + \underbrace{2e^{-800t}\delta(t)}_{\text{eval at } t=0}$$

$$2\delta(t)$$

(b) Determine the output signal $y(t)$ when the input signal has a Fourier transform given by

$$X(j\omega) = 2\pi\delta(\omega - 800)$$

Once again assume that the parameters of the system are $a = 800$ and $d = 2$. Determine a *simple* formula for the output $y(t)$, which is *complex-valued* in this case.

$y(t) = 1.414e^{j0.785}e^{j800t}$

$$Y(j\omega) = H(j\omega)X(j\omega) = 2\pi H(j800)\delta(\omega - 800)$$

$$H(j800) = \frac{2(j800)}{800 + j800} = \frac{2j}{1+j} = \sqrt{2}e^{j\pi/4} = 1.414e^{j0.785}$$

Inverse FT of $2\pi Ae^{j\theta}\delta(\omega - 800) \rightarrow Ae^{j\theta}e^{j800t}$

PROBLEM sp-09-F.2:

The two subparts of this problem are completely independent of one another.

(a) Determine the result of the following convolution:

$$y(t) = \{100e^{-4t}u(t-5)\} * u(t-23)$$

$y(t) = 25e^{-20}(1-e^{-4(t-28)})u(t-28)$

$y(t) = \{100e^{-4(t-5)}e^{-20}u(t-5)\} * u(t-23)$

$= 100e^{-20}\delta(t-5) * \underbrace{e^{-4t}u(t) * u(t)} * \delta(t-23)$

$e^{-at}u(t) * e^{-bt}u(t) = \frac{1}{b-a}(e^{-at}-e^{-bt})u(t)$

Use $b=4,\ a=0$

$= 100e^{-20}\delta(t-28) * \frac{1}{4}(1-e^{-4t})u(t)$

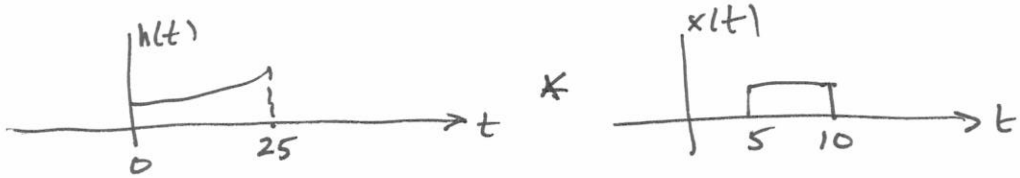
$= \frac{100}{4}e^{-20}(1-e^{-4(t-28)})u(t-28)$

(b) When two finite-duration signals are convolved, the result is a finite-duration signal. In this part,

$$h(t) = t^2[u(t-25)-u(t)] \qquad \text{and} \qquad x(t) = 9[u(t-5)-u(t-10)]$$

Determine the starting and ending times of the output signal $y(t) = x(t) * h(t)$, i.e., find T_1 and T_2 so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$. *Hint:* Visualize the flip-and-slide, but don't integrate.

$T_1 = 5$ sec. $T_2 = 35$ sec.



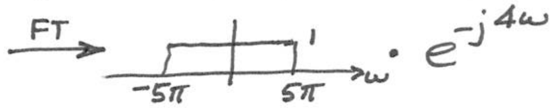
y(t) starts at $0+5=5$

ends at $25+10=35$

PROBLEM sp-09-F.3:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \delta(t-4)$

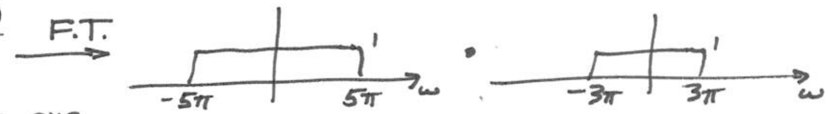


1

(b) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$

5

(c) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \dfrac{\sin(3\pi t)}{\pi t}$



