GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2023
Final

NAME: ______________________________________ GTemail: __________________________
FIRST LAST ex: gpBURDELL@gatech.edu

- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for two two-sided pages $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to received partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

Problem	*Value*	*Score*
1	30	
2	15	
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4	30	
5	10	
6	15	
Total		

Print Name (First Last) ________________________________

PROBLEM 1: Parts a, b, and c (10 points each) can be solved independently of each other.

(a) Find the unknowns A and B such that following equation is true for all time t. (NOTE: A and B are co-prime integers (i.e., share no common factors) and $A > 0$, $B > 0$.

$$\sin\left(20\pi t - \frac{A}{B}\pi\right) + \cos\left(20\pi t + \frac{\pi}{22}\right) = 0$$

$A =$	$B =$

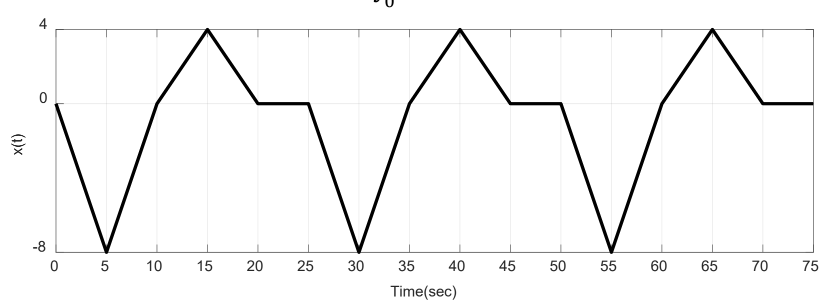
(b) Find the smallest value of M for $M > 0$ such that following equation is true for all time t.

$$\cos(67\pi t) = -\cos\left(67\pi t - \frac{\pi}{6}\right) + \sum_{k=0}^{M} \cos\left(67\pi t - \frac{k\pi}{6}\right)$$

$M =$

(c) The Fourier series coefficients of the periodic signal $x(t)$ below can be found with the following equation:

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-j2\pi k f_0 t} dt$$



Find f_0 and a_0

$f_0 =$	$a_0 =$

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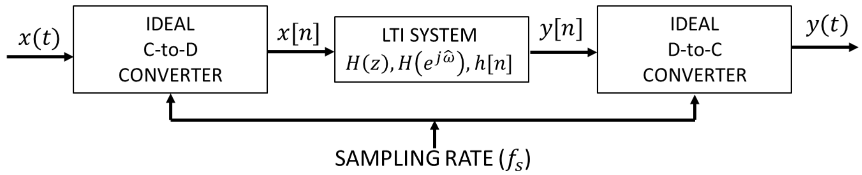
PROBLEM 2: Parts a, b, and c (5 points each) can be solved independently of each other.
Consider the following system where the continuous-time input $x(t)$ is sampled to produce $x[n]$ which is processed through the LTI system to produce $y[n]$ before being reconstructed to yield an overall output $y(t)$



The LTI system function is defined as $H(z) = 8 - 2z^{-4} + 8z^{-8}$.
Let $x(t) = \cos(30\pi t) + \cos\left(64\pi t + \frac{\pi}{3}\right)$

(a) Write the **difference equation** for the LTI system.

$y[n] =$

(b) If $f_s = 300\ Hz$. Find $y(t)$

$y(t) =$

(c) Find $f_s > 30$ so that $y(t)$ has the form $y(t) = A + B\cos(20\pi t + \varphi)$. Furthermore, specify the constants A, B, and φ (where in standard form $B > 0$ and $|\varphi| < \pi$).

$A =$ | $B =$ | $\varphi =$ | $f_s =$

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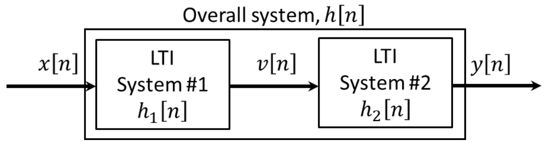
PROBLEM 3: Parts a, b, and c (5 points each) can be solved independently of each other.

