GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 2-May-05 COURSE: ECE-2025

NAME: ______________________________ GT #: ________________
LAST, FIRST (ex: gtz123a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	30	
4	30	
5	30	
6	30	
7	30	

PROBLEM sp-05-F.1:

For each part, pick a correct frequency[1] from the list and enter its letter in the answer box[2]:
Write a brief explanation of your answers to receive any credit.

Frequency

(a) 8000 Hz

(b) 4000 Hz

(c) 2000 Hz

(d) 1600 Hz

(e) 1200 Hz

(f) 1000 Hz

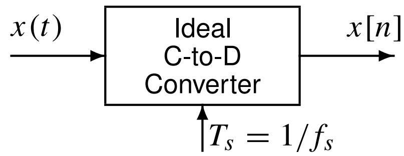
(g) 800 Hz

(h) 500 Hz

(i) 400 Hz

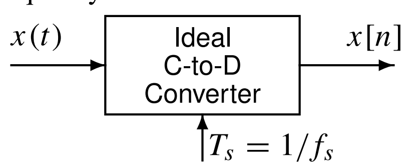
(a) If the output from an ideal C/D converter is $x[n] = 33\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value of the input frequency of $x(t)$:

ANS =



(b) If the output from an ideal C/D converter is $x[n] = 33\cos(0.5\pi n)$, and the input signal $x(t)$ defined by: $x(t) = 33\cos(3000\pi t)$ then determine one possible value of the sampling frequency of the C-to-D converter:

ANS =



(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = (A - B\cos(800\pi t))\sin(1200\pi t)$.

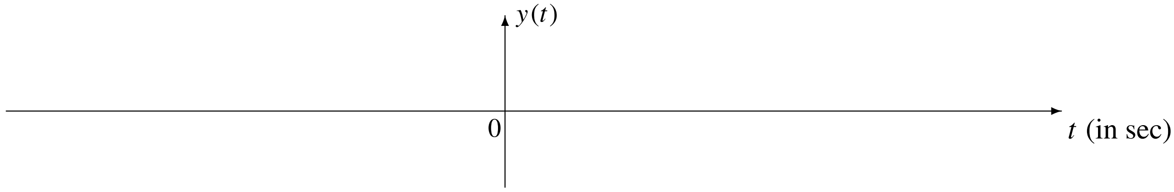
ANS =

[1] Some questions have more than one answer, but you only need to pick one correct answer from the list.
[2] It is possible to use an answer more than once.

PROBLEM sp-05-F.2:

The impulse response, $h(t)$, of a continuous-time linear, time-invariant system is: $h(t) = -7\delta(t) + A\delta(t - \Delta)$

(a) Find the output of the system, $y(t)$, when the input is $x(t) = u(t),\ A = 7$, and $\Delta = 0.025$. Give your answer as a plot on the axes below. Label your plot carefully.



(b) Find the output of the system, $y(t)$, when the input is $x(t) = \cos(100\pi t),\ A = -7$, and $\Delta = 0.025$. Express your answer as a single sinusoid.

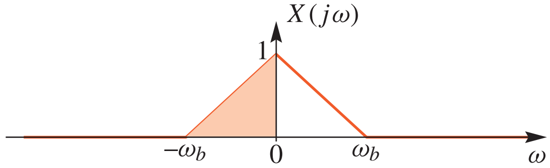
$y(t) =$

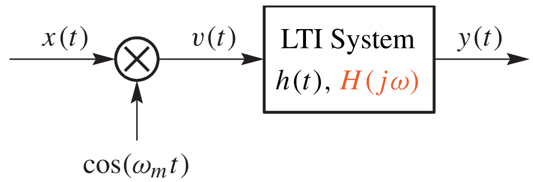
(c) Now assume that the input signal is $x(t) = \cos(100\pi t)u(t)$, i.e., a one-sided sinusoid that is zero for $t < 0$. Find values for A and Δ that will produce an output $y(t)$ that is ***exactly three and half periods*** of a 50-Hz sinusoid, and zero thereafter.

$A =$	$\Delta =$

PROBLEM sp-05-F.3:

The transmitter system below involves a multiplier followed by a filter:

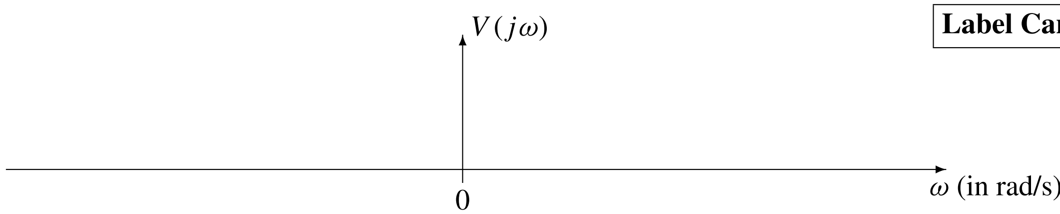




The Fourier transform of the input is $X(j\omega)$. For all parts below, assume that $\omega_m = 100\pi$, and $\omega_b = 20\pi$.

(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, when the input is $X(j\omega)$ shown above.