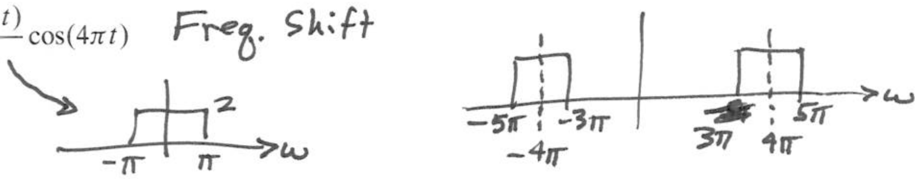
2 get the narrower one

(d) $x(t) = \left(\dfrac{\sin(\pi(t+1))}{\pi(t+1)}\delta(t+1)\right) * \delta(t-5)$ $= (1\delta(t+1)) * \delta(t-5) = \delta(t-4) \xrightarrow{F} e^{-j4\omega}$

eval at $t=-1$

4

(e) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t}\cos(4\pi t)$ Freq. Shift



9

Each of the time signals above has a Fourier transform that should be in the list below.

[0] $X(j\omega) = u(\omega+5\pi) - u(\omega-5\pi)$

[1] $X(j\omega) = e^{-j4\omega}[u(\omega+5\pi) - u(\omega-5\pi)]$

[2] $X(j\omega) = u(\omega+3\pi) - u(\omega-3\pi)$

[3] $X(j\omega) = j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

[4] $X(j\omega) = e^{-j4\omega}$

[5] $X(j\omega) = 0$

[6] $X(j\omega) = e^{-j4\omega}[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

[7] $X(j\omega) = [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)] * \delta(\omega-4\pi)$

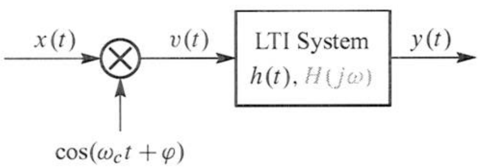
[8] $X(j\omega) = 2u(\omega+\pi) - 2u(\omega-\pi) + \pi\delta(\omega+4\pi) + \pi\delta(\omega-4\pi)$

[9] $X(j\omega) = u(\omega+5\pi) - u(\omega+3\pi) + u(\omega-3\pi) - u(\omega-5\pi)$

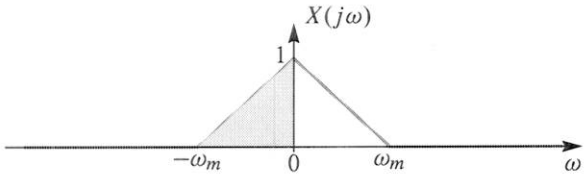
[None] $X(j\omega)$ not in the list above.

PROBLEM sp-09-F.4:

The system below involves a multiplier followed by a filter:



$\omega_c = 400$ rad/s, $\varphi = 0.2$ rads

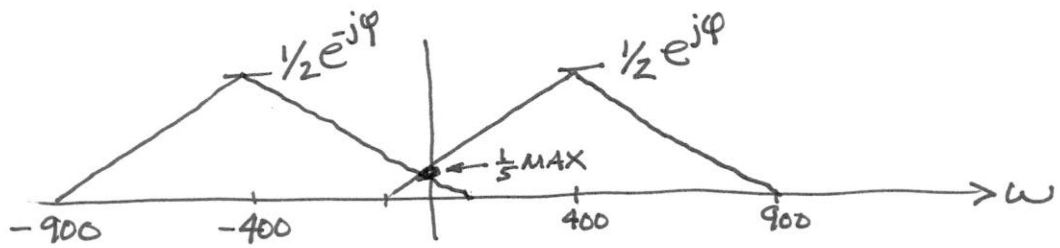


$\omega_m = 500$ rad/s

Suppose the frequency of the cosine in the multiplier above is $\omega_c = 400$ rad/s, and its phase is $\varphi = 0.2$ rads. The Fourier transform of the input, $X(j\omega)$, is also shown above.

(a) Determine the DC value of $V(j\omega)$, the Fourier transform of the signal $v(t)$ after the multiplier.

