GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
ECE2026 – Fall 2014
Final Exam

NAME: ______________________________ **GT Username:** ______________________
LAST **FIRST** **(e.g., gtinit1010a)**

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):
(Failing to circle the correct section will cost you 3 points)

	Mon	Tue	Wed	Thu
9:35-10:55				L06 (Bhatti)
12:05-13:25		L07 (Causey)		
13:35-14:55		L09 (Causey)		L10 (Bhatti)
15:05-16:25	L01 (Moore)	L11 (Davenport)	L02 (Juang)	L12 (Walkenhorst)
16:35-17:55	L03 (Aghasi)	L13 (Davenport)	L04 (Juang)	L14 (Walkenhorst)

Important Notes:

- Write your name on the front page ONLY. **DO NOT** UNSTAPLE the test.
- Closed book, but a calculator is permitted.
- Two sheets (8.5"x11") of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Partial credit for incorrect answers may be granted ONLY when you JUSTIFY your correct reasoning CLEARLY.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If more space is needed for scratch work, use the backs of previous pages.

Problem	Value	Score Earned	Problem	Value	Score Earned
1	25		5	25	
2	25		6	25	
3	25		7	25	
4	25		8	25	
No/Wrong Rec	-3		Total		

PROBLEM Fall-14-F.1:

(a) A continuous-time signal is given as $x(t) = \cos(5t + \pi/4) + 3\sin(5t + 3\pi/4) + 2\sin(5t + \pi/2)$. Find a local peak of the signal that is *closest to the time origin*. Determine this peak value, A_m, and the time it occurs, t_m .

$A_m = 5.596$	$t_m = -0.106$

$$x(t) = \text{Re}\{e^{j\pi/4}e^{j5t} + 3e^{j(3\pi/4-\pi/2)}e^{j5t} + 2e^{j(\pi/2-\pi/2)}e^{j5t}\}$$
$$= \text{Re}\{(e^{j\pi/4} + 3e^{j\pi/4} + 2)e^{j5t}\} = \text{Re}\{(4e^{j\pi/4} + 2)e^{j5t}\}$$

$$4e^{j\pi/4} + 2 = 5.596e^{j0.53}$$

$$x(t) = \text{Re}\{5.596e^{j0.53}e^{j5t}\} = 5.596\cos(5t + 0.53)$$

The signal attains peak values of 5.596 at $5t + 0.53 = 2k\pi,\ k = \cdots,-1,0,1,\cdots$ corresponding to $t =$ –1.3626, –0.1060 and 1.1506 near the time origin. Therefore, $t_m = -0.106$ is the answer.

(b) Another signal $y(t) = 2\cos(10t + \theta)$ is being added to the above signal $x(t)$. Find θ in $[0, 2\pi)$ such that the signal $x(t) + y(t)$ attains the maximum possible amplitude.

$\theta = 1.06\ (radian)$

At $t = -0.106$, $x(t)$ attains a maximum value of 5.596. If

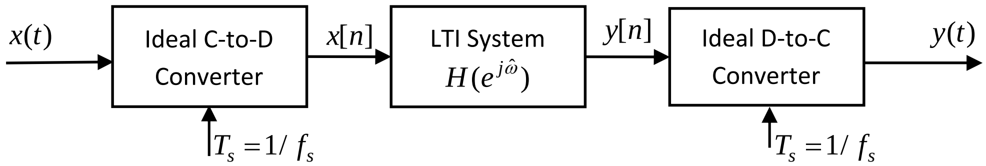
$$10(-0.106) + \theta = 2k\pi, k = \cdots,-1,0,1,\cdots$$

$y(t)$ will also attain a maximum value, which is 2. Therefore, pick k = 0 and

$$10(-0.106) + \theta = 0 \rightarrow \theta = 1.06 \quad \text{(radian)}$$

PROBLEM Fall-14-F.2:

Consider the following system for discrete-time filtering of a continuous-time signal:



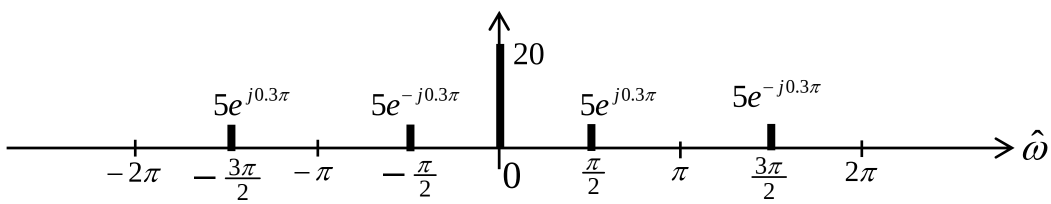
Assume that the discrete-time system is defined by the difference equation:

$$y[n] = 0.8y[n-1] + x[n] - x[n-1]$$

(a) The continuous-time input to the system is given as

$$x(t) = 12 + 10\cos(800\pi t + 0.3\pi) + 16\cos(3200\pi t + \pi/3) \quad \text{for } -\infty < t \le \infty$$

For a sampling rate of $f_s = 1600$ samples/s, draw the spectrum of $x[n]$, the discrete-time signal after the C-to-D converter, within the range $-2\pi < \hat{\omega} < 2\pi$. Make sure all values are properly marked.



$$10\cos(800\pi t + 0.3\pi) \rightarrow 10\cos\left(\tfrac{800\pi n}{1600} + 0.3\pi\right) = 10\cos\left(\tfrac{1}{2}\pi n + 0.3\pi\right)$$

$$16\cos(3200\pi t + \pi/3) \rightarrow 16\cos\left(\tfrac{3200\pi n}{1600} + \tfrac{\pi}{3}\right) = 16\cos\left(2\pi n + \tfrac{\pi}{3}\right) = 8$$

$$x[n] = 12 + 10\cos\left(\tfrac{\pi}{2}n + 0.3\pi\right) + 8 = 20 + 10\cos\left(\tfrac{\pi}{2}n + 0.3\pi\right)$$

$$= 20 + 5e^{j0.3\pi}e^{j\frac{\pi}{2}n} + 5e^{-j0.3\pi}e^{-j\frac{\pi}{2}n}$$

(b) For the same $x(t)$ as in the previous part, and the same sampling rate of 1600 samples/s at both C-to-D and D-to-C, obtain a formula for the output $y(t)$ for $-\infty < t \le \infty$.

$$y[n] = 0.8y[n-1] + x[n] - x[n-1] \quad \Rightarrow \quad H(z) = \frac{1 - z^{-1}}{1 - 0.8z^{-1}}$$

$$\text{For } \hat{\omega} = 0,\ H(z)\Big|_{z=1} = \frac{1-1}{1-0.8} = 0 \qquad \text{For } \hat{\omega} = \frac{\pi}{2},\ H(z)\Big|_{z=e^{j\frac{\pi}{2}}} = \frac{1 - e^{-j0.5\pi}}{1 - 0.8e^{-j0.5\pi}} = 1.1043e^{j0.1107}$$

$$y[n] = 20 \bullet 0 + 10 \bullet 1.1043\cos\left(\tfrac{\pi}{2}n + 0.3\pi + 0.1107\right) = 11.043\cos\left(\tfrac{\pi}{2}n + 1.053\right)$$