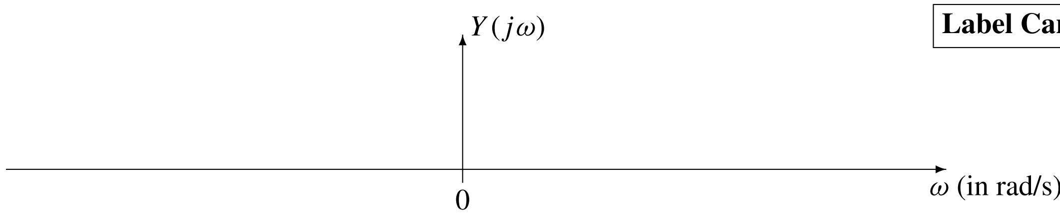
Label Carefully



(b) If the filter is an ideal filter defined by $H(j\omega) = \begin{cases} 0 & |\omega| < 100\pi \\ j & \omega \geq 100\pi \\ -j & \omega \leq -100\pi \end{cases}$

Make a sketch of $Y(j\omega)$, the Fourier transform of $y(t)$, when the input is $X(j\omega)$ shown above.

Label Carefully



(c) Now change the input to be $x(t) = A\cos(\omega_c t + \varphi)$. Determine the values of ω_c, φ, and A so that the output signal is $y(t) = \cos(110\pi t)$. *Note:* Use $\omega_m = 100\pi$ as above and $H(j\omega)$ from part (b).

$\omega_c =$

$\varphi =$

$A =$

PROBLEM sp-05-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

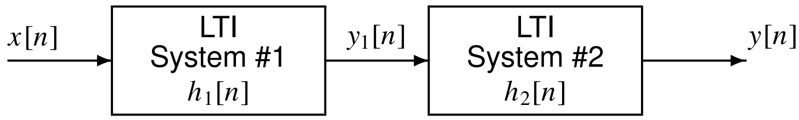


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{9z^{-2} - 9z^{-3}}{1 - 2z^{-1}}$$

but System #2 is unknown.

(a) Determine whether or not System #1 is stable. Give a reason to support your answer.

(b) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $y_1[n]$.

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n-2]$$

From this information, determine the system function $H_2(z)$ for the second system. ***Simplify the expression for*** $H_2(z)$***.***

PROBLEM sp-05-F.5:

In each case, make a sketch of the plot that MATLAB will produce. Label your sketches carefully.
Hint: Convert the MATLAB code to an equivalent mathematical function or operation to make the sketch.

(a)
```
xn = [1,zeros(1,4)];
yn = filter([1,-1], [1,2], xn);
stem(0:4,yn)
```

(b)
```
ww = -pi:pi/200:pi;
[HH,ww] = freekz([13,13], [1,-0.5], ww);
plot(ww, abs(HH))
```

(c)
```
tt = 1e-12 + 0.02*(-100:100);  %-avoid divide by zero
ht = sin(pi*tt)./tt/2;
plot(tt, ht)
```

PROBLEM sp-05-F.6:

In each of the following cases, determine the (inverse or forward) Fourier transform. The following Fourier transform pair will be needed for some parts:

$$x(t) = e^{-t^2} \quad \longleftrightarrow \quad X(j\omega) = \sqrt{2\pi}\, e^{-\omega^2/4}$$

Give your answer as a plot, or a $\boxed{\text{simple}}$ formula.

(a) Find $x(t)$ when $X(j\omega) = 14\cos(3\omega)$.

(b) Find $S(j\omega)$ when $s(t) = \left\{ \frac{d}{dt} e^{-t^2} \right\} \delta(t-3)$.

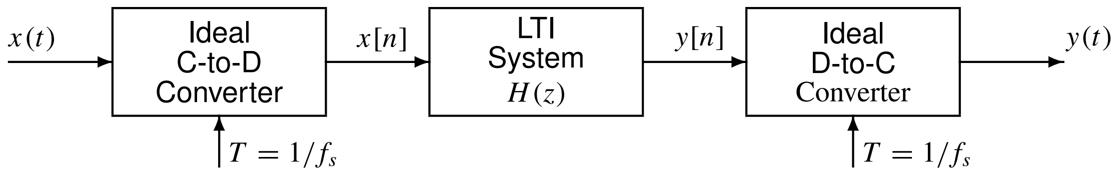
(c) Find $R(j\omega)$ when $r(t) = e^{-9t^2}$.

(d) Find $H(j\omega)$ when $h(t) = u(t-5)u(8-t)$.

(e) The convolution $y(t) = \dfrac{\sin(13\pi t)}{\pi t} * \left\{ \sum_{n=-\infty}^{\infty} \delta(t - n/8) \right\}$ evaluates to a constant, i.e., $y(t) = y_0$.

Find the value of y_0.

PROBLEM sp-05-F.7:



In all parts below, the sampling rates of the C-to-D and D-to-C converters **are equal to** $f_s = 180$ **samples/sec,** and the LTI system is an IIR filter with two poles at $0.9e^{\pm j\pi/3}$, and two zeros at $e^{\pm j\pi/3}$, i.e., the system function is $H(z) = \dfrac{(1 - q_1 z^{-1})(1 - q_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$, where $p_{1,2}$ are the poles and $q_{1,2}$ are the zeros.