$V_{DC} =$ 0.196



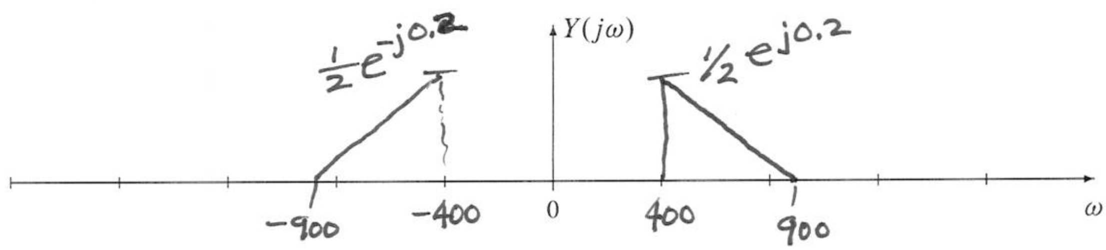
$\varphi = 0.2$

at $\omega = 0$ $\quad \frac{1}{5}\cdot\frac{1}{2}e^{j\varphi} + \frac{1}{5}\cdot\frac{1}{2}e^{-j\varphi} = \frac{1}{5}\cos\varphi = 0.196$

(b) Suppose the LTI system is an *ideal* HPF defined by

$$H(j\omega) = \begin{cases} 0 & |\omega| < 400\,\text{rad/s} \\ 1 & |\omega| \geq 400\,\text{rad/s} \end{cases}$$

Make a sketch of the Fourier transform of $y(t)$, called $Y(j\omega)$, when the input is $X(j\omega)$ shown above.



PROBLEM sp-09-F.5:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

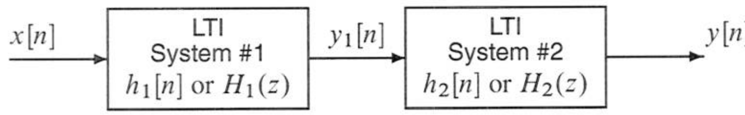
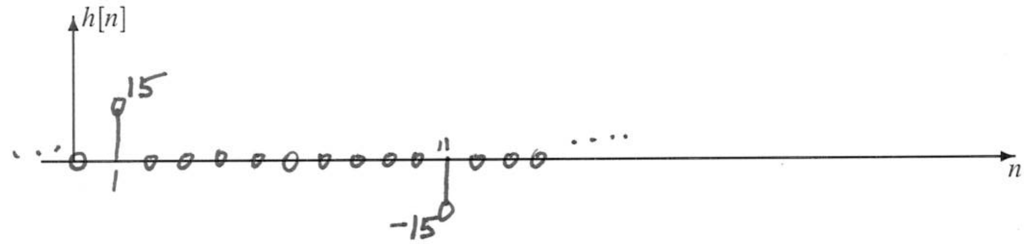


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that $H_1(z) = 3 - 3z^{-5}$ is the system function for System #1, and

System #2 is an FIR filter described by a difference equation: $y[n] = 5y_1[n-1] + 5y_1[n-6]$

(a) Determine the impulse response $h[n]$ of the overall system. ***Give your answer as a stem plot.***



$$H_2(z) = 5z^{-1} + 5z^{-6}$$

$$H(z) = H_1(z)H_2(z) = 3(1 - z^{-5})(5z^{-1})(1 + z^{-5})$$

$$= 15z^{-1}(1 - z^{-10}) = 15z^{-1} - 15z^{-11}$$

$$h[n] = 15\delta[n-1] - 15\delta[n-11]$$

(b) Determine the frequency response of System #2. Express your answer in the following form by finding values for α, β, and μ:

$$H(e^{j\hat{\omega}}) = \alpha e^{-j\beta\hat{\omega}} \cos(\mu\hat{\omega})$$

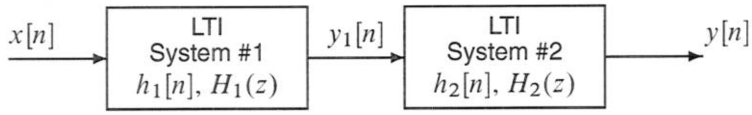
$\alpha = 10$
$\beta = 3.5$
$\mu = 2.5$

$$H_2(e^{j\hat{\omega}}) = H_2(z)\Big|_{z=e^{j\hat{\omega}}}$$

$$= 5e^{-j\hat{\omega}} + 5e^{-j6\hat{\omega}}$$

$$= 5e^{-j3.5\hat{\omega}}\underbrace{(e^{j2.5\hat{\omega}} + e^{-j2.5\hat{\omega}})}_{2\cos(2.5\hat{\omega})}$$