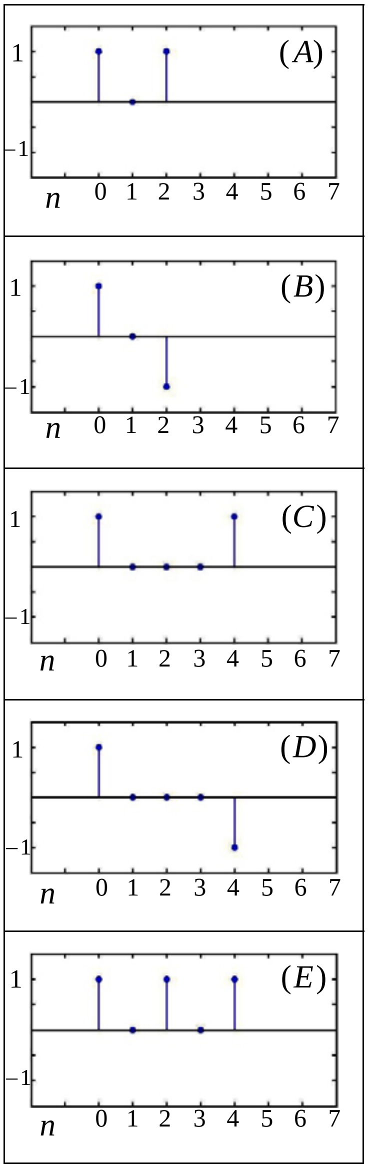
$$n \leftarrow f_s t = 1600t \qquad y(t) = 11.043\cos\left(\tfrac{\pi}{2}1600t + 1.053\right) = 11.043\cos(800\pi t + 1.053)$$

$$\boxed{y(t) = 11.043\cos(800\pi t + 1.053)}$$

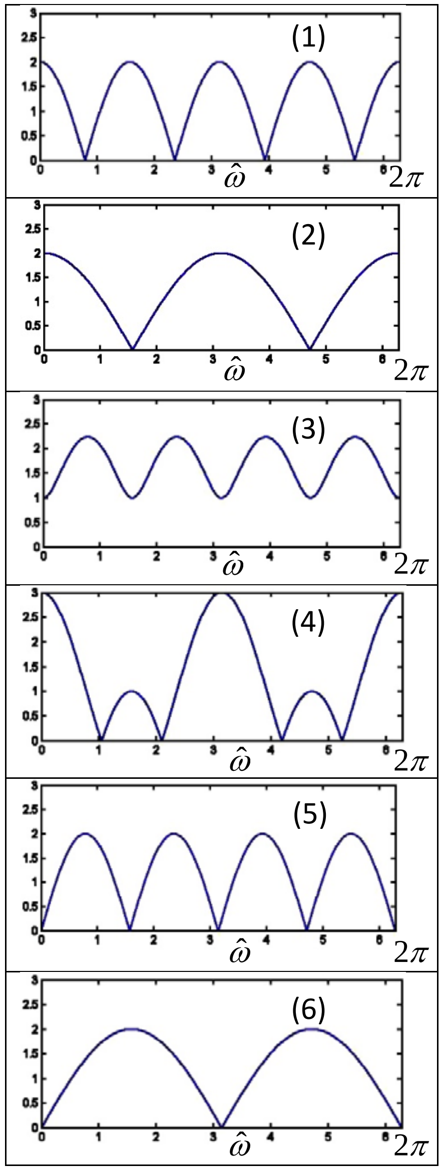
PROBLEM Fall-14-F.3:

Two sets of plots are provided below. On the left-hand side is a collection of impulse responses $h[n]$ (alphabetized A through E) and on the right-hand side are the corresponding magnitude responses $|H(e^{j\hat{\omega}})|$, plus an extra, plotted in linear scale for $0 \le \hat{\omega} \le 2\pi$ (plots numbered 1 through 6, randomized in order). (Magnitude response is the magnitude of the frequency response.) Fill in the blank below to match the impulse response and its frequency response.

$h[n]$



$|H(e^{j\hat{\omega}})|$



Work space:

(A) ↔ 2	(B) ↔ 6	(C) ↔ 1
(D) ↔ 5	(E) ↔ 4	

PROBLEM Fall-14-F.4:

Determine whether each of the following statements is true or false. Half of the statements are true; you must assign three *true*s and three *false*s to the six statements; any unbalance of answers will result in heavy penalty. For example, all true or all false will not earn you any score.