(a) Determine the filter coefficients of the IIR filter and fill in the IIR difference equation below.

$$y[n] = \boxed{}\, y[n-1] + \boxed{}\, y[n-2] + \boxed{}\, x[n] + \boxed{}\, x[n-1] + \boxed{}\, x[n-2]$$

(b) Determine the DC response of the digital filter, i.e., the output $y[n]$ when the input is $x[n] = 1$.

(c) If the input signal is a sinusoid of the form $x(t) = \cos(2\pi f_0 t + \phi)$, and the sampling rates are $f_s = 180$ samples/sec, determine a value for the input frequency f_0 so that the output signal is zero. ***Explain.***

$f_0 = \boxed{\text{Hz}}$

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LAST, FIRST (ex: gtz123a)

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PROBLEM sp-05-F.1:

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(b) 4000 Hz
(c) 2000 Hz
(d) 1600 Hz
(e) 1200 Hz
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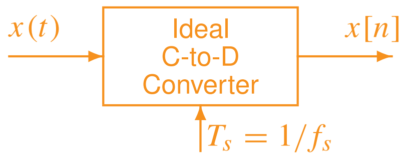
(h) 500 Hz
(i) 400 Hz

(a) If the output from an ideal C/D converter is $x[n] = 33\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value of the input frequency of $x(t)$:



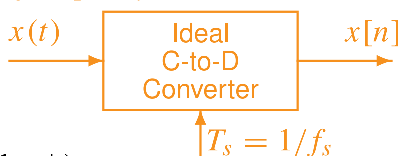
$\hat{\omega} = 0.5\pi$ rads, $f_s = 2000$ Hz

$$\hat{\omega} = \frac{2\pi f}{f_s} + 2\pi \ell \quad \Rightarrow f = \frac{\hat{\omega} - 2\pi\ell}{2\pi} f_s$$

For $\ell = 0,\ f = \left(\frac{0.5\pi}{2\pi}\right) 2000 = 500$ Hz

ANS = **(h)**

(b) If the output from an ideal C/D converter is $x[n] = 33\cos(0.5\pi n)$, and the input signal $x(t)$ defined by: $x(t) = 33\cos(3000\pi t)$ then determine one possible value of the sampling frequency of the C-to-D converter:



$$\hat{\omega} = \frac{2\pi f}{f_s} + 2\pi\ell \qquad (f \text{ can be } \pm)$$

$$\Rightarrow f_s = \frac{\pm 2\pi f}{\hat{\omega} - 2\pi\ell} = \frac{\pm 3000\pi}{0.5\pi - 2\pi\ell} = \frac{\pm 6000}{1 - 4\ell} \text{ Hz}$$

For $\ell = 0,\ f_s = 6000$ Hz

For $\ell = -1$, numerator $= 6000$, $f_s = 1200$ Hz

ANS = **(e)**

For $\ell = +1, +4$, numerator $= -6000$, $f_s = 2000, 400$ Hz

ANS = **(c),(i)**

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = (A - B\cos(800\pi t))\sin(1200\pi t)$.

$$y(t) = \left(A - \frac{B}{2}e^{j800\pi t} - \frac{B}{2}e^{-j800\pi t}\right)\left(\frac{1}{2j}e^{j1200\pi t} - \frac{1}{2j}e^{-j1200\pi t}\right)$$

$\Rightarrow$ highest frequency in $x(t)$ is $\omega_{\max} = 800\pi + 1200\pi = 2000\pi$ rad/s.

$$f_{\max} = 1000 \text{ Hz} \quad \Longrightarrow \quad f_s > 2 f_{\max} = f_{\text{Nyquist}} = 2000 \text{ Hz}$$

ANS = **(c)**

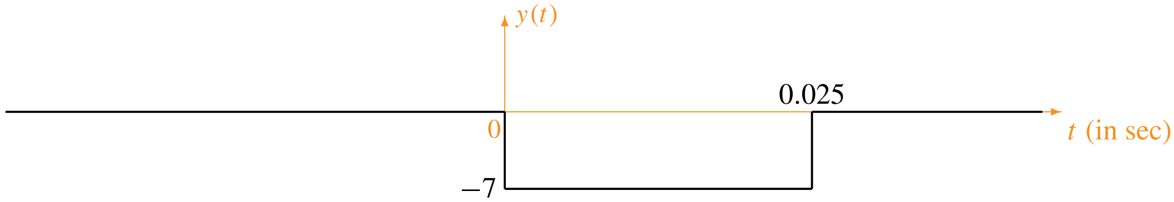
---

[1] Some questions have more than one answer, but you only need to pick one correct answer from the list.
[2] It is possible to use an answer more than once.

PROBLEM sp-05-F.2:

The impulse response, $h(t)$, of a continuous-time linear, time-invariant system is: $h(t) = -7\delta(t) + A\delta(t - \Delta)$

(a) Find the output of the system, $y(t)$, when the input is $x(t) = u(t)$, $A = 7$, and $\Delta = 0.025$. Give your answer as a plot on the axes below. Label your plot carefully.