Two causal LTI systems are connected in cascade as shown in the figure below:



LTI System #2: $H_2(z) = \dfrac{\left(1-0.7\cos\left(\frac{2\pi}{3}\right)z^{-1}\right)}{1-1.4\cos\left(\frac{2\pi}{3}\right)z^{-1}+0.49z^{-2}}$

(a) Is $H_2(z)$ a stable LTI system?

Stable? YES or NO (Circle one and explain below. Explanation required for any credit)

(b) Find $h_2[n]$

$h_2[n] =$

(c) Suppose that $y[n] = 6\delta[n-1]$ when $x[n] = \delta[n]$. Find the difference equation for LTI System #1 in terms of the output $v[n]$ and the input $x[n]$ under this condition.

$v[n]$=

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PROBLEM 4: Parts a, b, and c (10 points each) can be solved independently of each other.

(a) If $H(z) = \frac{z^{-3}(3+5z^{-2})}{1-0.6z^{-1}}$, find $h[6]$.

$h[6] =$

(b) Find $y[n] = w[n] * h[n]$ where (**NOTE: '*' is convolution**):

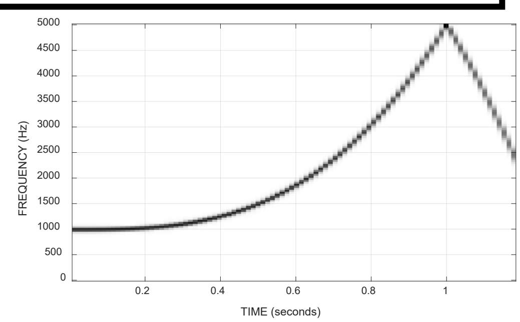
- $w[n] = \left[\cos\left(\frac{\pi}{2}n\right) + 2\cos\left(\frac{3\pi}{4}n\right)\right] * (\delta[n] + 2\delta[n-4] + \delta[n-8])$
- $h[n] = \left(5\delta[n] - 2\frac{\sin(0.6\pi n)}{\pi n}\right)$

$y[n] =$

(c) Consider the MATLAB code below for the spectrogram plot shown to the right over the range of 0 to 1.2 seconds.

```
tt=0:1/fs:1.2;
xx = pi*cos(A*(pi*tt.^4+pi.*tt));
plotspec(xx,fs,256)
```

From the plot, determine numerical values for **fs** and **A**. The spectrogram shows a curve that takes a value of about 2500 Hz around time 1.2. If we look into the future, what value, K, will this curve take at time 2?