(a) A periodic continuous-time signal may not result in a periodic discrete-time sequence after sampling.
(b) The frequency of a sinusoid cannot be changed by a linear time-invariant system but the amplitude and phase can.
(c) An infinite impulse response (IIR) system has an impulse response that is infinitely long and therefore cannot be practically implemented with processors using finite memory; approximation by truncation of the impulse response is always implied in practice.
(d) The discrete-time Fourier transform (DTFT) of a finite sequence, $\{x[n], n = 0,1,\cdots N\}$, is equal to the discrete Fourier transform (DFT) of infinite length computed on a periodic sequence in which the original finite sequence $\{x[n], n = 0,1,\cdots N\}$ is repeated and concatenated indefinitely.
(e) An M^{th} order FIR system will always have M poles at the origin.
(f) The Fourier series of a real periodic signal consists of sum of sinusoids, and to obtain the Fourier coefficients one performs the Fourier integral over an arbitrary duration of time as long as it is normalized by the chosen duration.

	True/False
(a)	**T**
(b)	**T**
(c)	**F**
(d)	**F**
(e)	**T**
(f)	**F**

Problem FALL-14-F.5:

Determine and pick the period of the following discrete-time sequences, numbered 1 through 5, from the numbers (in samples) in the table, labeled A through F. Use 0 samples if the sequence is not periodic. Note: do not assume that all these sequences have different periods.

(1) $x[n] = \cos(45n + 0.1) + \cos(81n)$
(2) $x[n] = \cos(0.14\pi n + 0.1) + \cos(0.42\pi n)$
(3) $x(t) = \cos(45\pi t + 0.1) + \cos(81\pi t) + 1$ sampled at $f_s = 90$ samples/s
(4) $x[n] = \cos(3.8\pi n + 0.1) + \cos(0.4\pi n)$
(5) $x(t) = \cos(45\pi t + 0.1) + \cos(81\pi t)$ sampled at $f_s = 30$ samples/s

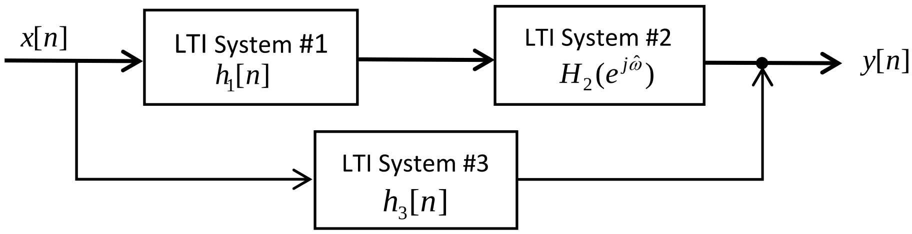
The following are expressed as number of samples.

(A) 10	(B) 50	(C) 0	(D) 100	(E) 25	(F) 20

Fill in your answers in the corresponding letter that has the right period number:

(1) $\leftrightarrow$ C	(2) $\leftrightarrow$ D	(3) $\leftrightarrow$ F	(4) $\leftrightarrow$ A	(5) $\leftrightarrow$ F

PROBLEM Fall-14-F.6:



A system as depicted in the above figure consists of three sub-systems, which are given the following specifications:

$$h_1[n] = 2\delta[n-1] - 2\delta[n-2], \quad H_2(e^{j\hat{\omega}}) = e^{-j2.5\hat{\omega}} \frac{\sin(2\hat{\omega})}{\sin(0.5\hat{\omega})},$$

$$h_3[n] = -2\delta[n-2] + 2\delta[n-4]$$

(a) Find the impulse response $h[n]$ of the overall system.