$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= -7u(t) + 7u(t - 0.025) \qquad \text{(This is a rectangular pulse)}
\end{aligned}
$$

(b) Find the output of the system, $y(t)$, when the input is $x(t) = \cos(100\pi t)$, $A = -7$, and $\Delta = 0.025$. Express your answer as a single sinusoid.

$$\boxed{y(t) = 7\sqrt{2}\cos(100\pi t + 3\pi/4)}$$

$$
\begin{aligned}
y(t) &= x(t) * h(t) = -7x(t) - 7x(t - 0.025) \\
&= -7\cos(100\pi t) - 7\cos(100\pi(t - 0.025)) \\
&= -7\cos(100\pi t) - 7\cos(100\pi t - 2.5\pi)
\end{aligned}
$$

Use Phasor Addition:

$$-7e^{j0} - 7e^{-j2.5\pi} = -7 + j7 = 7\sqrt{2}\,e^{+j3\pi/4}$$

(c) Now assume that the input signal is $x(t) = \cos(100\pi t)u(t)$, i.e., a one-sided sinusoid that is zero for $t < 0$. Find values for A and Δ that will produce an output $y(t)$ that is ***exactly three and half periods*** of a 50-Hz sinusoid, and zero thereafter.

$\boxed{A = -7}$ $\qquad$ $\boxed{\Delta = 0.07 \text{ secs.}}$

100π rad/s is the same as 50 Hz $\Rightarrow$ period $= 1/50$ secs.

3.5 periods $\Longrightarrow$ $3.5/50$ secs. $= 0.07$ secs.

Cancel the tail of $x(t)$ with the shifted version $x(t - \Delta)$

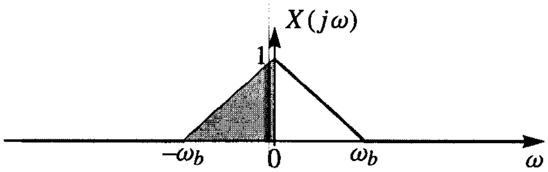
$$y(t) = x(t) * h(t) = -7x(t) + Ax(t - \Delta)$$

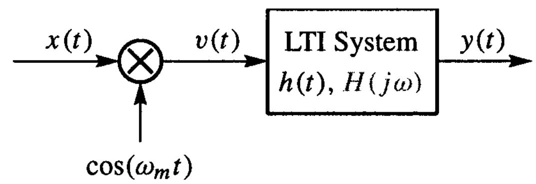
Need $y(0.07) = 0 \Longrightarrow -7x(0.07) + Ax(0) = 0$

Since $x(0) = 1$ and $x(0.07) = \cos(100\pi(0.07)) = -1$, $\quad A = 7x(0.07) = -7$

PROBLEM sp-05-F.3:

The transmitter system below involves a multiplier followed by a filter:

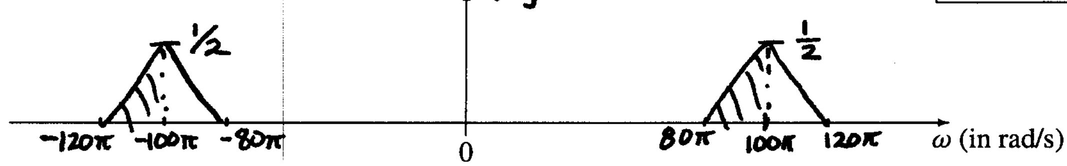




The Fourier transform of the input is $X(j\omega)$. For all parts below, assume that $\omega_m = 100\pi$, and $\omega_b = 20\pi$.

(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, when the input is $X(j\omega)$ shown above.

Label Carefully



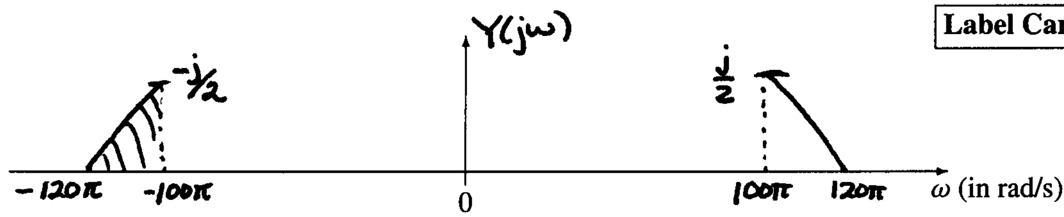
$V(j\omega) = \frac{1}{2}X(j(\omega-\omega_m)) + \frac{1}{2}X(j(\omega+\omega_m))$

(b) If the filter is an ideal filter defined by $H(j\omega) = \begin{cases} 0 & |\omega| < 100\pi \\ j & \omega \geq 100\pi \\ -j & \omega \leq -100\pi \end{cases}$

This is a HPF with phase

Make a sketch of $Y(j\omega)$, the Fourier transform of $y(t)$, when the input is $X(j\omega)$ shown above.

Label Carefully



$Y(j\omega) = H(j\omega)\,V(j\omega)$

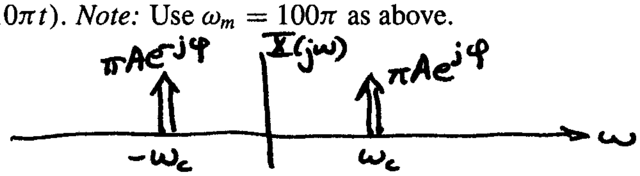
Since $H(j\omega)=0$ for $|\omega| < 100\pi$, the spectrum components below 100π rad/s are eliminated.