fs =	**A** =	$K =$

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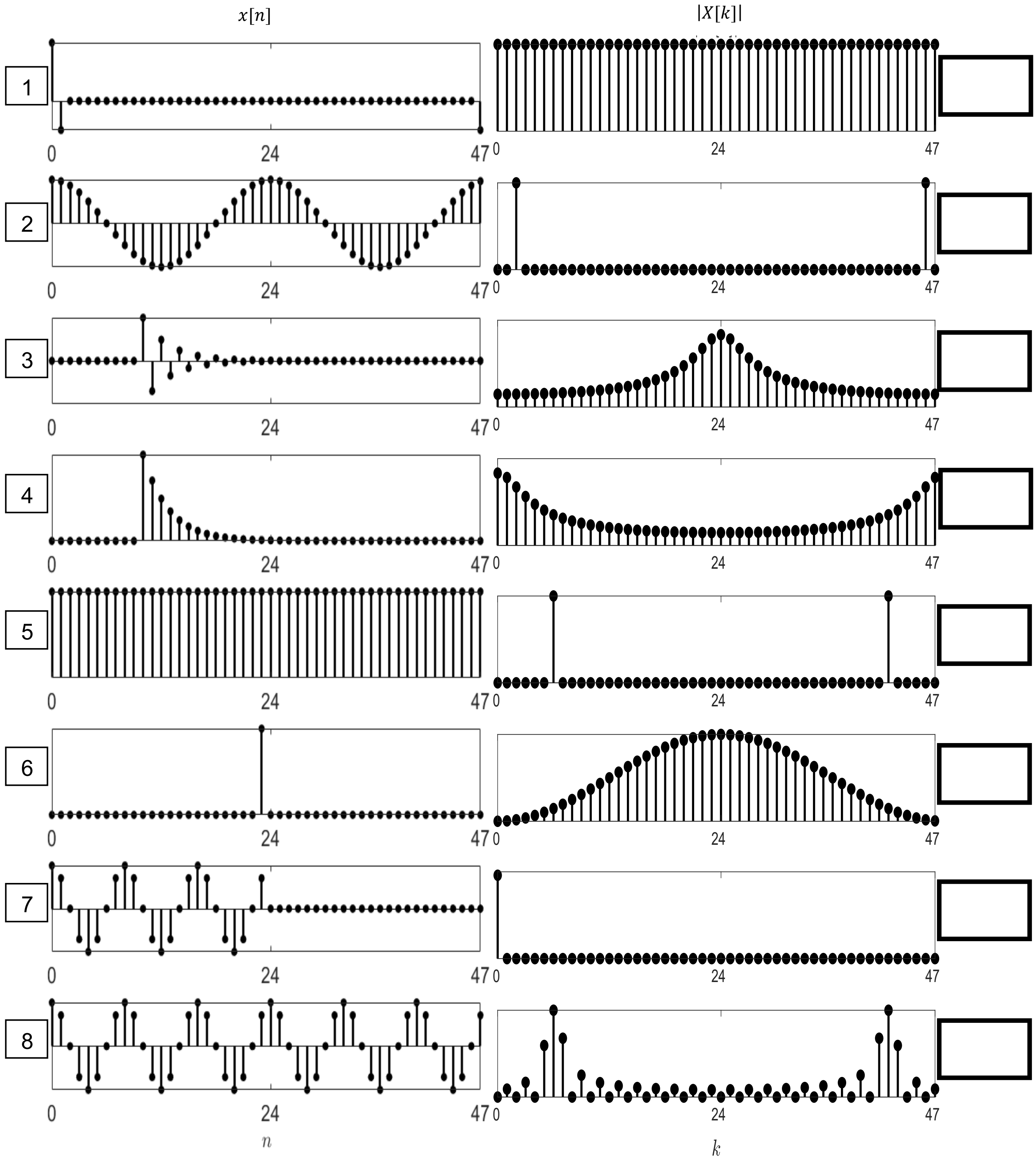
PROBLEM 5: 10 points

Match the plots (1)-(8) for $x[n]$ below with the plots for the corresponding 48-point DFT magnitudes $|X[k]|$ by placing the appropriate number in the boxes besides the plots for $|X[k]|$.