PROBLEM sp-09-F.6:
The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is a filter described by its impulse response: $h_1[n] = \frac{1}{2}\delta[n-1] - \delta[n-4]$

and System #2 is an FIR ***running sum*** filter whose system function is: $H_2(z) = \dfrac{1-z^{-7}}{1-z^{-1}}$

(a) When the input to the ***second*** system is $y_1[n] = 7\cos(2n+5.5)$, for all n, determine the output of the ***second*** system, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

$y[n] = 5.465\cos(2n - 0.5)$ — or, $\angle = 5.783$

2nd system is a 7-pt running sum.

$$H_2(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}} \frac{\sin(\frac{7}{2}\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

at $\hat{\omega} = 2$ $\quad H_2(e^{j2}) = 0.7808\, e^{j0.283}$ — or, 0.09π

Multiply amps $\qquad$ Add phases.

$7(0.7808) = 5.465 \qquad 5.5 + 0.283 = -0.5$
$\qquad\qquad\qquad\qquad\qquad\qquad = 5.783$

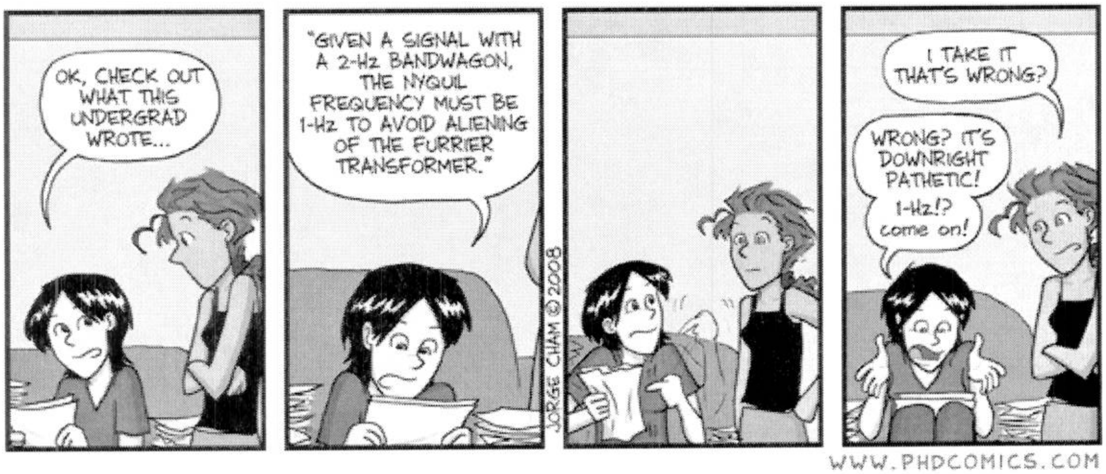
(b) When the input to the ***first*** system is $x[n] = 20$, for $-\infty < n < \infty$, determine the ***overall*** output, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

$y[n] = -70$

$y_1[n] = \frac{1}{2}x[n-1] - x[n-4] = 10 - 20 = -10$

$H_2(e^{j0}) = 7$ $\quad$ because -10 is D.C.

PROBLEM sp-09-F.7:
In ECE-2025 you've learned new vocabulary. Thus, the following comic strip (produced by a 1997 ME grad from Georgia Tech) now qualifies as humor:



(a) What is the correct answer that the GTA is looking for? In addition, ***explain why 1-Hz is wrong.***

The Nyquist rate for a 2-Hz bandlimited signal is twice the max frequency, i.e., 2×2Hz

Correct answer is 4 Hz

(b) ***Four*** technical terms from DSP are morphed into other words in the comic strip. Identify these ***four***, and give the correct DSP technical word for each.