$$H_2(e^{j\hat{\omega}}) = e^{-j2.5\hat{\omega}} \frac{\sin(2\hat{\omega})}{\sin(0.5\hat{\omega})} \to H_2(z) = z^{-1}\frac{1-z^{-4}}{1-z^{-1}}$$

$$h_1[n] = 2\delta[n-1] - 2\delta[n-2] \to H_1(z) = 2z^{-1}(1-z^{-1})$$

$$h_3[n] = -2\delta[n-2] + 2\delta[n-4] \to H_3(z) = -2z^{-2} + 2z^{-4}$$

$$H_1(z)H_2(z) + H_3(z) = 2(1-z^{-1})z^{-2}\frac{1-z^{-4}}{1-z^{-1}} - 2z^{-2} + 2z^{-4} = 2z^{-2} - 2z^{-6} - 2z^{-2} + 2z^{-4} = 2z^{-4} - 2z^{-6}$$

$$\boxed{h[n] = 2\delta[n-4] - 2\delta[n-6]}$$

(b) Determine the output $y[n]$ when the input is given as $x[n] = 3 + 2\cos(\frac{\pi}{2}n) + 2\cos(\pi n - \frac{\pi}{2})$.

$$h[n] = 2\delta[n-4] - 2\delta[n-6] \to H(e^{j\hat{\omega}}) = 2e^{-j4\hat{\omega}} - 2e^{-j6\hat{\omega}}$$

The input signal has three frequencies, $\hat{\omega} = 0, \frac{\pi}{2}$, and π

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = 2 - 2 = 0 \qquad H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\frac{\pi}{2}} = 2e^{-j4\frac{\pi}{2}} - 2e^{-j6\frac{\pi}{2}} = 2 + 2 = 4$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\pi} = 2e^{-j4\pi} - 2e^{-j6\pi} = 2 - 2 = 0$$

$$y[n] = 3 \bullet 0 + (2 \bullet 4)\cos(\tfrac{\pi}{2}n) + (2 \bullet 0)\cos(\pi n - \tfrac{\pi}{2}) = 8\cos(\tfrac{\pi}{2}n)$$

$$\boxed{y[n] = 8\cos(\tfrac{\pi}{2}n)}$$

PROBLEM Fall-14-F.7:

A real signal $x(t)$ is periodic with period T_0 = 4s and represented as a Fourier series of the form

$$x(t) = \sum_{k=-2}^{2} a_k e^{j(2\pi/4)kt}$$

where the Fourier coefficients have the value: $a_k = |k|\, e^{jk\pi/3},\ k \neq 0;\ a_0 = 5$.

a) The signal is being sampled by a C-to-D converter at an interval of 2s, resulting in the sequence $x[n]$. Obtain a formula for $x[n]$ in real functional form (i.e., no complex exponentials).

$$\begin{aligned} x(t) &= 2e^{-j(2\pi/3)}e^{-j\pi t} + e^{-j\pi/3}e^{-j(\pi/2)t} + 5 + e^{j\pi/3}e^{j(\pi/2)t} + 2e^{j(2\pi/3)}e^{j\pi t} \\ &= 5 + 2\cos\left(\tfrac{\pi t}{2} + \tfrac{\pi}{3}\right) + 4\cos\left(\pi t + \tfrac{2\pi}{3}\right) \end{aligned}$$

$$T_s = 2; \quad x[n] = x(nT_s) = x(t)\Big|_{t=2n}$$

$$\begin{aligned} x[n] &= x(2n) \\ &= 5 + 2\cos(\tfrac{\pi 2n}{2} + \tfrac{\pi}{3}) + 4\cos(\pi\, 2n + \tfrac{2\pi}{3}) \\ &= 5 + 2\cos(n\pi + \tfrac{\pi}{3}) + 4\cos(\tfrac{2\pi}{3}) = 5 + 2\cos(n\pi + \tfrac{\pi}{3}) - 2 \\ &= 3 + 2\cos(n\pi + \tfrac{\pi}{3}) \end{aligned}$$

$$\boxed{x[n] = 3 + 2\cos(n\pi + \tfrac{\pi}{3})}$$

b) The above sampled signal $x[n]$ is being filtered by a discrete time LTI system, defined by the difference equation: $y[n] = 0.6y[n-1] + x[n-1] + b_2 x[n-2] + 0.5x[n-3]$. Find b_2 such that the output $y[n]$ is a non-zero constant for all n.