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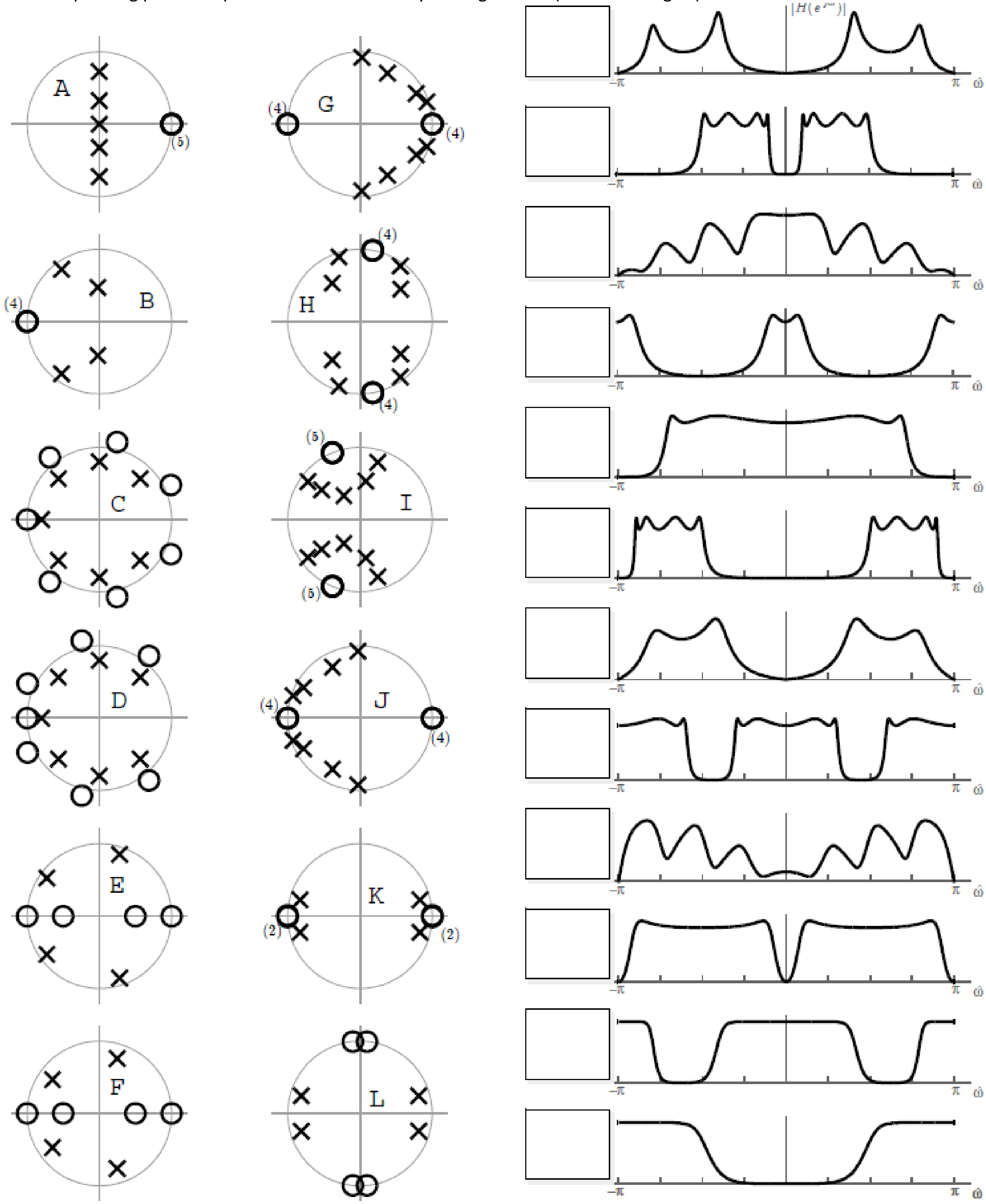
PROBLEM 6: 15 points

Shown below on the left are twelve pole-zero plots for $H(z)$, labeled A through L. Shown below on the right are the corresponding magnitude responses, $|H(e^{j\hat{\omega}})|$ plotted vs $\hat{\omega}$, but in a scrambled order. Match each magnitude response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through L) in to each answer box.



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(a) Find the unknowns A and B such that following equation is true for all time t. (NOTE: A and B are co-prime integers (i.e., share no common factors) and $A > 0$, $B > 0$.

$$\sin\left(20\pi t - \frac{A}{B}\pi\right) + \cos\left(20\pi t + \frac{\pi}{22}\right) = 0$$

$$\cos\left(20\pi t - \frac{A}{B}\pi - \frac{\pi}{2}\right) + \cos\left(20\pi t + \frac{\pi}{22}\right) = 0$$

$$e^{-j\left(\frac{A}{B}\pi - \frac{\pi}{2}\right)} + e^{j\frac{\pi}{22}} = 0 \rightarrow e^{-j\left(\frac{A}{B}\pi - \frac{\pi}{2}\right)} = e^{j\left(\frac{\pi}{22} - \pi\right)}$$

$$-\frac{A}{B}\pi - \frac{\pi}{2} = \frac{\pi}{22} - \pi \rightarrow -\frac{A}{B} = \frac{1}{22} - \frac{1}{2} \rightarrow \frac{A}{B} = \frac{10}{22} = \frac{5}{11}$$

$A = 5$	$B = 11$

(b) Find the smallest value of M for $M > 0$ such that following equation is true for all time t.

$$\cos(67\pi t) = -\cos\left(67\pi t - \frac{\pi}{6}\right) + \sum_{k=0}^{M} \cos\left(67\pi t - \frac{k\pi}{6}\right)$$

Set: $-\cos(67\pi t) - \cos\left(67\pi t - \frac{\pi}{6}\right) + \sum_{k=0}^{M} \cos\left(67\pi t - \frac{k\pi}{6}\right) = 0$