Bandwagon → Bandlimit
or, Bandwidth

Nyquil → Nyquist

Aliening → Aliasing

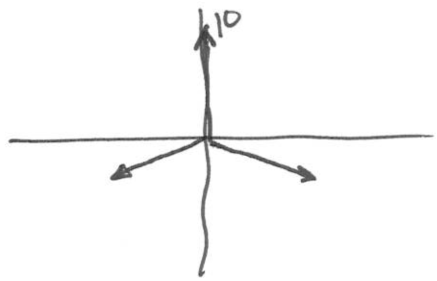
Furrier Transformer → Fourier Transform

PROBLEM sp-09-F.8:
Circle the correct answer in both parts.

(a) Determine the amplitude (A) and phase (ϕ) of the sinusoid that is the sum of the following three sinusoids, expressed in MATLAB:

```
10*cos(6*tt + pi/2) + 7*cos(6*tt - pi/6) + 7*cos(6*tt + 7*pi/6)
```

(a) $A = 10$ and $\phi = \pi/2$.
(b) $A = 7$ and $\phi = \pi/2$.
(c) $A = 0$ and $\phi = 0$.
(d) $A = 3$ and $\phi = \pi/2$.
(e) $A = 24$ and $\phi = \pi/2$.
(f) None of the above



$$10e^{j\pi/2} + 7e^{-j\pi/6} + 7e^{j7\pi/6}$$

$$10j + \frac{7\sqrt{3}}{2} + j\frac{7}{2} - \frac{7\sqrt{3}}{2} - j\frac{7}{2} = j10 - j7 = j3 = 3e^{j\pi/2}$$

(b) A sinusoidal signal $x(t)$ is defined by the vector xx in the following MATLAB code:

```
tt = 0:0.001:10; xx = real( (1+j)*exp(j*pi*t) ); plot(tt,xx)
```

When xx is plotted versus time (tt), its maximum value will be:

(a) $A = 1$
(b) $A = 1 + j$
(c) $A = \sqrt{2}$
(d) $A = 0$
(e) None of the above

$$\mathrm{Re}\{(1+j)e^{j\pi t}\} = \mathrm{Re}\{\sqrt{2}e^{j\pi/4}e^{j\pi t}\}$$

$$= \sqrt{2}\cos(\pi t + \pi/4)$$

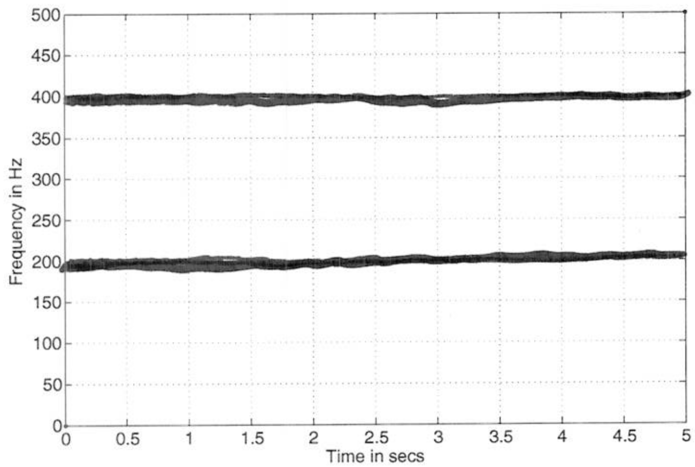
PROBLEM sp-09-F.9:

Consider the following snippet of code:

```
nn = 0:5000; fs = 1000; tt = nn/fs;
xx = ... %<-- this line is given below for each part
plotspec(xx,fs,128)   %-- same as:  specgram(xx,128,fs);
```

For two variations of the MATLAB code on the second line, sketch the resulting spectrogram.