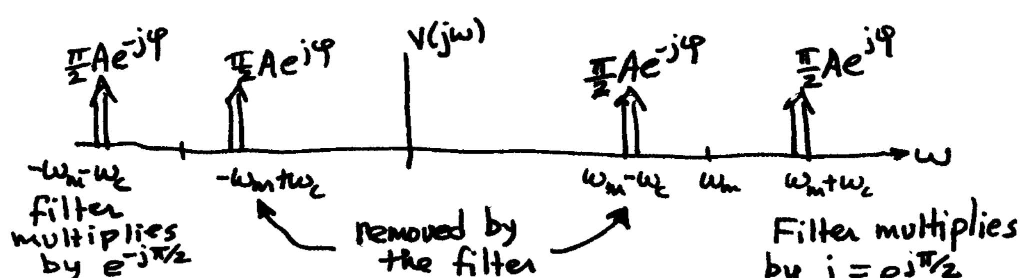
(c) Now change the input to be $x(t) = A\cos(\omega_c t + \varphi)$. Determine the values of ω_c, φ, and A so that the output signal is $y(t) = \cos(110\pi t)$. *Note:* Use $\omega_m = 100\pi$ as above.

$\omega_c = 10\pi$

$\varphi = -\pi/2$

$A = 2$





$\omega_m + \omega_c = 110\pi$

$\Rightarrow \omega_c = 110\pi - 100\pi$

$= 10\pi$ rad/s

$\Rightarrow y(t) = \frac{1}{2}A\cos((\omega_m+\omega_c)t + \underbrace{\varphi + \pi/2}_{=0}) \Rightarrow \varphi = -\pi/2$

PROBLEM sp-05-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

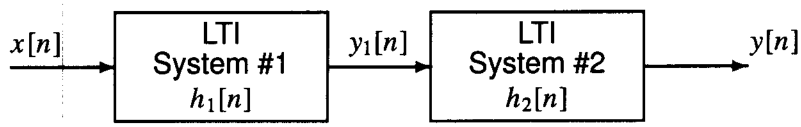


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{9z^{-2} - 9z^{-3}}{1 - 2z^{-1}}$$

but System #2 is unknown.

(a) Determine whether or not System #1 is stable. Give a reason to support your answer.

$H_1(z)$ has one pole at $z = 2$. Thus $h_1[n]$ acts like 2^n

$\therefore$ System #1 is unstable

(b) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $y_1[n]$.

$$x[n] = u[n] \Rightarrow X(z) = \frac{1}{1 - z^{-1}}$$

$$Y_1(z) = X(z)H_1(z) = \left(\frac{1}{1 - z^{-1}}\right)\left(\frac{9z^{-2}(1 - z^{-1})}{1 - 2z^{-1}}\right)$$

$$= \frac{9z^{-2}}{1 - 2z^{-1}}$$

$$\therefore y_1[n] = 9(2)^{n-2}u[n-2]$$

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n - 2]$$

From this information, determine the system function $H_2(z)$ for the second system. ***Simplify the expression for*** $H_2(z)$.

$$y[n] = \delta[n-2] \Rightarrow Y(z) = z^{-2} \qquad x[n] = \delta[n] \Rightarrow X(z) = 1$$

$$Y(z) = H_1(z)H_2(z)X(z)$$

$$H_2(z) = Y(z)/H_1(z)X(z) = \frac{z^{-2}}{\frac{9z^{-2}(1-z^{-1})}{1-2z^{-1}}} = \frac{1-2z^{-1}}{9(1-z^{-1})}$$

PROBLEM sp-05-F.5:

In each case, make a sketch of the plot that MATLAB will produce. Label your sketches carefully.
Hint: Convert the MATLAB code to an equivalent mathematical function or operation to make the sketch.