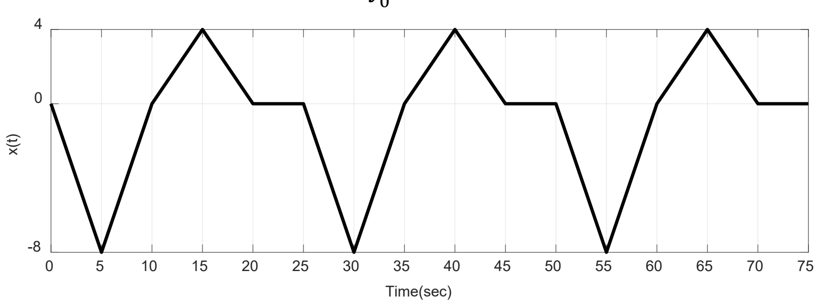
We know that: $\sum_{k=0}^{11} \cos\left(67\pi t - \frac{k\pi}{6}\right) = 0$; (using phasor addition) So we need to account for the terms $\cos(67\pi t) + \cos\left(67\pi t - \frac{\pi}{6}\right)$.

Using phasor addition we see that: $\cos(67\pi t) + \cos\left(67\pi t - \frac{\pi}{6}\right) \rightarrow e^{-\frac{j12\pi}{6}} + e^{-\frac{j13\pi}{6}}$. Therefore: M=13

$M = 13$

(c) The Fourier series coefficients of the periodic signal $x(t)$ below can be found with the following equation:

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-j2\pi k f_0 t} dt$$



Find f_0 and a_0

$$f_0 = \frac{1}{T} = \frac{1}{25}$$

$$a_0 = \frac{1}{25}\int_0^{25} x(t)\, dt = \frac{1}{25}\left(-\frac{1}{2}(10)(8) + \frac{1}{2}(8)(4)\right) = -\frac{4}{5}$$

$f_0 = \frac{1}{25}$	$a_0 = -\frac{4}{5} = -0.8$

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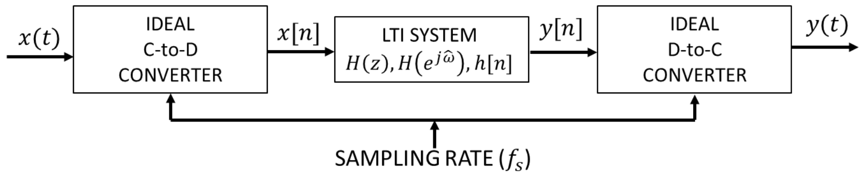
PROBLEM 2: Parts a, b, and c (5 points each) can be solved independently of each other.

Consider the following system where the continuous-time input $x(t)$ is sampled to produce $x[n]$ which is processed through the LTI system to produce $y[n]$ before being reconstructed to yield an overall output $y(t)$



The LTI system function is defined as $H(z) = 8 - 2z^{-4} + 8z^{-8}$.

Let $x(t) = \cos(30\pi t) + \cos\left(64\pi t + \frac{\pi}{3}\right)$

(a) Write the **difference equation** for the LTI system.

$$y[n] = 8 - 2[n-4] + 8x[n-8]$$

$y[n] = 8 - 2[n-4] + 8x[n-8]$

(b) If $f_s = 300\ Hz$. Find $y(t)$

$$x[n] = \cos\left(\frac{30\pi n}{300}\right) + \cos\left(\frac{64\pi n}{300} + \frac{\pi}{3}\right) = \cos\left(\frac{\pi n}{10}\right) + \cos\left(\frac{16\pi n}{75} + \frac{\pi}{3}\right)$$

$$H(e^{j\hat{\omega}}) = e^{-j4\hat{\omega}}(16\cos(4\hat{\omega}) - 2) \rightarrow H\left(e^{j\left(\frac{\pi}{10}\right)}\right) = 2.94e^{-j0.4\pi};\ H\left(e^{j\left(\frac{16\pi}{75}\right)}\right) = 16.33e^{j0.1467\pi}$$

$$y(t) = 2.94\cos(30\pi t - 0.4\pi) + 16.33\cos(64\pi t + 0.48\pi)$$

(There is no aliasing)