(a) `xx = cos(2*pi*100*tt) .* cos(2*pi*300*tt);`



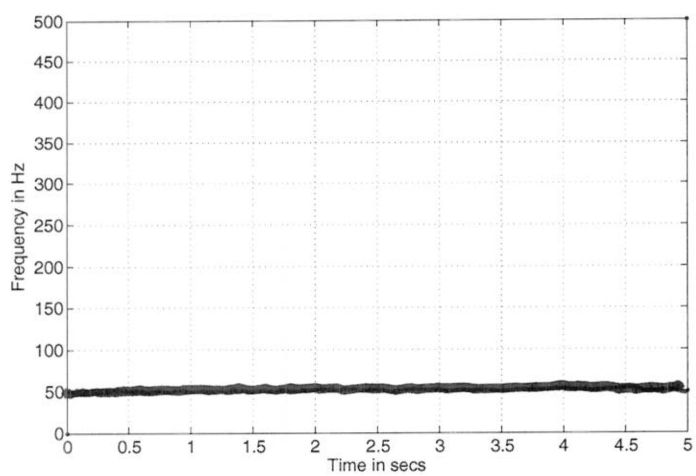
$\frac{1}{2}(e^{j200\pi t} + e^{-j200\pi t}) \cdot$

$\frac{1}{2}(e^{j600\pi t} + e^{-j600\pi t})$

$= \frac{1}{4}e^{j800\pi t} + \frac{1}{4}e^{j400\pi t}$ + neg freqs

400Hz   200Hz

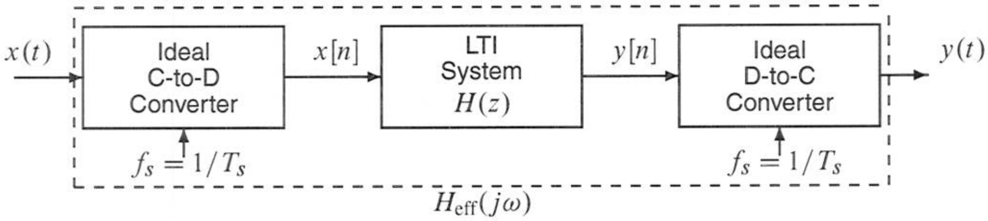
(b) `xx = cos(2*pi*950*tt);`



950 Hz aliases to 50 Hz when $f_s = 1000$

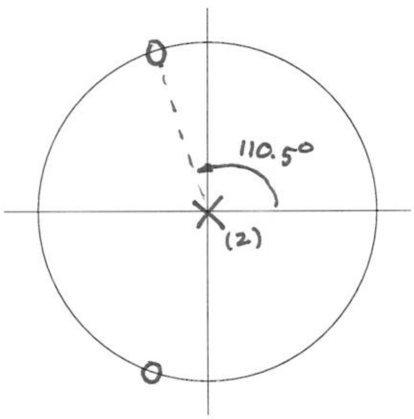
PROBLEM sp-09-F.10:

Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time FIR system has a system function $H(z)$ defined as: $H(z) = 1 + b_1 z^{-1} + z^{-2}$

(a) For the case where $b_1 = 0.7$, determine the poles and zeros of the system, and then give your answer as a pole-zero plot. Account for ***all*** poles and zeros.



Roots of $\dfrac{z^2 + 0.7z + 1}{z^2}$

Zeros at $e^{\pm j1.928}$ ← $\pm 0.614\pi$ or $\pm 110.5°$

poles at $z = 0$

(b) The effective frequency response of this system (using the $H(z)$ above) is able to null out one sinusoid. It is similar to the system used in the lab[1] to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient b_1 controls the (frequency) location of the null. If the sampling rate is $f_s = 8000$ Hz, determine the value of b_1 so that the overall effective frequency response has a null at 2000 Hz.

$b_1 = 0$

With zeros at $e^{\pm j\theta}$ we get

$$(1 - e^{j\theta} z^{-1})(1 - e^{-j\theta} z^{-1})$$

$$= 1 - 2\cos\theta \, z^{-1} + z^{-2}$$

θ is $\hat{\omega}$ when doing the frequency response.

$$\hat{\omega} = 2\pi \frac{f_{NULL}}{f_s} = 2\pi \frac{2000}{8000} = \frac{\pi}{2} \qquad 2\cos(\pi/2) = 0$$

[1]This is *not* an IIR notch filter; it is an FIR *nulling* filter.