(a)
```
xn = [1,zeros(1,4)];
yn = filter([1,-1], [1,2], xn);
stem(0:4,yn)
```

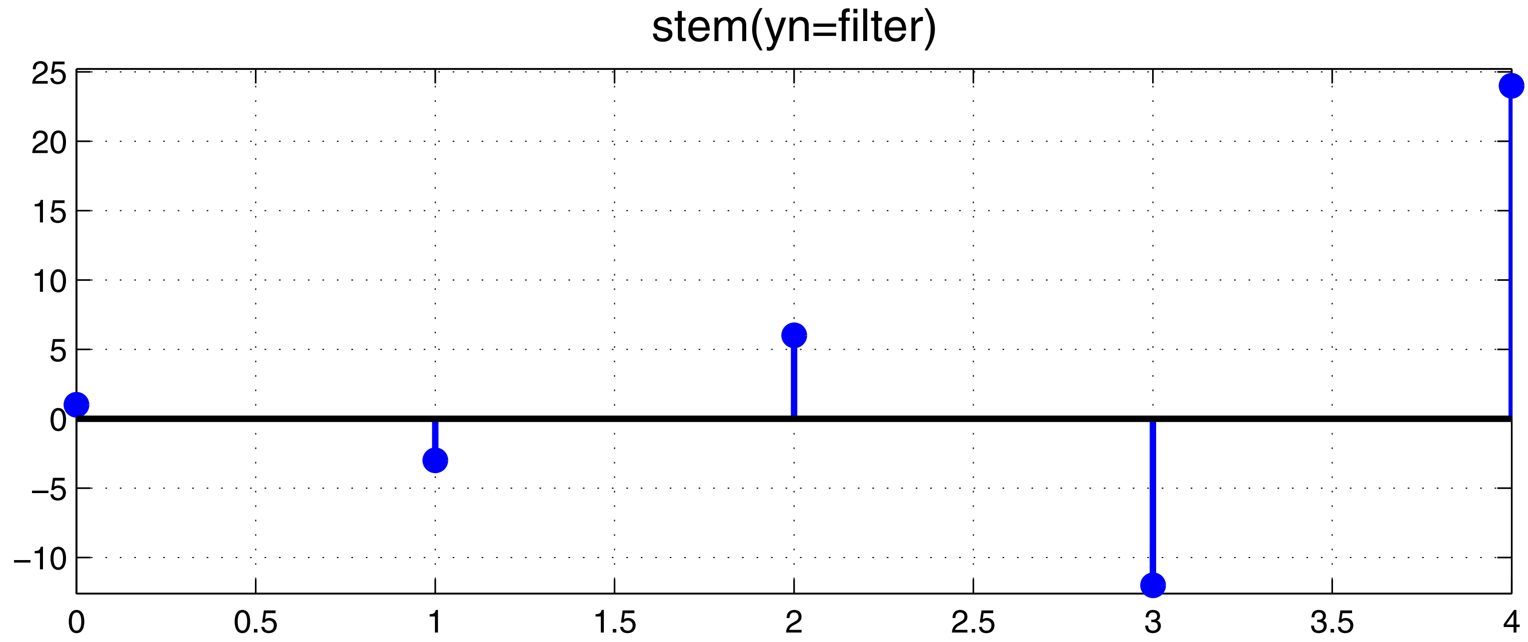
Plot the solution of $y[n] = -2y[n-1] + x[n] - x[n-1]$ for $n = 0, 1, 2, 3, 4$. See the plot on the next page.

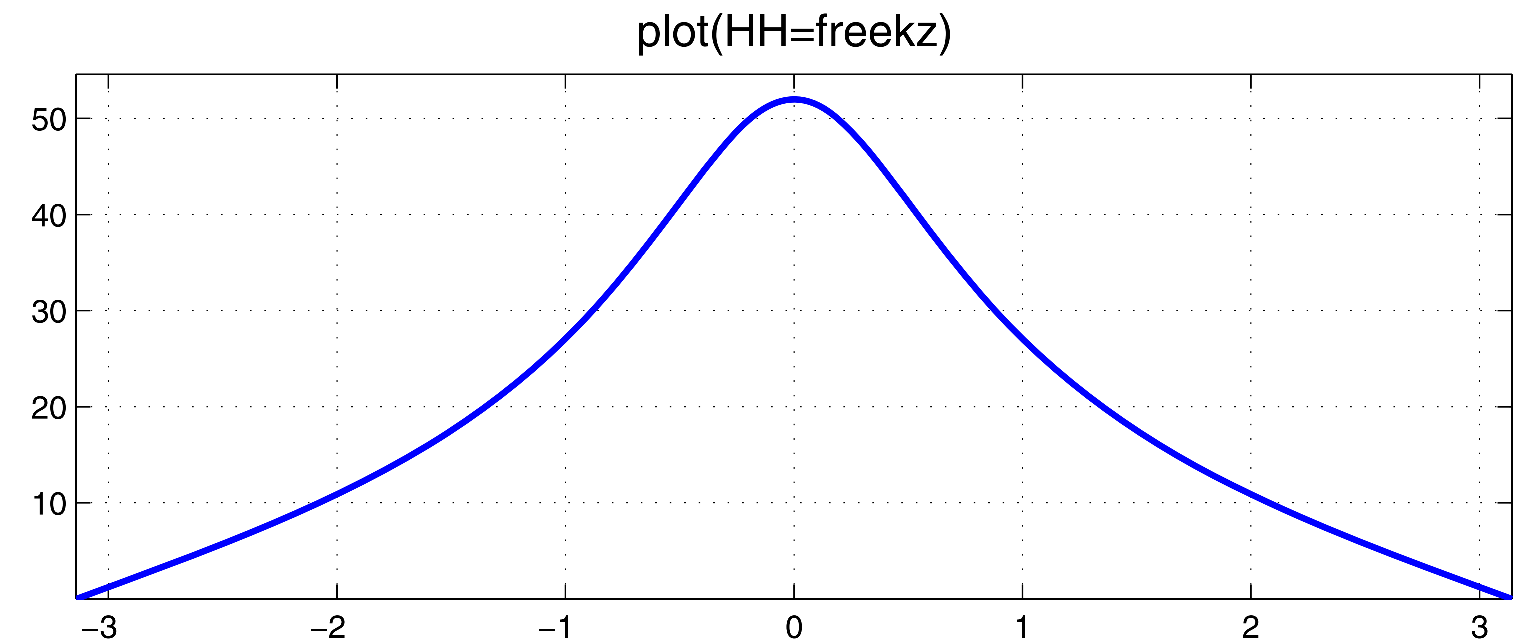
(b)
```
ww = -pi:pi/200:pi;
[HH,ww] = freekz([13,13], [1,-0.5], ww);
plot(ww, abs(HH))
```