$y(t) = 2.94\cos(30\pi t - 0.4\pi) + 16.33\cos(64\pi t + 0.48\pi)$

(c) Find $f_s > 30$ so that $y(t)$ has the form $y(t) = A + B\cos(20\pi t + \varphi)$. Furthermore, specify the constants A, B, and φ (where in standard form $B > 0$ and $|\varphi| < \pi$).

$$y(t) = A + B\cos(20\pi t + \varphi)$$

The highest frequency is 15 Hz so $f_s > 30$ and the 32 Hz signal from x(t) is now missing so $f_s < 64$. Additional information is the presence of a DC term (A). This implies that the 32Hz signal is DC. Given the restrictions on f_s, the sample rate that can work is $f_s = 32$.

$$x[n] = \cos\left(\frac{30\pi n}{32}\right) + \cos\left(\frac{64\pi n}{32} + \frac{\pi}{3}\right) = \cos\left(\frac{15\pi n}{16}\right) + \cos\left(2\pi n + \frac{\pi}{3}\right) = \cos\left(\frac{15\pi n}{16}\right) + \cos\left(\frac{\pi}{3}\right)$$

$$H(e^{j\hat{\omega}}) = e^{-j4\hat{\omega}}(16\cos(4\hat{\omega}) - 2) \rightarrow H\left(e^{j\left(\frac{15\pi}{16}\right)}\right) = 9.31e^{j0.25\pi};\ H(e^{j(0)}) = 14$$

$$y(t) = 8 * \cos\left(\frac{\pi}{3}\right) + 6.1\cos(20\pi t + 0.1818\pi) = 7 + 9.31\cos(20\pi t + 0.25\pi)$$

$A = 7$	$B = 9.31$	$\varphi = 0.25\pi$	$f_s =$ 32Hz

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PROBLEM 3: Parts a, b, and c (5 points each) can be solved independently of each other.

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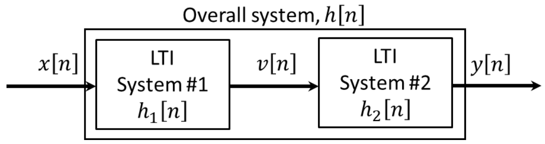

LTI System #2: $H_2(z) = \dfrac{\left(1-0.7\cos\left(\frac{2\pi}{3}\right)z^{-1}\right)}{1-1.4\cos\left(\frac{2\pi}{3}\right)z^{-1}+0.49z^{-2}}$

(a) Is $H_2(z)$ a stable LTI system?

Stable? YES or NO (Circle one and explain below. Explanation required for any credit)

All poles are inside the unit circle

(b) Find $h_2[n]$

From Table and properties: $h_2[n] = (0.7)^n \cos\left(\frac{2\pi}{3}n\right) u[n]$

$h_2[n] = (0.7)^n \cos\left(\frac{2\pi}{3}n\right) u[n]$

(c) Suppose that $y[n] = 6\delta[n-1]$ when $x[n] = \delta[n]$. Find the difference equation for LTI System #1 in terms of the output $v[n]$ and the input $x[n]$ under this condition.

IF: $H_2(z) = \dfrac{\left(1-0.7\cos\left(\frac{2\pi}{3}\right)z^{-1}\right)}{1-1.4\cos\left(\frac{2\pi}{3}\right)z^{-1}+0.49z^{-2}}$ and $Y(z) = 6z^{-1} = X(z)H_1(z)H_2(z)$ for $X(z) = 1$

Then: $H_1(z) = \dfrac{6z^{-1}}{H_2(z)} = \dfrac{6z^{-1}}{\dfrac{\left(1-0.7\cos\left(\frac{2\pi}{3}\right)z^{-1}\right)}{1-1.4\cos\left(\frac{2\pi}{3}\right)z^{-1}+0.49z^{-2}}} = \dfrac{6z^{-1}\left(1+0.7\cos\left(\frac{2\pi}{3}\right)z^{-1}+0.49z^{-2}\right)}{\left(1-0.7\cos\left(\frac{2\pi}{3}\right)z^{-1}\right)} = \dfrac{6z^{-1}+4.2z^{-2}+2.94z^{-3}}{1+0.35z^{-1}}$

Then the difference equation is:

$$v[n] = -0.35v[n-1] + 6x[n-1] + 4.2x[n-2] + 2.94x[n-3]$$

$v[n] = -0.35v[n-1] + 6x[n-1] + 4.2x[n-2] + 2.94x[n-3]$

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PROBLEM 4: Parts a, b, and c (10 points each) can be solved independently of each other.