The input $x[n]$ has two sinusoids with frequencies at $\hat{\omega} = 0$ and $\hat{\omega} = \pi$, respectively.

$$y[n] = 0.6y[n-1] + x[n-1] + b_2 x[n-2] + 0.5x[n-3]$$

$$H(z) = \frac{z^{-1}(1 + b_2 z^{-1} + 0.5z^{-2})}{1 - 0.6z^{-1}} = \frac{z^2 + b_2 z + 0.5}{z^2(z - 0.6)}$$

To block the sinusoid with $\hat{\omega} = \pi$, one needs a zero at $z = -1$.

$$z^2 + b_2 z + 0.5 = (z+1)(z+0.5) = z^2 + 1.5z + 0.5 \rightarrow b_2 = 1.5$$

$$\boxed{b_2 = 1.5}$$

PROBLEM Fall-14-F.8:

Two periodic sequences are given

n	…	0	1	2	3	4	5	6	7	8	9	10	11	12	…
$x_1[n]$	…	1	$\frac{1}{2}$	$-\frac{1}{2}$	–1	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	–1	$-\frac{1}{2}$	$\frac{1}{2}$	1	…
$x_2[n]$	…	0	2	0	–2	0	2	0	–2	0	2	0	–2	0	…

and added together to form a new sequence $x[n] = x_1[n] + x_2[n]$. A 12-point DFT is taken on the sequence $\{x[n],\ n = 0,1,\cdots,11\}$. Obtain these DFT coefficients and write your results in the table below.

k	0	1	2	3	4	5
$X[k]$	0	0	6	$12e^{-j\frac{\pi}{2}}$	0	0
k	6	7	8	9	10	11
$X[k]$	0	0	0	$12e^{j\frac{\pi}{2}}$	6	0

$x_1[n] = \cos(\frac{\pi}{3}n)$

$x_2[n] = 2\cos(\frac{\pi}{2}(n-1))$

$x[n] = \cos(\frac{\pi}{3}n) + 2\cos(\frac{\pi}{2}n - \frac{\pi}{2})$

$\frac{\pi}{2}$ and $\frac{\pi}{3}$ share a common factor of $\frac{\pi}{6}$ and the sequence $x[n]$ has a period of 12 samples

Thus, a 12-point DFT can be easily obtained from the DFS of the sequence.

$$x[n] = \cos(\tfrac{\pi}{3}n) + 2\cos(\tfrac{\pi}{2}n - \tfrac{\pi}{2}) = \frac{1}{2}\left[e^{-j\frac{\pi}{3}n} + e^{j\frac{\pi}{3}n}\right] + \left[e^{j\frac{\pi}{2}}e^{-j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{2}n}\right]$$

$$= \frac{1}{2}\left[e^{-j\frac{2\pi}{12}2\bullet n} + e^{j\frac{2\pi}{12}2\bullet n}\right] + \left[e^{j\frac{\pi}{2}}e^{-j\frac{2\pi}{12}3\bullet n} + e^{-j\frac{\pi}{2}}e^{j\frac{2\pi}{12}3\bullet n}\right]$$

$$= \frac{1}{12}\left\{6\left[e^{j\frac{2\pi}{12}2\bullet n} + e^{j\frac{2\pi}{12}10\bullet n}\right] + 12\left[e^{-j\frac{\pi}{2}}e^{j\frac{2\pi}{12}3\bullet n} + e^{j\frac{\pi}{2}}e^{j\frac{2\pi}{12}9\bullet n}\right]\right\}$$