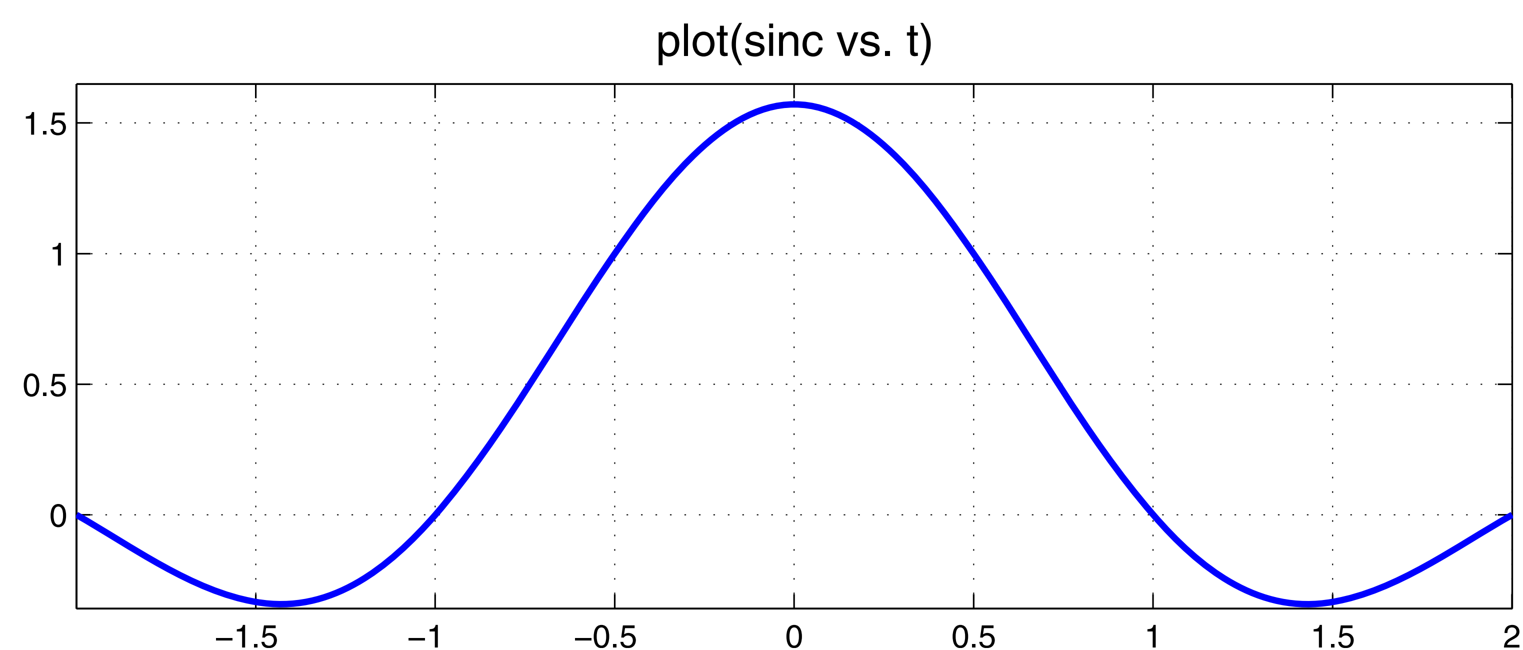
Plot $H(e^{j\omega}) = \dfrac{13 + 13e^{-j\hat{\omega}}}{1 - 0.5e^{-j\hat{\omega}}}$ for $-\pi \leq t \leq \pi$. See the plot on the next page.

(c)
```
tt = 1e-12 + 0.02*(-100:100);  %-avoid divide by zero
ht = sin(pi*tt)./tt/2;
plot(tt, ht)
```

Plot $\dfrac{\sin(\pi t)}{2t}$ for $-2 \leq t \leq 2$. See the plot on the next page.


stem(yn=filter)
25
20
15
10
5
0
−5
−10
0
0.5
1
1.5
2
2.5
3
3.5
4


plot(HH=freekz)
50
40
30
20
10
−3
−2
−1
0
1
2
3


plot(sinc vs. t)
1.5
1
0.5
0
−1.5
−1
−0.5
0
0.5
1
1.5
2

PROBLEM sp-05-F.6:

In each of the following cases, determine the (inverse or forward) Fourier transform. The following Fourier transform pair will be needed for some parts:

$$x(t) = e^{-t^2} \quad \longleftrightarrow \quad X(j\omega) = \sqrt{2\pi}\, e^{-\omega^2/4}$$

Give your answer as a plot, or a simple formula.

(a) Find $x(t)$ when $X(j\omega) = 14\cos(3\omega)$.

$X(j\omega) = 7e^{j3\omega} + 7e^{-j3\omega}$

use $e^{-j\omega t_d} \longleftrightarrow \delta(t-t_d)$

$x(t) = 7\delta(t+3) + 7\delta(t-3)$

(b) Find $S(j\omega)$ when $s(t) = \left\{\frac{d}{dt}e^{-t^2}\right\}\delta(t-3)$.