(a) If $H(z) = \frac{z^{-3}(3+5z^{-2})}{1-0.6z^{-1}}$, find $h[6]$.

From Table : $H(z) = \frac{z^{-3}(3+5z^{-2})}{1-0.6z^{-1}} = \frac{3}{1-0.6z^{-1}}z^{-3} + \frac{5}{1-0.6z^{-1}}z^{-5}$

$$\rightarrow h[n] = 3(0.6)^{n-3}u[n-3] + 5(0.6)^{n-5}u[n-5]$$

$$\rightarrow h[6] = 3(0.6)^3 + 5(0.6) = 3.648$$

$h[6] = 3.648$

(b) Find $y[n] = w[n] * h[n]$ where (**NOTE: '*' is convolution**):

- $w[n] = \left[\cos\left(\frac{\pi}{2}n\right) + 2\cos\left(\frac{3\pi}{4}n\right)\right] * (\delta[n] + 2\delta[n-4] + \delta[n-8])$
- $h[n] = \left(5\delta[n] - 2\frac{\sin(0.6\pi n)}{\pi n}\right)$

- $(\delta[n] + 2\delta[n-4] + \delta[n-8]) \rightarrow e^{-j4\hat{\omega}}(2 + 2\cos(4\hat{\omega})) = V(e^{-j\hat{\omega}})$

$$\rightarrow V\left(e^{\frac{j\pi}{2}}\right) = 4; V\left(e^{\frac{j3\pi}{4}}\right) = 0;$$

$$\rightarrow w[n] = 4\cos\left(\frac{4\pi}{5}n\right)$$

- $h[n] = \left(5\delta[n] - 2\frac{\sin(0.6\pi n)}{\pi n}\right) \rightarrow H(e^{j\hat{\omega}}) = \begin{cases} 3 & |\hat{\omega}| < 0.6\pi \\ 5 & 0.6\pi < |\hat{\omega}| < \pi \end{cases}$

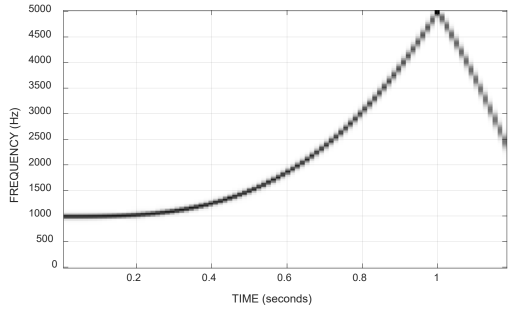
$$\rightarrow y[n] = 12\cos\left(\frac{\pi}{2}n\right)$$

$y[n] = 12\cos\left(\frac{\pi}{2}n\right)$

(c) Consider the MATLAB code below for the spectrogram plot shown to the right over the range of 0 to 1.2 seconds.

```
tt=0:1/fs:1.2;
xx = pi*cos(A*(pi*tt.^4+pi.*tt));
plotspec(xx,fs,256)
```

From the plot, determine numerical values for **fs** and **A**. The spectrogram shows a curve that takes a value of about 2500 Hz around time 1.2. If we look into the future, what value, K, will this curve take at time 2?



$$x(t) = \pi\cos(A\pi t^4 + A\pi t)$$

$$f_i(t) = \frac{1}{2\pi}(4A\pi t^3 + A\pi) = 2At^3 + \frac{A}{2}$$

$$f_i(0) = 1000 = \frac{A}{2} \rightarrow A = 2000$$

$$f_i(2) = 2(2000)2^3 + 1000 = 33000 - 30000 = 3000$$

fs $= 10000$

A $= 2000$

$K = 3000$

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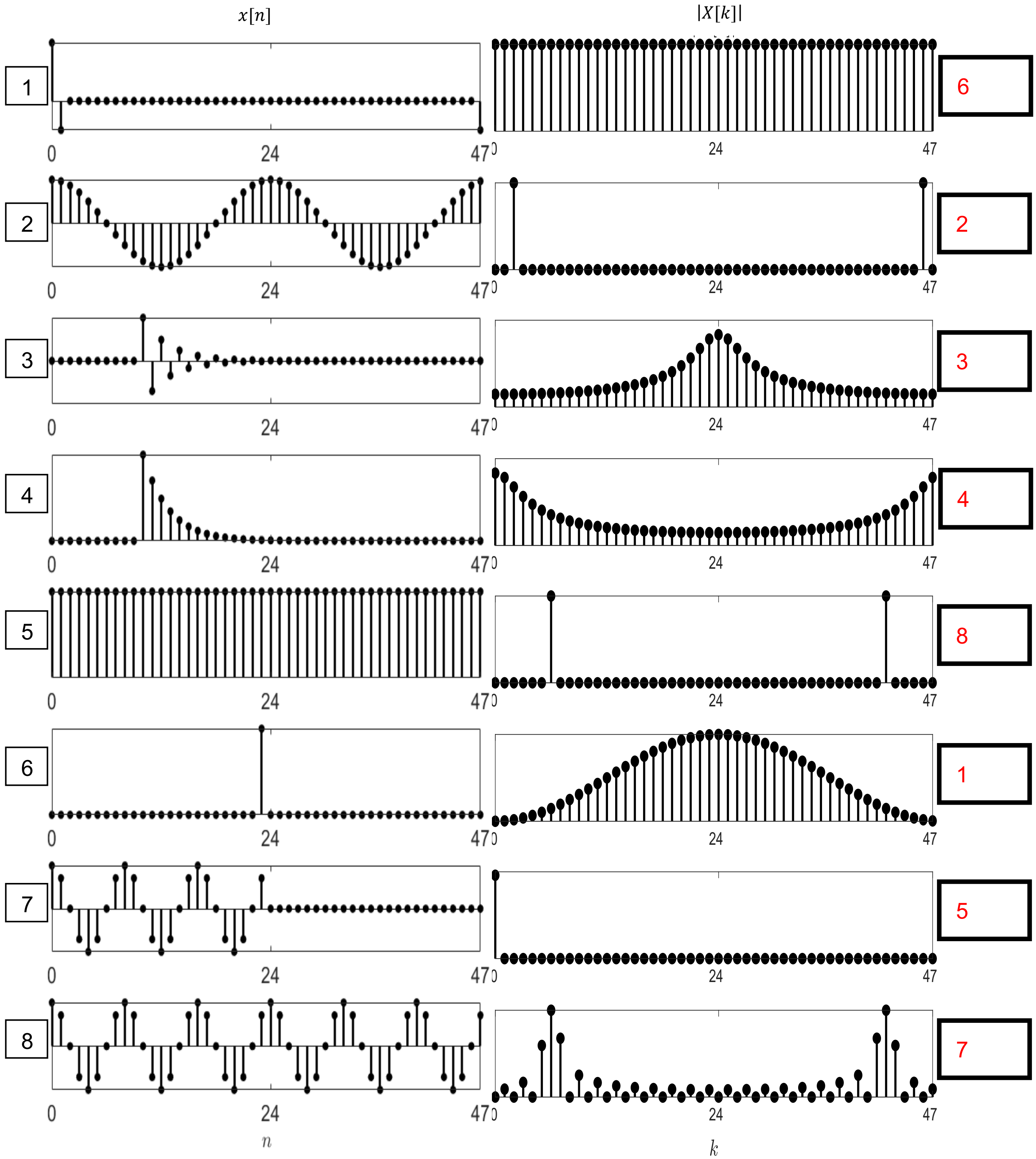
PROBLEM 5: 10 points

Match the plots (1)-(8) for $x[n]$ below with the plots for the corresponding 44-point DFT magnitudes $|X[k]|$ by placing the appropriate number in the boxes besides the plots for $|X[k]|$.



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PROBLEM 6: 15 points

Shown below on the left are twelve pole-zero plots for $H(z)$, labeled A through L. Shown below on the right are the corresponding magnitude responses, $|H(e^{j\hat{\omega}})|$ plotted vs $\hat{\omega}$, but in a scrambled order. Match each magnitude response to its corresponding pole-zero plot. Indicate answers by writing a letter (from A through L) in to each answer box.

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