$\frac{d}{dt}e^{-t^2} = -2te^{-t^2}$ ← eval at $t=3$.

$s(t) = (-2te^{-t^2})\delta(t-3) = -6e^{-9}\delta(t-3)$

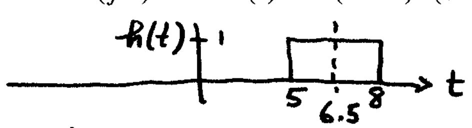
$S(j\omega) = -6e^{-9}e^{-j3\omega}$

(c) Find $R(j\omega)$ when $r(t) = e^{-9t^2}$. $= e^{-(3t)^2}$

Use scaling property: $x(at) \longleftrightarrow \frac{1}{|a|}X(j\omega/a)$ $\quad a=3$

$R(j\omega) = \frac{1}{3}\sqrt{2\pi}\, e^{-(\omega/3)^2/4} = \frac{\sqrt{2\pi}}{3}e^{-\omega^2/36}$

(d) Find $H(j\omega)$ when $h(t) = u(t-5)u(8-t)$.



$H(j\omega) = e^{-j6.5\omega}\frac{\sin(3\omega/2)}{\omega/2}$

rect ↔ sinc

delayed by 6.5

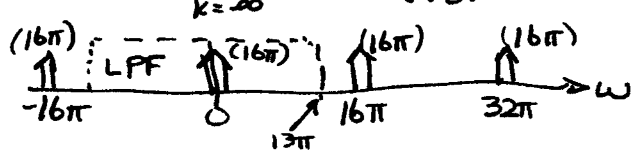
duration of pulse = 3 secs.

(e) The convolution $y(t) = \frac{\sin(13\pi t)}{\pi t} * \left\{\sum_{n=-\infty}^{\infty}\delta(t-n/8)\right\}$ evaluates to a constant, i.e., $y(t) = y_0$.

Find the value of y_0.

F.T. of impulse train is $2\pi(8)\sum_{k=-\infty}^{\infty}\delta(\omega - \frac{2\pi k}{(1/8)})$

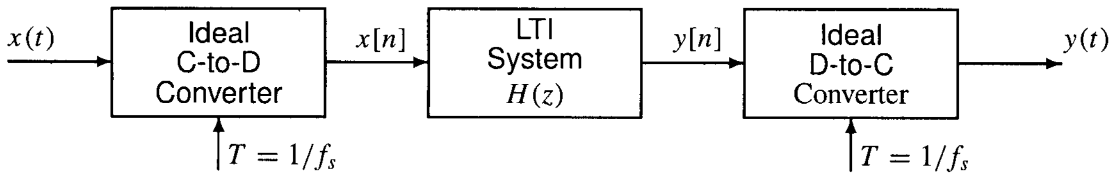
F.T. of $\frac{\sin(13\pi t)}{\pi t}$ is LPF, cutoff $\pm 13\pi$



$\Rightarrow Y(j\omega) = 16\pi\,\delta(\omega)$

$\Rightarrow y(t) = 8 = y_0$

PROBLEM sp-05-F.7:



In all parts below, the sampling rates of the C-to-D and D-to-C converters **are equal to** $f_s = 180$ **samples/sec,** and the LTI system is an IIR filter with two poles at $0.9e^{\pm j\pi/3}$, and two zeros at $e^{\pm j\pi/3}$, i.e., the system function is $H(z) = \dfrac{(1-q_1z^{-1})(1-q_2z^{-1})}{(1-p_1z^{-1})(1-p_2z^{-1})}$, where $p_{1,2}$ are the poles and $q_{1,2}$ are the zeros.

(a) Determine the filter coefficients of the IIR filter and fill in the IIR difference equation below.

$y[n] = \boxed{0.9}\, y[n-1] + \boxed{-.81}\, y[n-2] + \boxed{1}\, x[n] + \boxed{-1}\, x[n-1] + \boxed{1}\, x[n-2]$

$$H(z) = \frac{(1-e^{j\pi/3}z^{-1})(1-e^{-j\pi/3}z^{-1})}{(1-.9e^{j\pi/3}z^{-1})(1-.9e^{-j\pi/3}z^{-1})} = \frac{1 - z^{-1} + z^{-2}}{1-0.9z^{-1}+0.81z^{-2}}$$

(b) Determine the DC response of the digital filter, i.e., the output $y[n]$ when the input is $x[n] = 1$.

DC is $H(e^{j0})$, or $H(z)$ at $z=1$

$$H(1) = \frac{1-1+1}{1-0.9+0.81} = 1.0989$$

(c) If the input signal is a sinusoid of the form $x(t) = \cos(2\pi f_0 t + \phi)$, and the sampling rates are $f_s = 180$ samples/sec, determine a value for the input frequency f_0 so that the output signal is zero. ***Explain.***

$f_0 = \boxed{30 \text{ Hz}}$

The filter will null out $\hat{\omega} = \pi/3$

Also, $\hat{\omega} = \dfrac{2\pi f}{f_s} \Rightarrow f_0 = \dfrac{\hat{\omega} f_s}{2\pi}$

$$\Rightarrow f_0 = \frac{(\pi/3)180}{2\pi} = 30 \text{ Hz}$$